TEXT-BOOKS OF SCIENCE.

TEXT-BOOKS OF SCIENCE

ADAPTED FOR THE USE OF

ARTISANS AND STUDENTS IN PUBLIC AND SCIENCE SCHOOLS

MACHINE DESIGN

PART II

THE ELEMENTS

OF

MACHINE DESIGN

Part II

CHIEFLY ON ENGINE DETAILS

BY

W. CAWTHORNE UNWIN, F.R.S.

B.Sc., Mem. Inst. Civil Engineers, Hon. Mem. Inst. Mechanical Engineers
Hon. Mem. Am. Soc. Mechanical Engineers
Foreign Hon. Mem. Am. Academy of Arts and Sciences

PROFESSOR OF ENGINEERING AT THE CENTRAL TECHNICAL COLLEGE;
FORMERLY PROFESSOR OF HYDRAULIC AND MECHANICAL ENGINEERING AT THE
ROYAL INDIAN ENGINEERING COLLEGE

NEW EDITION

REVISED AND ENLARGED

LONGMANS, GREEN, AND CO.
39 PATERNOSTER ROW, LONDON
NEW YORK AND BOMBAY
1902

PREFACE TO PART II

The present volume deals chiefly, though not exclusively, with steam-engine details. The subjects discussed form, it is believed, a definitely related group, embracing most of the mechanical problems which occur in machinery for generating power; problems which are, on the whole, more complex than those examined in the First Part of this treatise.

The general data of the strength of materials and the laws relating to the dependence of dimensions on straining actions are given in Part I. But in the cases to which these laws are there applied the straining actions are generally steady or statical, or may for practical purposes be treated as being so. In the design of Transmissive Machinery, in addition to questions of strength, kinematical problems are those most commonly presented. In Part II., which relates to machines the object of which is to supply mechanical energy for doing work, the parts of which move at varying speeds and are subjected to varying forces of inertia which are too large to be neglected, dynamical principles find a more frequent application.

A treatise such as this has a double function. It is partly a collection of designs, data, and rules, of use in the workshop and engineering drawing-office. But it differs

from mere collections of precedents or pocket-books in this, that the reasons for different arrangements or proportions are, as far as possible, discussed in connection with the examples given. In the less complicated cases fairly definite rules can be given, based on scientific principles, as a guide to the practical engineer. In the more complicated cases, where conditions are too indefinite for any precise theory, at least an indication can be given of the extent to which approximate solutions are to be distrusted. It is not the object of machine science to reduce every problem of construction to a rule of thumb. Rather it furnishes limits within which a free judgment, based on experience, has to be used, and alternatives amongst which choice is to be made.

It is another function of such a book as this to serve as a text-book for engineering students, furnishing a series of practical problems. By working through such problems a student becomes acquainted with the various requirements involved in machine construction, and the considerations which guide an engineer in designing machines.

It is with a view to students that some problems are treated here in a fairly complete and systematic way, although necessarily with less completeness than in special treatises on particular parts of engineering. Such special treatises are generally too detailed to be convenient for students' use, and to understand them a larger knowledge of practical details is required than a student has time to master. It was specially from an experience of the want of some account of valve gears better suited to students than the large and admirable monographs already published, that the Author was led to extend the chapters on that subject.

During the last quarter of a century there has been

a remarkable development of engineering schools, and, if present indications can be trusted, there will be in the near future a still larger increase in the number of students in engineering classes of various types. There is now a Faculty of Engineering in the University of London, and examinations are held by the Institution of Civil Engineers. It is at last pretty generally recognised, even in this country—which has been the slowest to make provision for technical education—that some training in a technical school is necessary, as the first step in the professional education of engineers. In any scheme of instruction for engineers, Machine Designing must have a place.

It is in the nature of the case that, in school instruction and in text-books for students, theoretical considerations occupy relatively a larger space than in the ordinary routine of the drawing-office. The experienced designer of machinery has no need to recur constantly to first principles, although no doubt he has systematised his experience by the aid of all the science he possesses. The inexperienced student, much as he will be assisted by theory in dealing with practical problems, will no doubt be liable to misapply it, for a knowledge of the limits within which theory can be trusted, or more accurately a knowledge of the extent to which a theory covers the data in any given case, is only arrived at gradually. Meanwhile, he is likely to meet rebuffs which seem to imply that theory is held in very low esteem by practical engineers. To such a student may be commended the saying of Carlyle: 'There was once a man called Jean Jacques Rousseau. He wrote a book called the "Social Contract." It was a theory and nothing but a theory. The French nobles laughed at the theory, and their skins went to bind the second edition of the book.'

The present edition has been largely rewritten throughout, partly in view of more recent researches, but chiefly with the object of better adapting it as a reference-book for draughtsmen, and a text-book for students.

It may be useful to repeat here that, where there is no express statement to the contrary, the units adopted are as follows :

Dimensions in inches.

Loads or forces in lbs.

Stresses in lbs. per sq. in.

Velocities in feet per sec.

Accelerations in feet per sec. per sec.

Work in foot lbs.

Speeds in revolutions per minute.

Angular velocities in radians per sec.

Statical moments in inch lbs.

The subject of Machine Designing is so extensive that a very much larger treatise than this would be required for its full explanation. All that the Author could hope to do was to make a reasonably careful selection of what to include and what to reject. Shortcomings there must be. It can only be hoped that, taking into account the limitations imposed, they are not excessively numerous or serious.

July 1902.

CONTENTS

INTRODUCTION

CHAPTER I

CHAPTER II

CHAPTER IX

CHAPTER X

CHAPTER XI

CHAPTER XII

CHAPTER XVII

ELEMENTS

OF

MACHINE DESIGN

PART II

INTRODUCTION

IN the first two chapters of Part I. of this work, the qualities and properties of the ordinary materials of construction are discussed, and also the general nature of the straining actions to which machines are subjected and the principles on which working limits of stress are determined. The third chapter contains a statement of most of the formulæ connecting the stress and dimensions of a structure when subjected to tension, bending, torsion, and other straining actions. It is not necessary to repeat what has been there said, but the following tables of the properties of materials are so often wanted for reference that it is desirable to reprint them.

TABLE I.—*Ultimate and Elastic Strengths of*

Material	Breaking Strength		
	Tension	Pressure	Shearing
Cast iron	30,500	130,000	12,000
	17,500	95,000	10,500
	10,800	50,000	8,700
Wrought-iron bars	67,000	—	49,000[2]
	57,600	50,000	40,000
	33,500	—	22,400
Iron ship plates ‖	49,000	—	—
Iron boiler plates ‖	47,000	—	36,000
,, ,, ⊥	41,500	—	—
Steel plates, ¼% carbon	65,000	—	50,000
,, ,, ½% ,,	78,000	—	56,000
,, ,, 1% ,,	110,000	—	83,000
Steel boiler plates	66,000	—	56,000
Rivet steel	65,000	—	55,600
Cast steel untempered	150,000	—	—
	120,000	—	—
	84,000	—	—
Cast steel tempered	—	—	—
Steel castings from	63,000	—	—
to	34,000	—	—
Copper cast from	19,000	—	—
to	23,000	58,000	—
,, rolled plates	31,000	—	—
,, annealed wire	45,000	—	—
,, hard drawn wire	58,000	—	—
Brass from	17,500	—	—
to	29,000	—	—
Gunmetal or bronze	52,000	—	—
	27,000	—	—
	23,000	—	—
Delta metal, cast	36,000	—	—
,, ,, rolled	74,000	—	—
Phosphor bronze	58,000	—	43,000
Muntz metal	49,000	—	—
Cast zinc	7,500	—	—
Lead	2,500	7,000	—
Tin	4,700	—	—
Wood, pine from	6,700	3,400	—
to	13,000	7,000	630[3]
,, oak	15,000	10,000	2,240[3]
Leather	4,200	—	—

[1] Cast iron has properly no elastic limit.
[3] These are along the fibres of the wood.

Materials and Co-efficients of Elasticity, in lbs. per sq. in.

Elastic Strength			Co-efficient of Elasticity	
Tension	Pressure	Shearing	Direct E	Transverse C
—	—	—	23,000,000	7,600,000
10,500[1]	21,000[1]	8,000[1]	17,000,000	6,300,000
—	—	—	14,000,000	5,000,000
—	—	—	31,000,000	—
30,000	30,000	22,000	29,000,000	10,500,000
—	—	—	27,000,000	—
—	—	—	—	—
24,000	24,000	15,000	26,000,000	14,000,000
—	—	—	27,000,000	—
42,000	38,000	—		—
47,000	49,000	—	31,000,000	13,000,000
67,000	71,000	—		—
36,000	—	—	30,000,000	13,500,000
46,000	—	—	30,000,000	13,000,000
—	—	—	—	—
80,000	80,000	64,000	30,000,000	12,000,000
—	—	—	—	—
190,000	190,000	145,000	36,000,000	14,000,000
34,000	—	—	30,000,000	—
20,000	—	—	20,000,000	—
—	—	—	12,000,000	—
—	—	—	—	—
5,600	4,000	3,000	15,000,000	5,600,000
—	—	—	16,000,000	—
—	—	—	17,000,000	—
—	—	—	13,500,000	—
—	—	—	—	—
—	—	—	—	—
6,200	—	4,150	13,500,000	—
—	—	—	—	—
17,000	—	—	12,000,000	—
51,000	—	—	13,000,000	—
19,700	—	14,500	14,000,000	5,250,000
—	—	—	—	—
3,200	—	—	—	—
1,500	—	—	2,500,000	—
—	—	—	—	—
—	—	—	1,000,000	—
—	—	—	1,600,000	—
—	—	—	1,450,000	—
—	—	—	25,000	—

[1] The shearing resistance of wrought iron is much less along the fibre than across it.

Table II.—*Ordinary Working Stress*

Case A. *The straining action a permanent one.*

Material	Kind of Stress				
	Tension	Compression	Bending	Shear	Torsion
Cast iron . .	4,200	12,000	6,000 to 8,000	2,300	4,000 to 6,000
Wrought iron :—					
,, Bar or forged	15,000	15,000	15,000	12,000	7,500
,, Plate $\parallel$.	15,000	—	—	—	—
,, ,, $\perp$.	12,000	—	—	10,000	—
Mild steel . .	13,000 to 17,000	13,000 to 17,000	13,000 to 17,000	10,000 to 13,000	8,000 to 12,000
Cast steel . .	17,000 to 21,000	17,000 to 21,000	17,000 to 21,000	13,000 to 17,000	12,000 to 16,000
Steel castings .	8,000 to 12,000	12,000 to 16,000	10,000 to 14,000	7,000 to 12,000	7,000 to 12,000
Phosphor bronze .	10,000	—	—	7,000	4,200
Gunmetal . .	4,200	—	—	—	—
Rolled copper .	4,000	—	—	2,400	—
Brass . . .	3,000	—	—	—	—

Case B. *Straining action producing stress of one kind only, varying from zero to a greatest value frequently.*

Material	Tension	Compression	Bending	Shear	Torsion
Cast iron . .	2,800	8,000	4,000 to 5,300	1,500	2,600 to 4,000
Bar iron . .	10,000	10,000	10,000	8,000	5,000
Plate iron $\parallel$.	10,000	—	—	—	—
,, ,, $\perp$.	8,000	—	—	6,500	—
Mild steel . .	8,600 to 11,400	8,600 to 11,400	8,600 to 11,400	6,500 to 8,600	5,300 to 8,000
Cast steel . .	11,400 to 14,000	11,400 to 14,000	11,400 to 14,000	8,600 to 11,400	8,000 to 10,600

CASE B. *Straining action producing stress of one kind only, varying from zero to a greatest value frequently.*

Material	Tension	Compression	Bending	Shear	Torsion
Steel castings	5,300 to 8,000	8,000 to 10,600	6,600 to 9,400	4,700 to 8,000	4,700 to 8,000
Phosphor bronze	6,600	—	—	4,600	2,800
Gunmetal	2,800	—	—	—	—
Rolled copper	2,600	—	—	1,600	—
Brass	2,000	—	—	—	—

CASE C. *Straining action producing equal stresses of opposite sign alternately.*

Material	Tension and Compression	Bending	Shear	Torsion
Cast iron	1,400	2,000 to 2,700	770	1,300 to 2,000
Bar iron	5,000	5,000	4,000	2,500
Mild steel	4,300 to 5,700	4,300 to 5,700	3,300 to 4,300	2,700 to 4,000
Cast steel	5,700 to 7,000	5,700 to 7,000	4,300 to 5,700	4,000 to 5,300
Steel castings	2,700 to 4,000	3,300 to 4,700	2,300 to 4,000	2,300 to 4,000
Gunmetal	1,400	—	—	—

TABLE IIA.—*Working Stresses in Building Construction*

	Tension lbs. per sq. in.	Compression lbs. per sq. in.		Tension lbs. per sq. in.	Compression lbs. per sq. in.
Wrought iron	14,000	14,000	Brickwork, cement mortar	—	72 to 108
Cast iron	3,360	10,080	Rubble masonry, lime mortar	—	58
Oak	1,400	940	,, ,, cement mortar	—	72
Pine	1,120	800	Portland cement concrete	—	100
Brickwork, lime mortar	—	36 to 72	Pressed bricks in cement mortar	—	114 to 128

Table III.—*Heaviness of Materials*

	lbs. per c. foot	lbs. per c. inch
Gaseous Bodies.		
Air at 32° and one atm.	·0807	—
Steam at 212° and one atm.	·0378	—
Liquids.		
Pure water at 39°·1 F.	62·425	0·0361
,, ,, 60° F.	62·373	—
River water, mean	63·0	—
Sea water, mean	64·05	0·0371
Mercury, at 32	849	0·491
Timber.		
Oak or teak	45 to 55	·026 to ·032
Pine or fir	30 to 44	·017 to ·025
Greenheart	72	·042
Brick and Stone.		
Ordinary brick	112	0·065
Brickwork (mean)	106	0·062
Rubble masonry (mean)	140	0·081
Concrete (mean)	150	0·087
Ashlar, sandstone	150	0·087
,, limestone	165	0·096
Granite	175	0·102
Earths.		
Chalk	150	0·087
Moist clay	163	0·094
Mud or moist sand	102	0·059
Compact moist soil	140	0·081
Metals.		
Wrought iron	490	0·283
Cast iron	468	0·271
Sheet lead	711	0·411
Copper	555	0·321
Zinc	449	0·260
Cast steel	496	0·287
Soft steel	480	0·278
Gunmetal	546	0·316
Cast brass	518	0·299
Rolled brass	526	0·305

CHAPTER I

ON PIPES AND CYLINDERS

PIPES and cylinders subjected to internal pressure form parts of many machines, and are extensively used in the conveyance of water, gas, steam, or oil. The proportions of these, and the modes of joining them, form the subject of the present chapter.

CAST-IRON PIPES

1. Cast-iron pipes appear to have been first made about 1780 at Coalbrookdale. They are made to be joined by flanges or by a socket, the latter being more easily cast and less costly. Ordinarily they are made in lengths of 9 feet up to 12 ins. (internal) diameter, and 12 feet long when larger. They should be cast vertically to insure soundness. When used in large quantities they are carefully inspected and tested, and are expected to be uniform in thickness and of normal weight. A variation of thickness in different parts of more than $\frac{1}{8}$ in., or a variation of more than 2 to 5 per cent. from the normal weight, would cause their rejection. They are usually tested by water pressure to double their intended working pressure.

2. *Thickness of cast-iron pipes for water mains.*—The rule for the thickness of a cylindrical vessel necessary to resist an internal or bursting pressure in given in I. § 26.

Let t = thickness of cylinder in inches.
d = diameter " "
p = excess of internal over external pressure in lbs. per sq. in.

Let H = pressure measured in feet of water.
f = safe limit of stress in lbs. per sq. in.

Then $p = 0{\cdot}4333$ H.
$\text{H} = 2{\cdot}308\,p$.

$$t = \frac{p\,d}{2f} = 0{\cdot}2166\,\frac{\text{H}\,d}{f} \qquad . \qquad . \; (1)$$

The average tenacity of the cast iron used for pipes may be taken at 18,500 lbs. per sq. in. Taking the factor of safety at $3\frac{1}{3}$, the highest safe tension is 5,500 lbs. per sq. in. Allowance must, however, be made—(*a*) for the irregular thickness of cast-iron pipes, which are often slightly thinner on one side than on the other; (*b*) for stresses due to hydraulic shock in the pipe, and to bending in consequence of pressure of the earth above, or settlement of the earth beneath, the pipe. A sufficient allowance will be made if the pipe is calculated for three times the actual working pressure, or, what amounts to the same thing, if the limit of stress is taken at one-third the value given above. Hence, the apparent factor of safety for pipes is $3 \times 3\frac{1}{3} = 10$, and the greatest safe stress, due to the actual pressure in the pipe, is 1,850 lbs. per sq. in.

In the mains used for the conveyance of water the external pressure is 1 atmosphere, or 33 feet of water pressure, and the greatest internal pressure is generally less than 7 atmospheres, or 231 feet of water. Hence, the excess of internal over external pressure may be taken at 6 atmospheres, or 90 lbs. per sq. in. Putting this value in the formula above, we get

$$t = \frac{90\,d}{2 \times 1850} = {\cdot}0231\,d. \qquad . \qquad . \; (2)$$

3. The following table shows that some of the thicknesses given by the above rule, although ample margin of

strength has been allowed, are so small that the pipes could not be cast with any certainty of success.

Internal diameter of pipe in ins. (eq. 2.)								
4	8	12	16	20	24	30	36	42
Thickness of pipe in ins.								
·0924	0·185	0·277	0·370	0·462	0·554	0·693	0·832	0·970
Thickness to nearest sixteenth of an inch								
$\frac{1}{8}$	$\frac{3}{16}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{11}{16}$	$\frac{13}{16}$	1

Many years ago the following rule for the thickness of water-mains was given by Mr. Hawksley :—

$$t = 0{\cdot}18\sqrt{d}$$

That rule represents, very fairly, the least thickness which it is desirable to attempt to cast. The following rule agrees still better with practical experience. Let $t_{\text{min.}}$ be the least thickness which should be adopted for a cylindrical pipe casting, of ordinary length and of diameter d, in order that there may be no special difficulty in getting it cast. Then

$$t_{\text{min.}} = 0{\cdot}11\sqrt{d} + 0{\cdot}1 \quad . \quad . \quad . \quad (3)$$

Diameter of pipe in ins. (eq. 3.)											
4	8	12	16	20	24	30	36	42	48	54	60
Least thickness of pipe in ins.											
·320	·411	·481	·540	·592	·639	·703	·760	·813	·862	·908	·953
Thickness to nearest sixteenth of an inch											
$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{15}{16}$	1

Pipes of the thicknesses here given will in general be safe for pressures not exceeding 6 atmospheres, or 90 lbs. per sq. in., when under 20 ins. diameter, and for 5 atmospheres, or 75 lbs. per sq. in., when under 60 ins. diameter. When pipes are subjected to greater pressure, it

is desirable to use the more exact formula given in I. § 26 (eq. 3) in calculating the thickness. Putting in that formula $f = 1850$, it becomes

$$\frac{t}{d} = \frac{1}{2}\left\{\sqrt{\frac{2775+p}{2775-2p}} - 1\right\} \quad . \quad . \quad (4)$$

From this formula the following table has been calculated :—

Excess of Internal over External Pressure.		Ratio of thickness to diameter of pipe
In lbs. per square inch	In feet of head of water	$\frac{t}{d}$
75	173	·021
90	208	·026
105	242	·030
120	277	·035
135	311	·039
150	346	·044
165	381	·048
180	415	·053
195	450	·058
210	484	·063
225	519	·068
250	577	·077
275	634	·085
300	692	·095
350	808	·114
400	923	·134
450	1039	·156
500	1154	·179
750	1731	·332
1000	2308	·603

4. In the following table the second and third columns give the least thickness of pipe which it is practicable to cast (from eq. 3), or the thickness to be adopted when equations (1) or (4) give a less value. The other columns give thicknesses calculated by equation (4). It should be remembered that in obtaining these thicknesses an allowance has been made for bending stress, and hence somewhat less thick-

Internal diameter of pipe in ins.	Least thickness of pipe by eq. (3)		Thickness necessary for strength, by eq. (4), for working internal pressures in lbs. per sq. in. and ft. of head amounting to							
	Exact	Correct to nearest sixteenth	75 lbs. 173 ft.	90 lbs. 208 ft.	105 lbs. 242 ft.	120 lbs. 277 ft.	150 lbs. 346 ft.	180 lbs. 415 ft.	210 lbs. 484 ft.	250 lbs. 577 ft.
2	·256	$\frac{1}{4}$	...	...	...	...	...	...	...	...
3	·291	$\frac{5}{16}$	...	...	...	...	...	...	...	...
4	·320	,,	...	...	...	...	...	...	...	...
5	·346	$\frac{3}{8}$	...	...	...	...	...	...	...	·385
6	·369	,,	...	...	...	...	...	...	·378	·462
7	·391	,,	...	...	...	...	...	...	·441	·539
8	·411	$\frac{7}{16}$	...	...	...	...	...	·424	·504	·616
9	·430	,,	...	...	...	...	...	·477	·567	·693
10	·448	$\frac{1}{2}$	...	...	...	...	...	·530	·630	·770
12	·481	,,	...	...	...	...	·528	·636	·756	·924
14	·512	$\frac{9}{16}$	...	...	...	...	·616	·742	·882	1·08
16	·540	,,	...	...	...	·560	·704	·848	1·01	1·23
18	·567	,,	...	...	...	·630	·792	·954	1·13	1·39
20	·592	$\frac{5}{8}$	...	...	...	·700	·880	1·060	1·26	1·54
22	·616	,,	...	...	·66	·770	·968	1·166	1·39	1·69
24	·639	,,	...	...	·72	·840	1·056	1·272	1·51	1·85
30	·702	$\frac{11}{16}$	...	·780	·90	1·05	1·320	1·590	1·89	2·31
36	·760	$\frac{3}{4}$	...	·936	1·08	1·26	1·584	1·908	2·27	2·77
42	·813	$\frac{13}{16}$	·882	1·092	1·26	1·47	1·848	2·226	2·65	3·23
48	·862	$\frac{7}{8}$	1·008	1·248	1·44	1·68	2·112	2·544	3·02	3·70
54	·909	$\frac{15}{16}$	1·134	1·404	1·62	1·89	2·376	2·862	3·40	4·16
60	·952	,,	1·260	1·560	1·80	2·10	2·640	3·180	3·78	4·62
72	1·033	1	1·512	1·872	2·16	2·52	3·168	3·816	4·54	5·54
84	1·108	$1\frac{1}{8}$	1·764	2·184	2·52	2·94	3·696	4·452	5·29	6·47

nesses may be adopted in pipes so supported as to be protected from any bending. To convert feet of head of water into lbs. per sq. in., multiply by 0·4333. In water-mains for towns, a thickness about 25 per cent. greater than that given in this table is often adopted in practice.

5. *Velocity of flow and loss of head in water-mains.*—The diameter d of water-mains is determined with reference to the volume Q in cubic feet per second to be delivered. If v is the velocity of flow in feet per second,

$$Q = \frac{\pi}{4} d^2 v$$

where the diameter d is in feet. Generally in water-mains the velocity does not exceed 3 feet, or at most 4 feet, per second, in order that the loss of head and the shocks due to the momentum of the water when its velocity changes may not be too serious. In cases where it is desirable to lose as little energy as possible, the velocity is restricted to 1½ or 2 feet per second.

In a large number of cases it is important to determine what loss of head occurs in long water-mains. The head lost in friction may be a loss of level or a loss of pressure, and in either case is most conveniently reckoned in feet of water. Let h be the head lost in feet, in a main of length l feet, when water flows through it with a velocity v in feet per second. The most exact relation between these quantities is—

$$\frac{h}{l} = \frac{m}{d^x} \cdot \frac{v^n}{2g} \quad . \quad . \quad . \quad . \quad (5)$$

where m, n, and x are constants depending on the roughness of surface of the pipe. The equation is, however, inconvenient for practical use. More commonly, therefore, engineers use the equation,

$$\frac{h}{l} = \frac{k}{d} \cdot \frac{v^2}{2g} \quad . \quad . \quad . \quad . \quad (6)$$

where k has a considerable range of values in different cases. In 1886 the author communicated to *Industries* a very careful determination of the constants m, n, and x, for all the most trustworthy experiments on flow in pipes. From these values it was then possible to calculate a series of values of k in equation (6) for such cases as commonly occur in practice. With the values given in the following tables, the loss of head in pipes can be calculated much more accurately than with a constant value for k, and nearly as accurately as if the more awkward equation (5) were used.

Clean Wrought-Iron Pipes

When d in feet is	The value of k for the following velocities in feet per second is			
	1 to 2	2 to 3	3 to 4	4 to 5
0·5 to 0·75	·0230	·0200	·0184	·0172
0·75 to 1·0	·0214	·0186	·0171	·0160
1·0 to 1·5	·0199	·0173	·0159	·0149
1·5 to 2·0	·0185	·0161	·0148	·0139
2·0 to 3·0	·0172	·0150	·0137	·0129
3·0 to 4·0	·0160	·0139	·0128	·0120

Asphalted Cast-Iron Pipes

When d in feet is	The value of k for the following velocities in feet per second is			
	1 to 2	2 to 3	3 to 4	4 to 5
0·5 to 0·75	·0257	·0236	·0224	·0216
0·75 to 1·0	·0246	·0226	·0215	·0207
1·0 to 1·5	·0235	·0216	·0206	·0198
1·5 to 2·0	·0225	·0207	·0197	·0189
2·0 to 3·0	·0215	·0198	·0188	·0181
3·0 to 4·0	·0206	·0190	·0180	·0173

New Cast-Iron Pipes

When d is in feet	The value of k for the following velocities in feet per second is			
	1 to 2	2 to 3	3 to 4	4 to 5
0·5 to 0·75	·0230	·0224	·0220	·0217
0·75 to 1·0	·0216	·0210	·0206	·0204
1·0 to 1·5	·0204	·0199	·0195	·0193
1·5 to 2·0	·0193	·0188	·0184	·0182
2·0 to 3·0	·0182	·0177	·0174	·0172
3·0 to 4·0	·0172	·0167	·0164	·0162

Incrusted Cast-Iron Pipes

When d in feet is	k is for all velocities
0·5 to 0·75	·0475
0·75 to 1·0	·0450
1·0 to 1·5	·0426
1·5 to 2·0	·0403
2·0 to 3·0	·0381
3·0 to 4·0	·0361

At first sight the co-efficients do not seem to differ greatly; but, for each kind of pipe, k varies by at least 20 per cent. within the restricted limits chosen. Also the resistance for incrusted cast iron is quite double that of clean wrought iron, and for the larger pipes and higher velocities more than this.

6. *Thickness of steam pipes and steam cylinders of cast iron.*—This is determined in precisely the same way as the thickness of water-mains. For steam pipes the thicknesses given in the preceding table will answer. For steam cylinders an allowance has to be made for re-boring. The cylinder thickness may be obtained from the following equation :—

$$t = 0{\cdot}0005\, p\, d + 0{\cdot}3 \qquad . \quad . \quad . (7)$$

The last constant may be regarded as an allowance for re-boring. p is the excess in lbs. per sq. in. of the

greatest steam pressure over the external pressure. Further particulars of steam pipes are given below.

WROUGHT-IRON AND STEEL PIPES

7. Wrought-iron welded tubes came into extensive use for the distribution of gas. A strip of wrought-iron plate heated to welding heat was bent to cylindrical form and then welded by hammers or rolls or by passing through dies. Such tubes are always weakest at the weld, and when the diameter exceeds 6 or 7 ins. are difficult to manufacture. For hydraulic purposes, where pressures of often 2 tons per sq. in. have to be transmitted, solid drawn weldless tubes

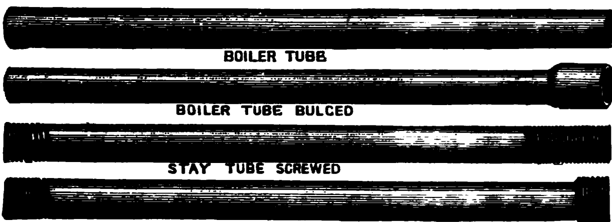


Fig. 1

have been produced in small sizes. Quite lately an entirely new process, invented by Messrs. Mannesmann, has been introduced, and seems likely to revolutionise the manufacture of tubes. It is not well adapted for wrought iron, but with ductile steel, copper, delta metal, or lead, weldless tubes are produced of almost any size by a peculiar process of rolling from a solid bar.[1]

Wrought-iron and steel pipes or tubes are obtainable of the following descriptions :—

(1) Butt-welded tubes used for gas, water, and steam, of wrought iron. These are made of from ⅛ in. to 4 ins. internal diameter, and in lengths usually not exceeding

[1] See a paper by Mr. J. G. Gordon, 'Journal of Society of Arts,' vol. xxxviii.

Wrought-Iron Tubes

Internal diameter in ins.	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4	4¼	4½	4¾	5	5¼
Thickness B. W. G.	9	9	9	9	8	8	8	8	7	7	7	7	7	6	6	6	6	5
Thickness in ins.	0·158	·158	·158	·158	·166	·166	·166	·166	·187	·187	·187	·187	·187	·208	·208	·208	·208	·217
Greatest working pressure in lbs. per sq. in.	1720	1370	1145	980	930	825	745	675	745	690	640	595	560	615	580	550	520	530
Greatest length	Usual length 12 to 14 feet. Can be made to 20 feet.																	

Internal diameter in ins.	5½	5¾	6	6¼	6½	6¾	7	7½	8	8½	9	9½	10	10½	11	12
Thickness B. W. G.	5	5	5	4	4	4										
Thickness in ins.	·217	·217	·217	·239	·239	·239	0·25	0·25	0·25	0·3125	0·3125	0·3125	0·375	0·375	0·375	0·375
Greatest working pressure in lbs. per sq. in.	505	485	460	505	485	470	485	455	425	525	500	470	565	540	515	480
Greatest length	As above.		Length 16 or 17 feet.													

14 feet. They can be obtained up to 20 feet in length if necessary. The steam tubes are two gauges thicker than the gas tubes.

(2) Lap-welded wrought-iron steam tubes, which are stronger than butt-welded tubes. They are proved to 400 lbs. per sq. in. before being sent out. The preceding table gives the usual dimensions of these tubes.

The thickness of such tubes will not be exactly regular. Suppose that when t is the nominal thickness, $t - \frac{1}{16}$ is the effective thickness which can be relied on in calculating the strength. Then by equation (2) I., p. 55, for an internal bursting pressure,

$$f = \frac{p\,d}{2(t - \frac{1}{16})};$$

and taking the greatest safe stress at 4 tons or 8,960 lbs. per sq. in.

$$t = \cdot 0000558\,p\,d + \tfrac{1}{16} \quad . \qquad . \quad (8)$$

By this rule the working pressures p for different diameters d, given in the table above, have been calculated. Lap-welded steam tubes of the dimensions given above are proved to 400 lbs. per sq. in. before being sent out.

Fig. 1 shows the ordinary forms in which wrought-iron and steel tubes can be obtained.

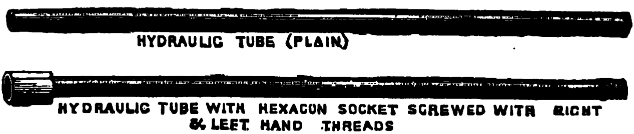


Fig. 2

(2) Weldless solid drawn steel tubes (fig. 2) are also manufactured, which are used for boiler tubes, hydraulic pipes, and occasionally for other purposes, such as hollow shafting and boring rods. The tables on p. 18 give the usual dimensions of such tubes.

The steel of which these tubes are made has a tensile strength of about 30 tons per sq. in. For hydraulic tubes

Weldless Steel Boiler Tubes

External diameter in ins.	$1\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{3}{4}$	$1\frac{7}{8}$	2	$2\frac{1}{8}$	$2\frac{1}{4}$	$2\frac{3}{8}$	$2\frac{1}{2}$	$2\frac{5}{8}$	$2\frac{3}{4}$	$2\frac{7}{8}$	3	$3\frac{1}{8}$	$3\frac{1}{4}$	$3\frac{3}{8}$	$3\frac{1}{2}$
Thicknesses B. W. G.	16 to 10	16 to 10	16 to 10	16 to 10	16 to 10	15 to 10	15 to 10	14 to 10	14 to 10	14 to 10	14 to 10	13 to 10	13 to 10	13 to 10	13 to 10	13 to 10	13 to 10
Length in feet	13 feet.											10 feet.					

Weldless Steel Boiler and Hydraulic Tubes

External diameter in ins.		$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{3}{4}$	$1\frac{7}{8}$	2	$2\frac{1}{8}$	$2\frac{1}{4}$	$2\frac{3}{8}$	$2\frac{1}{2}$	$2\frac{5}{8}$	$2\frac{3}{4}$	$2\frac{7}{8}$	3	$3\frac{1}{8}$	$3\frac{1}{4}$	$3\frac{3}{8}$	$3\frac{1}{2}$
Thicknesses in ins.	Least	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{32}$
	Greatest	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
Maximum length 15 feet.																										

the working strength may be taken at 8 tons per sq. in. Putting this for the value of f in equation (2), I., p. 55, we get

$$t = 0{\cdot}000028\,p\,d \quad . \quad . \quad (9)$$

where p is the excess of internal over external pressure in lbs. per sq. in. and d the internal diameter of the tube in ins.

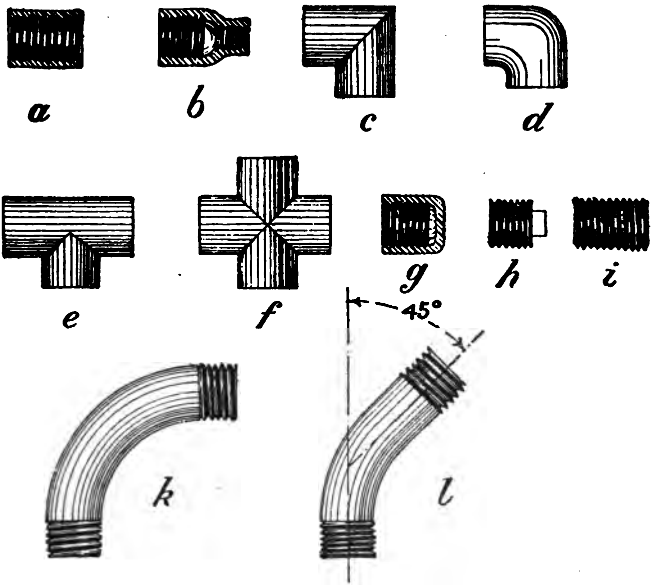


Fig. 3

8. *Tube fittings.*—Fig. 3 shows various connecting pieces for wrought iron and steel tubes as ordinarily manufactured. *a* is a coupling; *b*, a diminishing piece; *c*, elbow; *d*, round elbow; *e*, tee; *f*, cross; *g*, cap; *h*, plug; *i*, nipple; *k*, *l*, bends or springs.

9. *Wrought-iron and steel welded or riveted water pipes.*—Water-mains of comparatively large size of thin riveted wrought iron seem first to have been used in California in

hydraulic mining. An account of these pipes will be found in a paper by Mr. Hamilton Smith ('Trans. Am. Soc. of Civil Engineers,' 1884). These pipes are single or double riveted, and very often are put together stove-pipe fashion. That is, the pipe is made slightly conical; a piece of canvas is wrapped round the small end of one pipe and this is then forced into the large end of another by hydraulic jacks. Leakage is stopped if it occurs by small pine wedges. The pipes are sometimes only $\frac{1}{12}$ in. thick, and they are used under pressures which strain the metal at the riveted joint up to the singularly high-working stress of 16,000 to 18,000 lbs. per sq. in. They are protected against corrosion by a coating of tar, and no serious corrosion seems to occur. At bends they are strongly braced to prevent movement.

Water-mains made of mild steel plates are now largely used. Up to 15 or 20 ins. in diameter they can be obtained with a longitudinal welded joint. In sizes from 12 ins. diameter upwards they are more commonly riveted, the longitudinal joint being double riveted and the cross joints single riveted. The steel is of 25 to 28 tons tensile strength, with an extension of 16 per cent. in 8 ins. The minimum thickness of welded pipes is $\frac{1}{4}$ in. In riveted pipes the rings are sometimes alternately of greater and less diameter, the larger overlapping the ends of the smaller rings. Made so, caulking the joints is easier.

Such water-mains may be calculated for a working stress of $7\frac{1}{2}$ tons per sq. in., but an allowance of $\frac{1}{16}$th is usually made for corrosion. The longitudinal double riveted joint has about 0·7 of the strength of solid plate. If d is the internal diameter, t the thickness, p the working pressure in lbs. per sq. in. (= pressure height in feet of water $\div$ 2·3), f the working stress,—

$$t = \frac{p\,d}{2f} + \frac{1}{16}$$

$$= 0{\cdot}00003\; p\,d + \tfrac{1}{16} \text{ for welded pipes,}$$

$$= 0{\cdot}000042\; p\,d + \tfrac{1}{16} \text{ for riveted pipes.}$$

10. *Cylinders for compressed gas.*—Compressed gases are now supplied in cylinders for refrigerating and limelight purposes, and similar cylinders are used for compressed air in some traction engines. These cylinders are made of lap-welded wrought iron, or more usually of seamless or welded steel. The steel should not contain more than 0·25 per cent. of carbon or 0·5 per cent. of manganese. Strips cut from the cylinders should have a tenacity of 28 to 33 tons per sq. in. The cylinders should be carefully annealed. Such cylinders are tested hydraulically to double their working pressure. During the test the cylinder is placed in a water jacket, and the rise of a water column in a tube communicating with the jacket shows whether the cylinder is expanded by the test pressure. The small temporary elastic expansion is easily distinguished from a permanent expansion or set, due to the yield point of the steel having been exceeded by the test stress. There should be only a very small set in any cylinder accepted as satisfactory.

To allow some margin for wear and tear and for unavoidable irregularity of thickness, the stress in the cylinder due to the working pressure should not exceed $6\frac{1}{2}$ tons (14,560 lbs.) per sq. in. for lap-welded wrought iron; $7\frac{1}{2}$ tons (16,800 lbs.) for lap-welded steel; or 8 tons (17,920 lbs.) for seamless steel. Let d = external diameter of cylinder in ins., p = working pressure in lbs. per sq. in., t = thickness of cylinder in ins., f = permissible stress in material of cylinder in lbs. per sq. in., then by a simple modification of the ordinary rule

$$t = \frac{p\,d}{2\,(f + p)}.$$

The ordinary compression pressure of oxygen, hydrogen, and coal gas cylinders is 120 atmospheres, or 1,800 lbs. per sq. in. For liquid carbonic acid the maximum pressure (which varies greatly with temperature) should be taken at 120 atmospheres, or 1,800 lbs. per sq. in. No cylinder should contain more than $\frac{3}{4}$ lb. of carbonic acid per lb. of

water capacity. For liquid ammonia the working pressure should be taken at 1,000 lbs. per sq. in., and the cylinders should not contain more than $\frac{1}{2}$ lb. per lb. of water capacity.[1]

Pipes of other Materials

11. *Copper pipes* with a brazed longitudinal joint have been largely used for steam and feed pipes for engines, partly because they can be constructed in bent forms. They usually have flanges brazed on, the flange thickness being at least 4 times the pipe thickness, and the flange width $2\frac{1}{4}$ times the diameter of the flange bolts. It should be remembered that, at the high steam temperatures now common, copper loses a good deal of its strength (I., p. 21). *Lead pipes* are largely used for water pipes in houses, chiefly because of the facility with which they can be bent.

Lead pipes, $t = 0{\cdot}0025\, p\, d + \frac{3}{16}$.
Copper steam pipes, t not less than $\frac{1}{4}$ in.
" brazed, $t = 0{\cdot}00017\, p\, d + \frac{1}{16}$.
" solid drawn, $t = 0{\cdot}00017\, p\, d + \frac{1}{32}$.
" feed pipes, $t = 0{\cdot}00013\, p\, d + \frac{1}{8}$.

On account of accidents which have occurred with copper-brazed steam pipes, steel pipes are now generally used. But in some cases copper pipes are very convenient, and means have been sought to strengthen them. Brazed-copper pipes over 8 ins. in diameter have been served with a close wrapping of steel or delta metal wire, wound on under tension. The copper pipe is made strong enough for the ordinary hydraulic test pressure, and the wire serving about equally strong.

Pipe Joints

12. Cast-iron pipes are connected by flange joints, or by spigot and faucet joints. The former are stronger, easier

[1] Report of Committee appointed by the Home Office on Causes of Explosion of Cylinders of Compressed Gas. 1896.

to connect or disconnect, and are always used when the pipes are placed vertically. The latter are less costly, and are better for pipes laid in the ground, because they permit the pipes to adapt themselves to the inequalities of the ground while being laid, and the line of pipes retains a slight flexibility.

13. *Flanged joints.*—The proportions of flanges have been to some extent given in I. § 96.

The flanges are drawn together by bolts with some joint material interposed. For high pressures the joint packing should be thin, as the pressure acting on the internal surface tends to displace it. The packing may be in a groove, but then dismounting is less easy. The number of bolts is

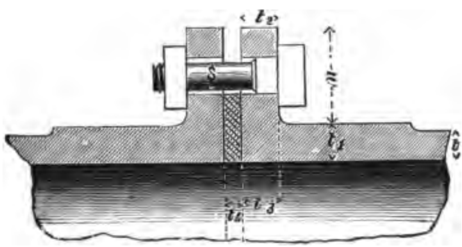


Fig. 4

generally not less than four, and the distance centre to centre of bolts not more than 5 ins. The bolt diameter should not be less than $\frac{5}{8}$ in. Strength must be allowed both to resist the internal pressure acting on the section of the pipe and to maintain a sufficient compression of the joint packing. Fig. 4 shows one form of flanged joint for pipes, for which the following proportions may be used :—

$$\text{Thickness } t_1 = t_2 = \tfrac{5}{4}\, t \text{ to } \tfrac{5}{4}\, t + \tfrac{1}{4}$$
$$t_3 = \tfrac{3}{2}\, t$$
$$t_4 = \tfrac{3}{8} \text{ in.}$$
$$\text{Width} = w = 2\,\delta + \tfrac{1}{2} \text{ to } 2\,\delta + \tfrac{3}{4}$$

$$\text{Diam. of bolts} = \delta = 0{\cdot}014\, d\sqrt{\frac{p}{n}} + \frac{1{\cdot}6}{d}$$

$$\text{Number of bolts} = n = 0{\cdot}8\, d + 4$$

$$\text{Diam. of bolt hole} = \delta + \tfrac{1}{8}$$

The joint shown is made with a lead ring. The joint may be made by facing the flanges and bringing them together with a string smeared with red lead, or an india-rubber or gutta-percha ring interposed. A rough joint is made with a ring of wrought iron, covered with tarred rope, the space between the flanges being filled up with rust cement. For steam pipes a ring of asbestos millboard is often used. Mr. Emery for steam pipes has used a ring of corrugated copper.

14. *Graphic diagram of pipe-joint proportions.*—Fig. 5 shows a very convenient graphic way of settling the proportions of simple machine parts for a series of different sizes. The curve or straight line which represents the proportions given by rule is first laid down as shown, for instance, by the dotted line *a b*. This is then broken up into a stepped line, the steps increasing by eighths of an inch or any other required difference. With a little dexterity such a diagram may be made to give all the required dimensions for any series of sizes without confusion. It is best drawn full size when that is possible. The method may be applied in many cases and is more convenient than a table.

15. *Standard proportions for cast-iron flanged pipes.*—The following table gives standard proportions for pipes and flanges adopted in the United States. The pipe thickness for a pressure p in lbs. per sq. in. and a pipe of d inches internal diameter is calculated by the empirical rule :—

$$t = \frac{p + 100}{7200} d + 0{\cdot}333 \left(1 - \frac{d}{100}\right)$$

Up to 22 ins. diameter the dimensions in the table are sufficient for pressures of 200 lbs. per sq. in. For the larger pipes, where two dimensions are given the smaller is

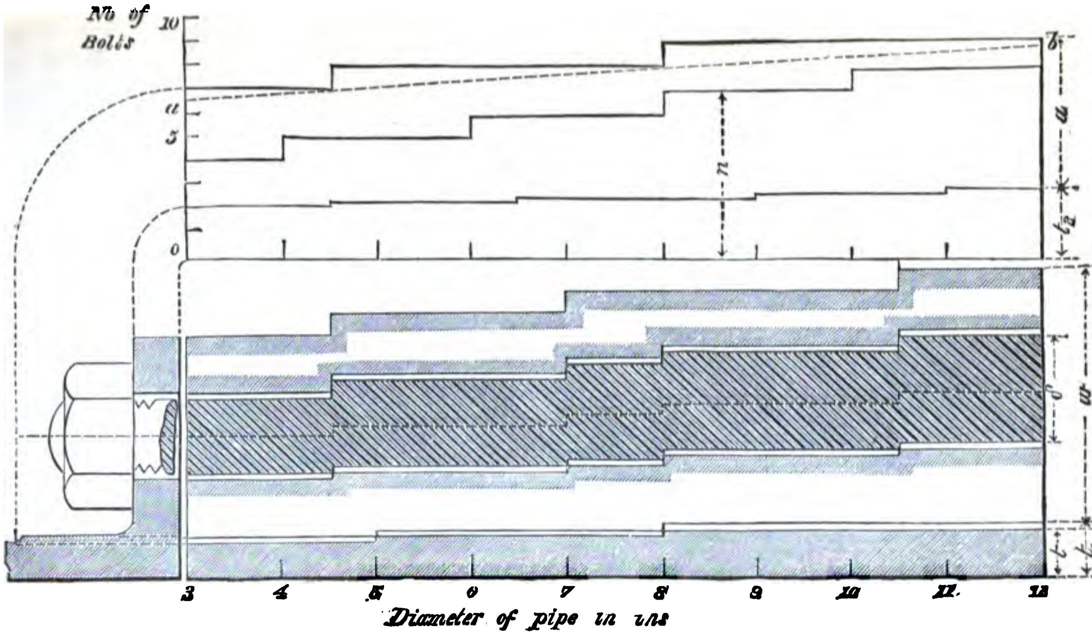
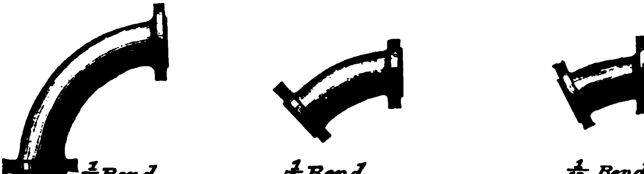


Fig. 5.—Half full size

Standard Pipe Flanges. (Committee of Am. Soc. Mech. Eng.)

Pipe size, ins.	Pipe thickness	Thickness, nearest fraction, ins.	Stress on pipe per square inch at 200 lbs.	Radius of fillet, ins.	Flange diameter, ins.	Flange thickness at hub for iron pipe, ins.	Flange thickness at edge, ins.	Width flange face, ins.	Bolt circle diameter, ins.	Number of bolts	Bolt size diameters, ins.	Bolt length, ins.	Stress on each bolt, per square inch at bottom of thread at 200 lbs.
2	.409	7/16	460	1/8	6	1	5/8	2	4 3/4	4	1/2 5/8	2	825
2 1/2	.429	7/16	550	1/8	7	1 1/8	11/16	2 1/4	5 1/2	4	1/2 5/8	2 1/4	1,050
3	.448	7/16	690	1/8	7 1/2	1 1/4	3/4	2 1/4	6	4	1/2 5/8	2 1/2	1,330
3 1/2	.466	1/2	700	1/8	8 1/2	1 1/4	13/16	2 3/8	7	4	1/2 5/8	2 1/2	2,530
4	.486	1/2	800	1/8	9	1 3/8	15/16	2 1/2	7 1/2	4	5/8 3/4	2 3/4	2,100
4 1/2	.498	1/2	900	1/8	9 1/4	1 3/8	15/16	2 5/8	7 3/4	8	5/8 3/4	3	1,430
5	.525	1/2	1,000	1/8	10	1 1/2	15/16	2 5/8	8 1/2	8	5/8 3/4	3	1,630
6	.563	9/16	1,060	1/8	11	1 1/2	1	2 5/8	9 1/2	8	5/8 3/4	3	2,360
7	.60	5/8	1,120	1/8	12 1/2	1 1/2	1 1/16	2 3/4	10 3/4	8	5/8 3/4	3 1/4	3,200
8	.639	5/8	1,280	1/8	13 1/2	1 3/4	1 1/8	2 3/4	11 3/4	8	5/8 3/4	3 1/2	4,190
9	.678	11/16	1,310	3/16	15	1 3/4	1 1/8	3	13 1/4	12	5/8 3/4	3 1/2	3,610
10	.713	3/4	1,330	3/16	16	2	1 3/16	3	14 1/4	12	3/4 7/8	3 5/8	2,970
12	.79	13/16	1,470	3/16	19	2	1 1/4	3 1/2	17	12	3/4 7/8	3 3/4	4,280
14	.864	7/8	1,600	3/16	21	2	1 3/8	3 1/2	18 3/4	12	7/8 1	4 1/4	4,210
15	.904	15/16	1,600	3/16	22 1/4	2	1 3/8	3 5/8	20	16	7/8 1	4 1/4	3,660
16	.946	1	1,600	3/16	23 1/2	2 1/4	1 7/16	3 3/4	21 1/4	16	7/8 1	4 1/4	4,2[illegible]0
18	1.02	1 1/16	1,690	3/16	25	—	1 9/16	3 1/2	22 3/4	16	1 1 1/8	4 3/4	4,540
20	1.09	1 1/8	1,780	3/16	27 1/2	—	1 11/16	3 3/4	25	20	1 1 1/8	5	4,490
22	1.18	1 3/16	1,850	1/4	29 1/2	—	1 13/16	3 3/4	27 1/4	20	1 1 1/4	5 1/2	4,320
24	1.25	1 1/4	1,920	1/4	31 1/2 32	—	1 1/4 1 7/8	3 3/4 4	29 1/4 29 1/2	20	1 1 1/4	5 1/2	5,130
26	1.30	1 5/16	1,980	1/4	33 3/4 34 1/4	—	1 3/8 2	3 7/8 4 1/8	31 1/4 31 3/4	24	1 1 1/4	5 3/4	5,030
28	1.38	1 3/8	2,040	1/4	36 36 1/2	—	1 7/16 2 1/16	4 4 1/4	33 1/2 34	28	1 1 1/4	6	5,000
30	1.48	1 1/2	2,000	1/4	38 38 3/4	—	1 1/2 2 1/8	4 4 3/8	35 1/2 36	28	1 1/8 1 3/8	6 1/4	4,590
36	1.71	1 3/4	1,920	1/4	44 1/2 45 3/4	—	1 5/8 2 3/8	4 1/4 4 7/8	42 42 3/4	32	1 1/8 1 3/8	6 1/2	5,790
42	1.87	2	2,100	1/4	51 52 3/4	—	1 7/8 2 5/8	4 1/2 5 3/8	48 1/2 49 1/2	36	1 1/4 1 1/2	7 1/4	5,700
48	2.17	2 1/4	2,130	1/4	57 1/2 59 1/2	—	2 2 3/4	4 3/4 5 3/4	54 3/4 56	44	1 3/8 1 1/2	7 3/4	6,090

suitable for pressures up to 100 lbs. per sq. in., the larger for pressures from 100 to 200 lbs. per sq. in.



$\frac{1}{4}$ *Bend* $\frac{1}{8}$ *Bend* $\frac{1}{12}$ *Bend*

Fig. 6

Fig. 6 shows the ordinary forms and denominations of pipe bends.

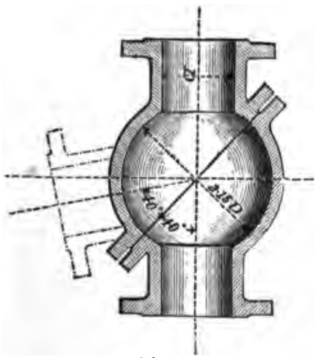


Fig. 7

Fig. 8

16. *Special forms of pipe joint.* Fig. 7 shows an adjustable form of pipe joint. When the angle between two pipes varies or is not definitely known beforehand a joint of this kind, which can be arranged at any angle between 80° and 180°, is convenient.

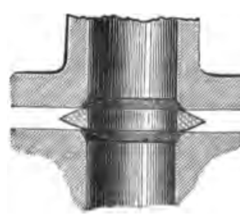


Fig. 9

Fig. 8 shows a pipe joint with one flange loose. The joint is made tight by a lead ring.

Fig. 9 shows what is termed a lens joint, which is easily

made tight and which adjusts itself to small changes of direction of the pipe. The joint is made by a gunmetal ring with spherical surfaces.

17. *Joints for hydraulic mains.*—Fig. 10 shows the joint used by Sir W. Armstrong for the pipes of his accumulator. These pipes are subjected to the enormous water pressure of 700 lbs. per sq. in. The pipes are of the best remelted cast iron, and are tested to 2,500 lbs. per sq. in. When 5 ins. diameter they are $\frac{7}{8}$ in. thick. Each end of the pipe has two strong elliptical flanges, with two bolts. One pipe slightly

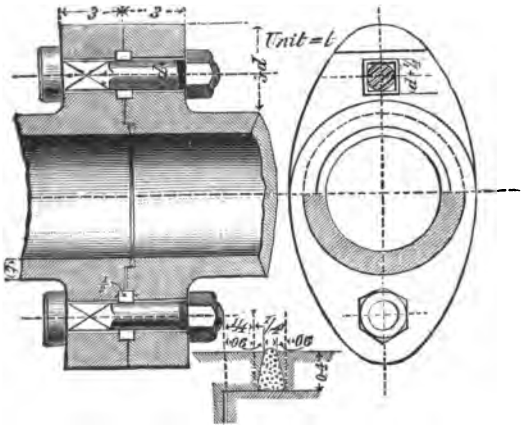


Fig. 10

enters into the other, forming a dovetailed recess, in which is placed a gutta-percha ring, $\frac{1}{4}$ in. thick. The thickness of these pipes may be calculated by the rule—

$$t = \cdot 000178 dp + \tfrac{1}{4},$$

the working stress in the cast iron being about 2,800 lbs. per sq. in., and $\frac{1}{4}$ in. being allowed for inequalities of casting and corrosion. The bolts are of such a diameter that the stress at reduced section of thread is 7,700 lbs. per sq. in.

18. *Cylinders for great internal pressure.*—In the construction of vessels for great hydraulic or other pressure special difficulties arise. If made of boiler plate, the riveted joints diminish the strength of the vessel and limit the thickness of the plates which can be used. If cast iron is adopted, the vessel must be ponderous in consequence of the low tenacity of the material, and the fluid sometimes escapes through porous parts of the casting. Dr. Siemens has described the construction of an air reservoir, to sustain 1,000 lbs. pressure per sq. in., of steel rings. These rings (40 ins. in diameter and 12 ins. in depth) were rolled out of ingots in a tyre mill, and had a slight flange (fig. 11) at their edges. The ends of the reservoir were made hemispherical, and beaten out of steel plate. The joints were made by turning a V-groove in the faces of the rings and placing in it a packing ring made of $\frac{5}{16}$ annealed copper wire. The rings and ends were held together by 20 steel longitudinal bolts, passing through two rings bearing on the hemispherical ends of the cylinder. These bolts were $1\frac{1}{4}$ in. in diameter with enlarged screwed ends. The rings were of steel, having a tenacity of 45 tons per sq. in., and were loaded to a working stress of 15 tons per sq. in.

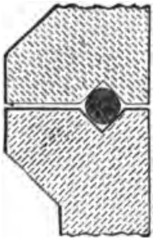


Fig. 11

19. *Paris joint.*—Compressed air-mains are liable to greater temperature changes than water-mains, and hence more care must be taken to provide for expansion and contraction. A joint first extensively used in the Paris air-mains (fig. 11A) permits great freedom of expansion, and has proved quite successful. It has since been used for water-mains. The lengths of main have plain spigot ends. A kind of double stuffing-box is formed over the two pipe ends, the packing consisting of two india-rubber rings. The rubber is compressed by two rings drawn together by bolts, and is so protected from access of air or water that it

appears to be nearly imperishable. Freedom of expansion and contraction is secured at every joint in the main. Experiments with this joint seem to show that there is little

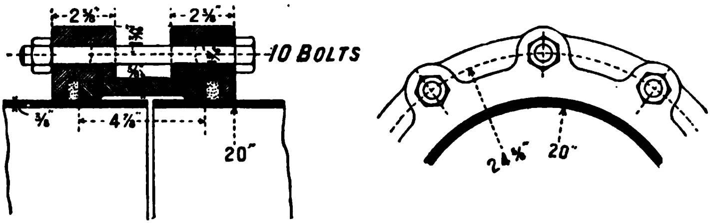


Fig. 11A

sliding of the pipes in the rubber, the movement of the pipe merely bending the rubber.

Pipes for compressed ammonia.—Liquid ammonia is largely used in refrigerating machinery, at pressures of 125 to 150 lbs. per sq. in. As ammonia escaping by leakage would be dangerous, special care has to be taken to secure absolutely tight joints. Ammonia has no chemical action on iron, and all fittings are of that material. Fig. 11B shows the pipe joint used by the De la Vergne Company. The pipe is screwed into a flange fitting of malleable iron or steel, but an annular space is left in the fitting round the pipe which is filled with solder. A lead ring makes the joint between the two flanges tight.

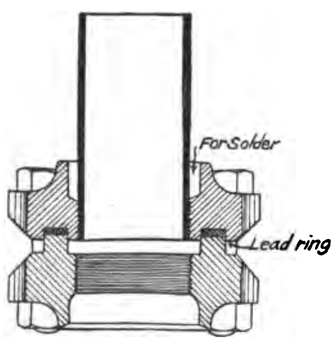


Fig. 11B

20. *Socket joints.*—Socket pipes should be cast vertically with the socket at bottom. Socket pipes are jointed either

with a gasket and lead joint, a rust joint, or a bored and turned joint Fig. 12 shows an ordinary lead joint. When the pipes are in place, a few coils of gasket or tarred rope are driven into the socket. Clay is then put round the outside of the socket and a lead ring is cast in it. The

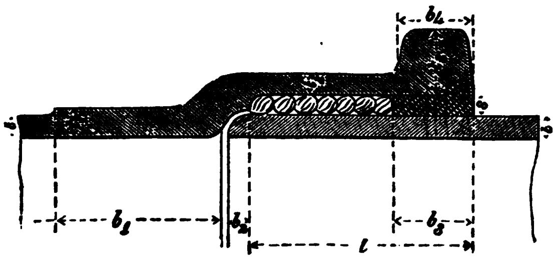


Fig. 12

clay is removed and the lead stemmed tightly into the socket. The proportions may be as follows: Let t = thickness, and d = diameter of pipe,

$$t_1 = 1{\cdot}07\ t + \tfrac{1}{16}$$
$$t_2 = 0{\cdot}025\ d + \tfrac{1}{4} \text{ to } 0{\cdot}025\ d + 0{\cdot}6$$
$$t_3 = 0{\cdot}045\ d + 0{\cdot}8$$
$$s = 0{\cdot}01\ d + {\cdot}25 \text{ to } 0{\cdot}01\ d + {\cdot}375$$
$$b_1 = 0{\cdot}075\ d + 2\tfrac{1}{4}$$
$$b_2 = t_2$$
$$l = 0{\cdot}09\ d + 2\tfrac{3}{4} \text{ to } 0{\cdot}1\ d + 3$$
$$b_4 = b_3 = 0{\cdot}03\ d + 1$$

A rust joint is very similar to a lead joint except that iron cement is stemmed in with a cold chisel in place of the lead. The iron cement consists of cast-iron borings or turnings, which should be passed through a sieve of eight meshes to the inch. One ounce of sal-ammoniac is added to each hundredweight of cast iron, and the mass is damped. When it has heated, it may be kept for some time in water.

Fig. 13 shows a different form of socket. The proportions may be the same as those just given.

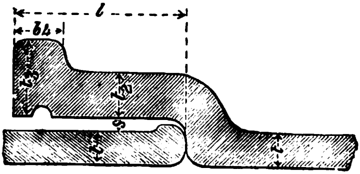


Fig. 13

Figs. 14 and 15 show two forms of bored and turned socket and spigot joints. When the bored and turned part is long the pipes are rigid, and are liable to be broken by the earth pressure. Hence, the fitting part is now often only $\frac{5}{8}$ in. in length, and has a very slight taper. The joint is made by painting over the faced part with red lead, or with fresh and liquid Portland cement. The pipe is then put in place,

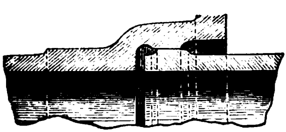


Fig. 14

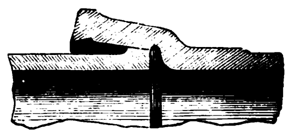


Fig. 15

and driven home by a wooden mallet, or by swinging the next length of pipe. The socket is filled up with cement. Joints of this kind are more easily and quickly made than lead joints, but lead joints are preferable in passing round curves. Socket pipes should be cast with the socket downwards, and about a foot of length should be allowed at the spigot end, into which the scoriæ may rise, and which is broken off when the pipe is cast.

21. With socket pipes care must be taken to provide against the end thrust due to the water pressure at bends. At a right-angled bend, the total pressure in the direction of the tangents to the bend on a pipe of diameter d feet, through which water is passing under a pressure head of h feet, and with a velocity of v feet per second, is

$$\frac{\pi}{4} d^2 G \left(h + \frac{v^2}{g}\right) \text{lbs.}$$

Where G = 62·4 lbs. Hence the resultant pressure bisecting the angle of the bend is

$$1\cdot414\frac{\pi}{4}d^2\text{G}\left(h+\frac{v^2}{g}\right)\text{lbs.} = 69\cdot29\,d^2\left(h+\frac{v^2}{g}\right).$$

This must be provided against by casting a foot on the pipe and abutting it against a block of masonry.

A good joint for pipes used for temporary purposes is shown in fig. 16. A loose tapered double socket is fitted over the ends of the pipes. The joints are made by india-rubber rings driven into the sockets.

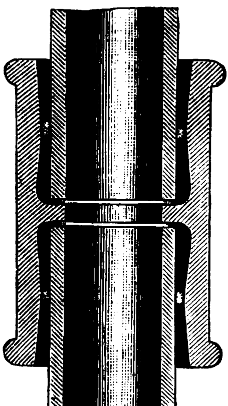


Fig. 16

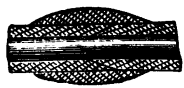


Fig. 17

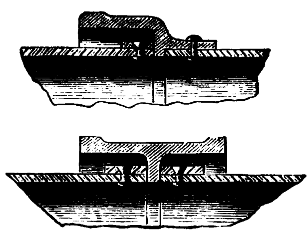


Fig. 18

22. Fig. 18 shows two ways of making a socket joint for wrought-iron pipes. The sockets are of cast iron. Wrought iron is chiefly used for very large or very small pipes. From its thinness, it is liable to more injury from corrosion than cast iron, and hence it has been suggested that large wrought-iron mains should be lined with a thin coating of Portland cement.

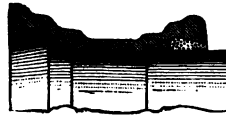


Fig. 19

Fig. 19 shows another form of ring joint for lead used

with wrought-iron pipes. Special rings and sockets are now made also of thin rolled steel.

23. *Joints for lead pipes.*—Lead pipes are useful, because they are easily bent. Joints may be made by flanging out the ends of the pipes, and compressing these flanges between two iron rings, with bolts. Commonly, the joint is made by soldering, and is termed a 'plumber's wiped joint' (fig. 17). The best joint is a 'burned' joint made by melting the lead at the joint with an oxyhydrogen blowpipe.

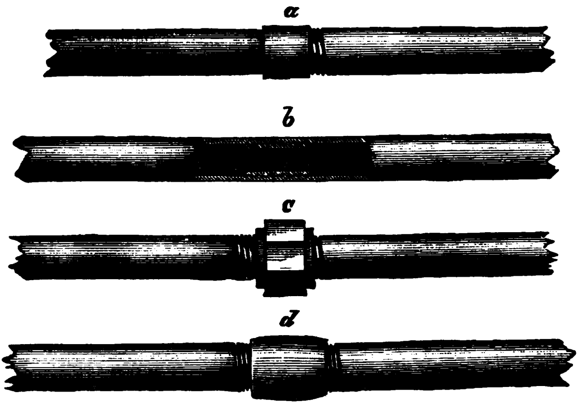


Fig. 20

24. *Joints for wrought-iron and steel tubes.*—Fig. 20 shows the ordinary forms of joints. At *a* one pipe is bulged and the other screwed into it. At *b* an internal screwed ferrule or socket piece is inserted. At *c* an external socket is shown screwed on both pipe ends, and at *d* a similar socket piece of lighter form.

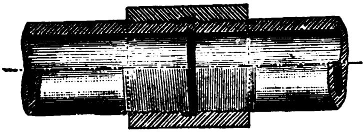


Fig. 21

A very good form of joint is shown in fig. 21, where security against leakage under great pressure is necessary.

The two pipe ends are screwed with right and left-handed threads. The end of one pipe is turned with a flat face and the other with a sharp V-shaped edge. The socket or coupling piece then draws the ends together into metallic contact. A union joint, shown in fig. 22, may

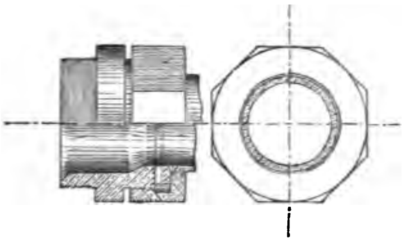


Fig. 22

also be used, with a packing ring of leather or india-rubber, if necessary.

25. *Boiler tubes.*—The mode of fixing boiler tubes in tube plates is shown in fig. 23. The lower figure shows an ordinary boiler tube, fixed at one end by slightly enlarging

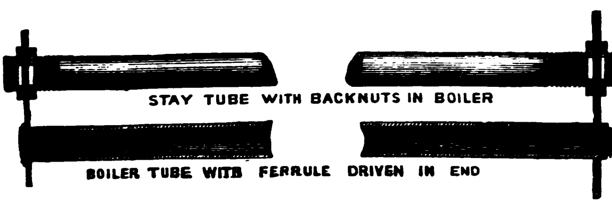


Fig. 23

the tube and riveting over the end. At the other end a wrought-iron ferrule is driven in. In American locomotives cast-iron ferrules have been used, and are said to be tighter than ferrules of wrought iron.

The ends of boiler tubes used to be expanded into the tube plate by a taper drift. Now a tube expander is always used. A taper mandril driven in forces out a set of rollers,

which enlarge the tube as the expander is turned round, and form a small shoulder inside the tube plate to resist the expansion lengthways of the tube when heated. The outer end of the tube is sometimes bell-mouthed a little to prevent the drawing out of the tube by the steam pressure.

The upper figure is a boiler tube adapted also to act as a longitudinal stay rod. For this purpose the ends are screwed and the tube fixed by a pair of thin nuts at each end. Sometimes one end of the stay tube is enlarged (fig. 1). This facilitates getting it into place.

Boiler tubes are exposed to an external pressure in fire-tube boilers, and to an internal pressure in water-tube boilers. For boiler tubes $2\frac{1}{2}$ to 4 ins. in diameter, and for pressures above 40 lbs. per sq. in., the thickness is about

$$t = \frac{6000}{d\,p} + \frac{1}{10}$$

Stay tubes are commonly $\frac{1}{4}$ in. thick to allow for cutting the screw threads. In locomotives, Muntz metal tubes were used at one time, now mild steel is more generally employed. These tubes are $1\frac{3}{4}$ to $2\frac{1}{2}$ ins. in diameter, and their length is about $4{\cdot}7\,d + 2$ in feet. They are spaced $\frac{3}{4}$ in. apart.

26. *Steam pipes.*—For gauge pressures up to 60 or 80 lbs. per sq. in., and in the case of pipes under 6 ins. diameter for greater pressures, cast iron pipes are still used, but they are heavy, liable to fracture by a blow, and rigid at bends. Copper steam pipes, once largely used, especially on board ship, have caused some serious failures. Now, important steam pipes are generally of mild steel, solid drawn up to 6 ins. in diameter, with flanges screwed on. Above that size welded pipes are used with flanges welded on for sizes up to 12 ins. diameter, and riveted on in larger sizes. The lap-welded joint has sometimes been found defective and a longitudinal covering strip is riveted over it by some engineers, but whether this really adds to the safety is doubtful. The bends for such pipes are of gunmetal.

The thickness of the pipes depends largely on manufac-

turing considerations. The following short table (MacCarthy, 'Proc. Inst. Mech. Eng.' 1896) gives ordinary practice :—

d Diameter, internal	Steam pressure p, lbs. per sq. in.			
	100	120	150	180
ins.				
6	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
9	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{5}{16}$
12	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{5}{16}$	$\frac{5}{16}$
16	$\frac{5}{16}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{3}{8}$
18	$\frac{5}{16}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{3}{8}$
Hydraulic test pressure	200	240	300	360

In no case probably should the stress exceed $2\frac{1}{2}$ tons per sq. in., or the thickness be less than $t = 0{\cdot}0001\,p\,d + {\cdot}04$.

Flanged joints for steel steam pipes.—Up to D = 12 ins. diameter of pipe, steel flanges can be welded on solid. The flange diameter is $D_f = 1{\cdot}25\,D + 4{\cdot}25$. The bolt circle diameter $D_b = 1{\cdot}2\,D + 2{\cdot}5$. The flange thickness $t = 0{\cdot}055\,D + 0{\cdot}6$. The number of bolts may be $n = 0{\cdot}66\,D + 4$, the nearest even number being taken, or better the nearest multiple of four. These dimensions are sufficient for pressures up to 200 lbs. per sq. in.

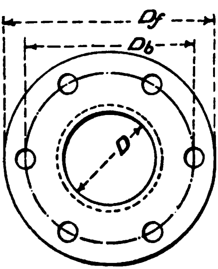


Fig. 24

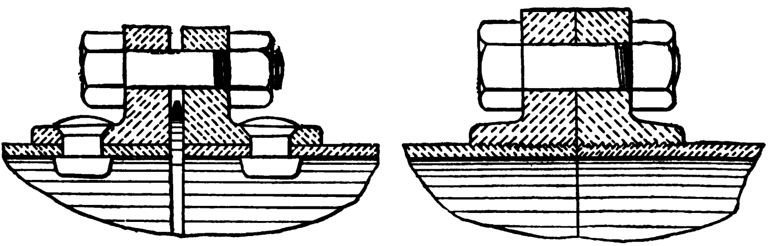
Let p be the steam pressure in lbs. per sq. in., n the number of bolts at a joint, D the

Diameter of pipe, internal	No. of bolts	Diameter of bolts
ins.		ins.
Up to $2\frac{1}{4}$	4	$\frac{1}{2}$ to $\frac{5}{8}$
$2\frac{1}{2}$ to 5	6	$\frac{5}{8}$ to $\frac{3}{4}$
6 to 8	8	$\frac{3}{4}$ to $\frac{7}{8}$
9 to 10	10	$\frac{3}{4}$ to $\frac{7}{8}$
11 to 12	12	$\frac{7}{8}$ to 1

internal diameter of pipe. Then the load due to steam pressure on each bolt is $Q = 0{\cdot}785\ D^2\ p/n$. Rules for the diameters of bolts are given in I., p. 194. But as there are bending and expansion stresses the bolts in practice are rather stronger, especially for the smaller sizes of pipe. The following rule agrees with practice for pipes up to 24 ins. diameter :—

$$d = 0{\cdot}011\sqrt{Q} + 0{\cdot}5.$$

For the larger steam pipes (fig. 25, *a*) the flanges are riveted on. The joints are sometimes made steamtight by a copper ring (fig. 25, *a*). The flange faces are machine finished,



a *b*

Fig. 25

and the joint made by a ring of triangular or rectangular section. With joints thus made pitting of the flange faces sometimes occurs from galvanic action, and the ring sometimes gets displaced by the expansion and contraction of the pipe. Fig. 25, *b*, shows a joint which may be made by scraping the flange faces true and interposing a thin layer of red lead and oil, or by leaving the flange faces machine finished and interposing a ring of asbestos paper soaked in oil.[1]

27. *Expansion joints.*—In steam and other pipes ex-

[1] See *Marine Engineering*, McKechnie, 'Proc. Inst. Mech. Eng.,' 1901.

posed to changes of temperature, provision must be made for expansion and contraction. In steam pipes a stuffing-box joint with gland is used, and provision is made against the drawing out of the pipe from the stuffing box by the action of the steam pressure. Neglect of this has led to serious accidents. In some cases a copper bend is introduced (fig. 26, *a*), and sometimes a copper corrugated pipe (fig. 26, *b*) But in both these cases, especially the latter, only a very small amount of movement of the pipe can be permitted without overstraining the metal of the bend or corrugated pipe.

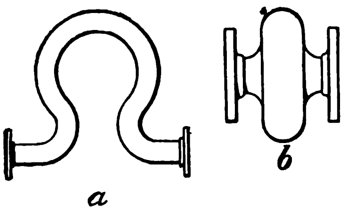


Fig. 26

28. *Condenser tubes.*—Condenser tubes are now generally made of brass. Sometimes they are tinned as a protection against the action of fatty acids. They are from ½ in. in diameter in small condensers to 1 in. in very large condensers and about 0·05 in. thick. They are fixed in the tube plate in different ways. Sometimes a simple soft wood ferrule is driven between tube and tube plate. The ends of the tubes are split and expanded to prevent creeping. Generally some very simple form of stuffing box is used, which can be easily and cheaply made. Perhaps the simplest method is that shown in fig. 27. The tube plate is drilled

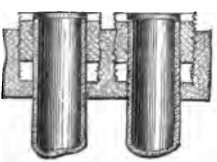


Fig. 27

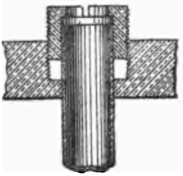


Fig. 28

with recesses round the tube ends which form stuffing boxes. The gland is simply a short piece of screwed tube cut off by a saw. The packing is a ring of tape. This method permits the free expansion of the condenser tube to occur without causing leakage. Fig. 28 shows a gland so formed as to prevent creeping of the condenser tubes.

CHAPTER II

ARRANGEMENT AND PROPORTIONS OF STEAM-ENGINE CYLINDERS

29. It is beyond the scope of this treatise to consider steam engines as machines for transforming heat into mechanical energy and the thermo-dynamic laws of the action of steam. Nevertheless, it is impossible to deal rationally with the details of steam engines unless the mechanical action of the steam in the cylinder is understood. The principal straining actions on which the dimensions of engine parts depend are due initially to the steam pressure on the piston, and the designer must be able to determine those straining actions, at least approximately, for any position of the mechanism. For a single-cylinder engine the problem is comparatively simple; but if a multiple-cylinder engine is used, then the problem is more difficult. Further, various problems in designing arise out of the action of the inertia of the moving parts, and cannot be attacked without a general knowledge of the arrangement of the mechanism of the engine. Hence it seems desirable to discuss, from a standpoint as purely mechanical as possible, the general arrangement and types of steam engines, as a basis for proportioning engine details.

30. *Arrangement of mechanism.*—(*a*) Beam engines, a type still retained in large pumping engines. In marine engines the beam engine took the form known as the side-lever engine, a type which has disappeared. (*b*) Ordinary direct-acting engines, with a connecting rod between the crosshead and crank. (*c*) Trunk engines, in which the connecting rod is attached direct to the piston inside a

trunk which forms the sliding guide. (*d*) Oscillating engines which kinematically are merely an inversion of the direct-acting engine, the connecting rod of the latter becoming the frame link of the former. (*e*) Return connecting-rod engines are a variety of type (*b*), in which space is saved by bringing the crank shaft close to the cylinder and placing the slide and cylinder on opposite sides of the crank shaft.

If s is the stroke and d the diameter of the cylinder, the length over all from centre of crank shaft to back of cylinder may be reduced to $3{\cdot}5s + 0{\cdot}6d$ in direct-acting engines ; $2{\cdot}6s + 0{\cdot}6d$ in trunk engines ; and $1{\cdot}6s + 0{\cdot}8d$ in return connecting-rod engines.

31. *Combined and compound engine arrangements.*—With simple engines it is least costly to use a single engine, and such an engine has the fewest working parts—a not inconsiderable advantage. But for various reasons a combined engine is often adopted. With two engines acting on cranks at right angles, and, still better, with three engines acting on cranks at 120°, the turning moment is much more uniform than with a single engine ; the cylinders are smaller in diameter and the working parts lighter. With a single engine there are two dead points in the revolution, and consequently positions in which the engine will not start by the action of the steam pressure. Pairs of simple engines may be placed parallel or inclined. The latter arrangement was at one time adopted because solid crank shafts with cranks at right angles were difficult to obtain, and the inclined or diagonal arrangement permitted both engines to act on a single overhung crank-pin, often on a crank arm keyed to the shaft.

Compound engines, or engines in which the steam expands successively in two cylinders, are of two general types : (1) the Woolf engine (or, more strictly, Roentgen engine), and (2) the Receiver engine. In the Woolf engine (*a*, fig 29) the steam passes as directly as possible from one end of the high-pressure to one end of the low-pressure cylinder.

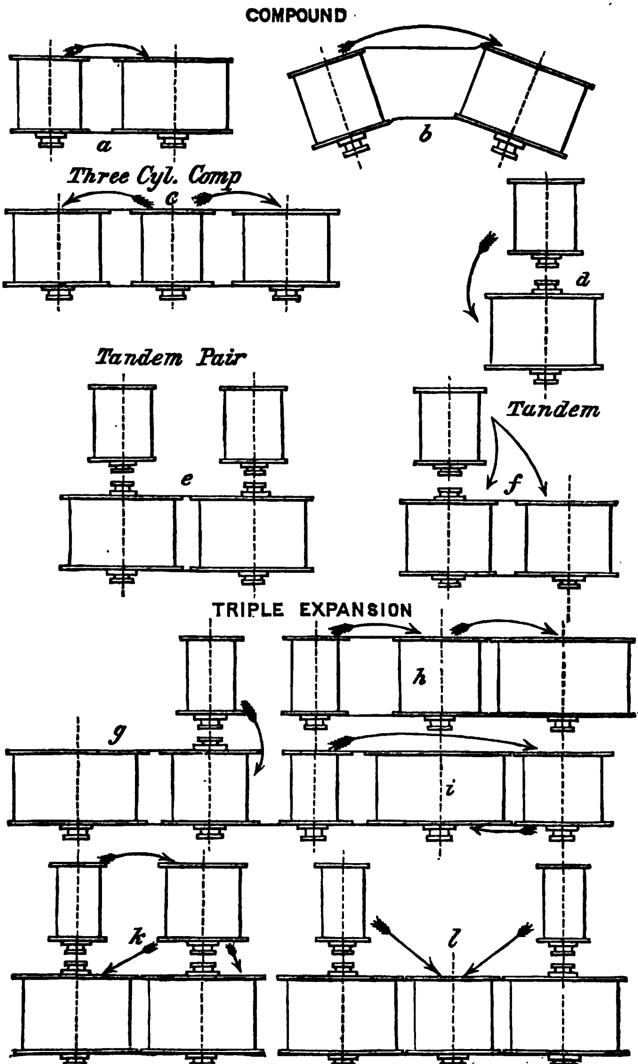

COMPOUND
a
b
Three Cyl. Comp
c
d
Tandem Pair
e
Tandem
f
TRIPLE EXPANSION
h
g
i
k
l

Fig. 29

In that case the cranks are usually at 180°. Some designers have attempted to reduce the action of the unavoidable passage between the cylinders by placing the cranks at 160° and allowing the HP cylinder to begin exhausting a little before the LP opens to steam. If the cylinders are placed tandem, as at *d*, there is only a single crank on which both pistons act. To get greater uniformity of twisting moment with a Woolf engine, a pair of tandem cylinders may be used, as at *e*. If in the arrangement *a* the cranks are at right angles, then there must be an intermediate reservoir into which the steam can expand till the LP cylinder opens. This is the simplest form of receiver engine. The arrangement *b*, with inclined cylinders at right angles, both pistons acting on a single crank, is an equivalent arrangement. In some cases it is convenient to divide the low-pressure cylinder into two cylinders, and then the arrangement *c* or *f* may be used.

The prejudicial action of the cylinder walls, causing condensation, increases rapidly with the range of temperature to which they are exposed. Hence, engineers have been driven, when using high pressures and high ratios of expansion, to carry the principle of compounding further, and to expand the steam in stages in three or even four successive cylinders. The simpler arrangements of triple-expansion engines are shown in fig. 29, at *g*, *h*, *i*, *k*, *l*.

32. *Working steam pressures.*—Of course no definite rule can be given, but the following are about the usual working boiler pressures used now in the most economical engines of given types :—

Type of Engine	Steam Pressure in lbs. per sq. in.	
	Non-condensing	Condensing
Simple	15	60
Compound	120	90
Triple	180	150
Locomotives, simple . . .	150	—
,, compound . .	180	—

At the present time the average (gauge) pressure for triple marine engines is about 180 lbs. per sq. in., and for quadruple engines 210 lbs. per sq. in. The maximum pressures used are about 250 lbs. per sq. in. with cylindrical boilers and 300 lbs. per sq. in. with water-tube boilers.

33. *Choice of type of engine.*—The selection of the type of engine for any given case requires judgment and experience. The dearer the coal in any given locality, the more reason there is for adopting an economical engine, which in general means somewhat greater cost and complication. The broad principle in deciding between two engines differing in cost is that, if the more costly is adopted, the saving in working expense should exceed the interest on the excess of cost, but it is not easy to apply this principle in all cases. Condensing engines are more economical than non-condensing engines, especially if the load is a variable one. Except for small engines under 100 H.-P. a compound engine, in spite of greater cost, is generally preferable to a simple engine. It is only in large engines and with high steam pressure that the use of triple expansion is justified in ordinary cases. Broadly, the higher the steam pressure the more economical is an engine, but it requires double or triple expansion to take full advantage of the highest pressures now adopted. The higher the piston speed the less is the cost of an engine for a given power; but there are limits of speed, depending on the type of engine and valve gear and on its size, which it is not desirable to exceed. The question of the relative advantages of long-stroke engines, making a moderate number of revolutions per minute, and short-stroke quick-revolution engines is difficult to decide. For direct driving of dynamos the quick-revolution engine is extremely convenient, and in this country is generally used. But there is much to be said on the ground of durability and ease of management for medium rotative speeds.

34. *Horse-power of engine.*—For any one engine

cylinder let d = diameter in ins.; A = piston area in sq. ins. $= \pi d^2/4$; s = the length of stroke in feet; N = the number of revolutions per minute; v = the piston speed in feet per minute $= 2sN$; p_m = the mean effective steam pressure during a stroke in lbs. per sq. in.; I.H.P. the indicated horse-power developed in the cylinder,—

$$\text{I.H.P.} = \frac{2 p_m A s N}{33{,}000} = \frac{p_m A v}{33{,}000}$$

In the case of a simple engine p_m is the m.e. pressure and A the piston area in the single cylinder. In the case of compound engines, the equation still applies if the I.H.P. is calculated for each cylinder separately. But it is usually more convenient to take for A the area of the LP cylinder and for p_m the mean effective pressure reduced to the LP cylinder. If p'_m, p''_m are the mean effective pressures of HP and LP cylinders, and A′ and A″ the corresponding piston areas (stroke and piston speed being the same), then the mean effective pressure reduced to LP piston is

$$p_m = p''_m + p'_m \frac{A'}{A''}.$$

In a preliminary determination of the size of cylinder a value of p_m must be assumed. This depends on the boiler pressure, the ratio of expansion, and the type of engine. Methods of determining p_m will be given later, but the following short table will be useful as a guide :—

	Mean effective pressure lbs. per sq. in.			
	Simple engine		Compound engine [1]	
Boiler Pressure	80	100	100	150
Non-condensing engine .	38	46	30	35
Condensing . . .	25	35	20	24

[1] In the case of the compound engine the mean effective pressures are those of HP and LP cylinders taken together and reduced to the LP piston.

The quantities in the products $A v$ or $A s N$ are settled with reference to—(1) the limit of piston speed permitted; (2) the number of revolutions required by the special work to be done; (3) the proportion between s and d which is usual for engines of given types. Overload is provided for, to the extent very commonly of one-third of the rated power, by arranging for a later cut off than in normal working.

In a compound engine, in which a greater ratio of expansion can be used than in simple engines, because of the less cylinder condensation, A is taken in the preliminary calculation as the area of the low-pressure piston, and the low-pressure cylinder is designed as if the engine were a simple engine. Then the proportions of the high-pressure cylinder are determined by making its volume such a fraction of the low-pressure cylinder volume as experience has shown to be desirable. The extent to which a compound engine will stand overloading depends on the HP cylinder volume and the lateness of cut off permitted in that cylinder.

35. *Rotative and linear engine speed.*—The term 'engine speed' is used rather ambiguously, sometimes meaning rotative speed (revs. p. m.), sometimes the linear piston speed (ft. p. m.). When all engines worked at moderate speeds, these were determined largely by considerations of local convenience and under a general belief that high speed involved rapid wear. For pumping engines, a moderate or rather slow piston speed is necessary to avoid hydraulic shock, and a low rotative speed is desirable also. With the introduction of electric machinery there has been a general demand for much higher *rotative* speeds, and this involves tolerably high piston speeds also. Increased accuracy of workmanship and attention to balancing has made engines of short stroke and quick rotative speed nearly as durable as slower engines, and such engines, which at first were only used for small powers, are now made of large size. With some valve gears there is a limit of rotative speed beyond which the action of the gear would be imperfect. Thus trip gears cannot be used for

speeds above 100 to 120 revs. p. m. Slide valves are heavy in large engines, and their inertia and friction introduce difficulty at high speeds, but with piston valves this difficulty is reduced. Vibration due to the inertia of the reciprocating parts is the greatest difficulty to be met in using high rotative speeds, but multiplication of cylinders and careful balancing reduces this difficulty.

The linear piston speed varies in different engines less than the rotative speed, and generally increases a little with the length of stroke. Let s = length of stroke in feet; v = piston speed in feet per min. Then,

$$v = a\sqrt[3]{s}$$

where a has the following values in different cases :—

Type of engine	$a =$
Direct-acting pumping engines (without flywheel)	80 to 120
Beam pumping engines	90 to 200
Horizontal Corliss engines	220 to 400
Horizontal compound mill engines	200 to 400
Small horizontal engines, ordinary	240 to 300
Short stroke, quick speed	400 to 550
Locomotive engines (highest speed)	800 to 1,000
Quick speed, short stroke, single-acting engines	550 to 650
Paddle marine engines	206
High-speed ocean steamers	500 to 550
Ocean passenger and cargo steamers	330 to 450

These figures give speeds of about 125 feet per minute in slow pumping engines; 240 to 360 in beam pumping engines; 300 to 450 in ordinary horizontal engines; 500 in single-acting quick-speed engines; 700 in marine screw engines; 1,000 in locomotives; that is, taking cases such as most ordinarily occur.

36. *Ratios of cylinder volumes in compound engines.*—

The cylinder volume is understood to mean the product of the area of piston and length of stroke or volume described by the piston in a single stroke. In proportioning the relative volumes of compound engine cylinders, designers have generally aimed at making either the range of temperature in each cylinder equal, or the effective work done in each cylinder equal. No very simple rule can be given to secure either result, because the clearance and receiver spaces affect the action of the steam so considerably. A means of testing any given arrangement will be described presently, meanwhile the following table of proportions usual in existing engines will be of service :—

ORDINARY RATIOS OF CYLINDERS IN COMPOUND AND MULTIPLE-EXPANSION ENGINES [1]

	Ratio of Cylinder Volumes			
				$\frac{LP}{HP}$
Compound				3 to 4
		$\frac{IP}{HP}$	$\frac{LP}{IP}$	$\frac{LP}{HP}$
Triple . . .		2 to 2·75	2 to 4	5 to 8
	$\frac{2nd}{1st}$	$\frac{3rd}{2nd}$	$\frac{4th}{3rd}$	$\frac{4th}{1st}$
Quadruple .	1·5 to 2	1·7 to $2\frac{1}{4}$	2 to 3	6 to 12

Clearly on theoretical grounds, and experience justifies the conclusion, the total ratio of expansion and the ratio of cylinder volumes should increase as the initial steam pressure increases. The most economical ratio of total expansion for any given type of engine depends on the extent to which by jacketing, high speed, or other means the cylinder condensation is checked. For engines in which

[1] Mr. McKechnie gives as the best practice at the present time, for triple-expansion engines working at 180 lbs. boiler pressure :—

$$IP/HP = 2{\cdot}76\,;\ LP/HP = 7{\cdot}55.$$

all ordinary arrangements for reducing cylinder condensation have been made, working at an initial absolute steam pressure p, the best ratio of expansion is $r = p/x$, where $x =$ 25 for non-condensing engines; 20 for simple condensing engines; 15 for compound condensing engines; and 12 for triple condensing engines.

37. *Construction and proportions of steam-engine cylinders.* A simple form of cylinder is shown in fig. 30. The valve chest is arranged for a slide valve, the steam and exhaust passages are cast in one with the barrel. The front end of cylinder and valve chest are closed by covers.

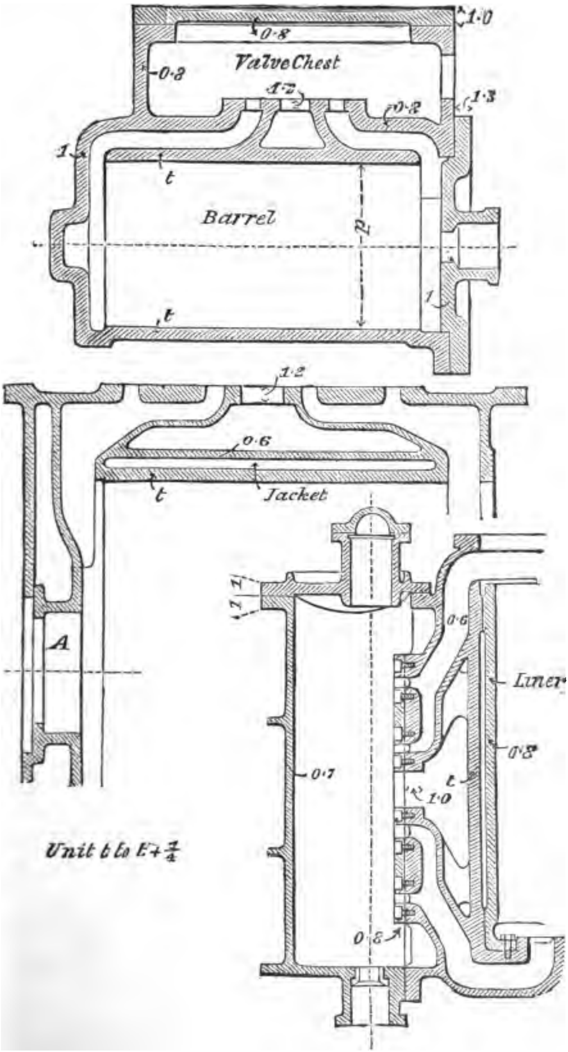
The thickness of the cylinder barrel must be determined so that it is (*a*) strong enough to resist the internal steam pressure; (*b*) rigid enough to prevent any sensible alteration of form; (*c*) it must be thick enough to insure a sound casting; and (*d*) thick enough to permit re-boring once or twice when worn. A rule has been already given (§ 6), which makes the thickness depend on the steam pressure. Generally other considerations than strength are of so much importance that the following empirical rule agrees better with practice:—

Thickness of barrel of cylinder

$$= t = 0{\cdot}02d + 0{\cdot}5 \text{ to } 0{\cdot}05d + 0{\cdot}5.$$

The flanges of the cylinder have a thickness $1{\cdot}3t$; the metal of the valve chest and passages $0{\cdot}7t$; valve chest flanges t; cylinder face in valve chest $1{\cdot}25t$. Sometimes a false face is used (fig. 32), screwed to the cylinder face with gunmetal screws. Then the cylinder face may have a thickness t, and the false face a thickness $0{\cdot}8t$ if of cast iron, and $0{\cdot}6t$ if of gunmetal.

Many engine cylinders have a steam jacket round the cylinder barrel, and this is either cast in one with the barrel (fig. 31), or formed by a liner inserted steamtight in the barrel (fig. 32). When cast in one with the barrel, the jacket is often omitted from the space below the steam passages,



Figs. 30, 31, 32

E 2

where it is shown in fig. 31. There is less danger of leakage if the jacket is cast with the cylinder barrel, but, of course, the moulding of the cylinder is more difficult. If the jacket is made by a liner, the liner may be of hard cast iron or forged steel. A cast-iron liner may have a thickness $0{\cdot}8t$, but a steel liner may have a thickness $pd/3000$. The liner may be fixed as shown in fig. 32, with a flange and studs at the front end of cylinder. At the back end it is turned to fit the barrel tightly, and a recess is formed packed with asbestos. Fig. 33 shows a liner held in place by the pressure of the cylinder cover.

The jacket is of great importance in slow engines with considerable expansion, as in pumping engines. Its importance diminishes as the size of the engine and the piston speed increase. In quick revolution short-stroke engines, and in large marine engines with high piston speed, the economy due to the jacket is small or may be zero.

The front end of the cylinder should be rigidly bolted to the engine framing. At the back end the bolt holes should be enlarged so that the cylinder can expand and contract. The expansion will not exceed 1-500th of the distance between the front and back bolts. The bolts may be calculated to resist the maximum pressure on the piston with a stress not exceeding 3,000 to 4,000 lbs. per sq. in.

When the cylinder covers are not very large they have a single thickness and are stiffened outside by radiating ribs. Larger covers are made hollow, with a double thickness of metal and stiffening ribs between. Ribs on a cover strengthen it when the pressure is on the ribbed side and the ribs are in compression. They may in some cases rather weaken than strengthen it when the pressure is on the plain side and the ribs are in tension.

The cylinder cover bolts or studs are never less than $\frac{3}{4}$ in. in diameter in ordinary practice. The number is

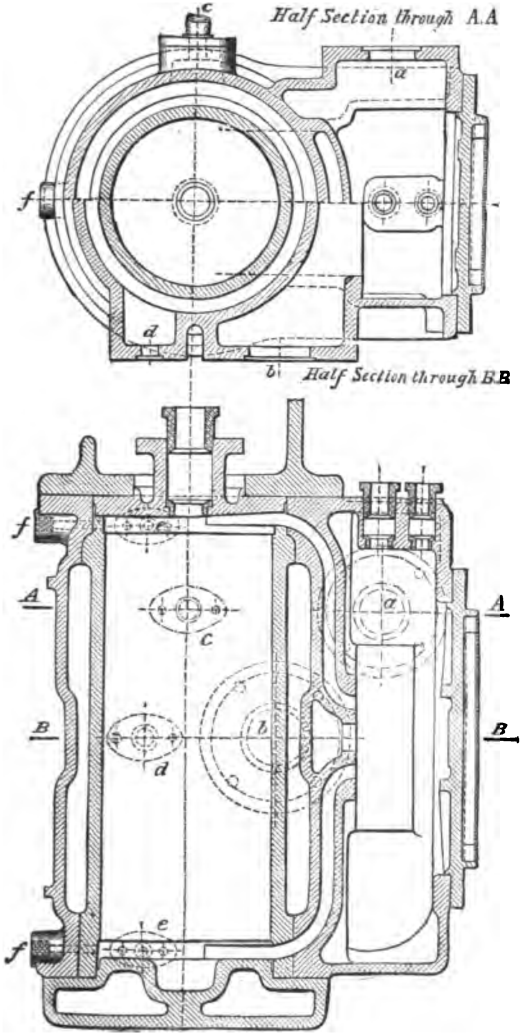


Fig. 33

a, Steam pipe; *b*, Exhaust; *c*, Jacket steam pipe; *d*, Jacket drain; *e*, Cylinder drain; *f*, Indicator.

about 0·7 *d*, where *d* is the cylinder diameter in ins. The nearest even number should be taken. The bolts may be

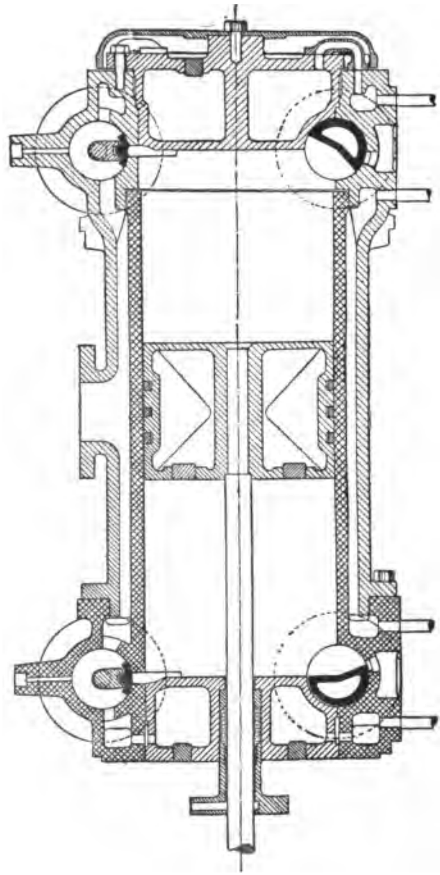


Fig. 34

calculated for a working stress of 4,000 lbs. per sq. in. of net section at the bottom of the threads, which allows a

margin against the shocks due to priming. Let Q be the load on a bolt in a cylinder cover in lbs., p the steam pressure in lbs. per sq. in., t the flange thickness in ins. Then the pitch of bolts should be about $40 \sqrt{(t/p)}$, and the bolt diameter should be $\delta = 0{\cdot}016 \sqrt{Q} + 0{\cdot}125$. For large bolts a somewhat greater working stress may be permitted.

The back end of large cylinders is cast with an aperture (A, fig. 31) to admit the boring bar, and this is closed by a cover.

38. *Jacketed cylinder with Corliss valve passages.*—When a cylinder and jacket of at all complicated form are cast in one piece there is risk of fracture from unequal expansion. In the case of cylinders with Corliss valve passages at the cylinder ends, the passages at the upper end in casting are liable to be spongy or unsound. Messrs. Bollinckx, of Brussels, have devised the method of casting the cylinder in two parts shown in fig. 34. For each part the valve passages are at the bottom of the mould in casting. The castings are much simpler than those of a cylinder and jacket, and harder iron can be used. At the same time the joints between jacket and cylinder can be made perfectly steam-tight, while at one end slight relative movement is possible. The hollow covers are connected by pipes with the jacket.

39. *Cylinder ports and steam passages.*—Let A be the area of piston and a the area of steam passage, in sq. ins.; s = the stroke in feet and N = the revolutions per min. of the engine. Then the mean piston speed is

$$V = S N / 30 \text{ ft. per sec.}$$

and the mean velocity of the steam in the steam passage is conventionally taken to be

$$u = V A / a \text{ ft. per sec.}$$

There is an error here arising from the fact that the flow of steam through the passage occurs usually during part only of the stroke, while the mean piston speed is reckoned

for the whole stroke. However, proceeding as is common in practice, the velocities allowed when u is calculated in this way are : in ports used alternately for admission and exhaust, $u = 80$ to 90 ; steam ports, $u = 90$ to 120 ; exhaust ports, $u = 70$ to 90 ; steam passages, $u = 100$; main steam pipe, $u = 100$ to 120 ; exhaust pipes, $u = 65$ to 70.

Generally the steam used per hour is known, and from this the weight of steam w used per stroke can be calculated. Also, when the cut-off is known, the time t in seconds during which steam flows in each stroke through the passage. If p is the absolute steam pressure in lbs. per sq. in., the volume per lb. is very nearly $v = 329/(p^{0·94})$ cubic feet. Hence the real mean velocity of steam in the passage is

$$u = \frac{w}{t} \cdot \frac{329}{p^{0·94}} \cdot \frac{144}{a} \text{ ft. per sec.}$$

40. *Diameter of steam pipe for a given flow of steam in lbs. per hour.*—It is common to reckon steam supplies in lbs. per hour. If the flow is x lbs. per hour, it is $x/3{,}600$ lbs. per second. The volume of x lbs. per hour is $(329\,x)/(3{,}600\,p^{0·94})$ cubic feet per second. Let d be the internal diameter of a steam pipe in inches, and v the velocity of the steam in feet per second. Then the volume of flow is

$$\frac{\pi}{4} \cdot \frac{d^2}{144} \text{ v cubic ft. per sec.}$$

$$\frac{\pi}{4}\,\frac{d^2}{144}\,\text{v} = \frac{329\,x}{3{,}600\,p^{0·94}}$$

$$\left.\begin{aligned} d^2 &= 16·76\,\frac{x}{\text{v}\,p^{0·94}} \\ x &= ·0597\ \text{v}\,d^2\,p^{0·94} \end{aligned}\right\}$$

Or if v = 100 ft. per sec.

$$\left.\begin{aligned} d &= 0·4094\,\sqrt{\left(\frac{x}{p^{0·94}}\right)} \\ x &= 5·97\,d^2\,p^{0·94} \end{aligned}\right\}$$

Written for logarithmic calculation, these formulæ become—

$$\log d = \bar{1}{\cdot}6121 + \tfrac{1}{2}(\log x - 0{\cdot}94 \log p)$$
$$\log x = 0{\cdot}7760 + 2 \log d + 0{\cdot}94 \log p.$$

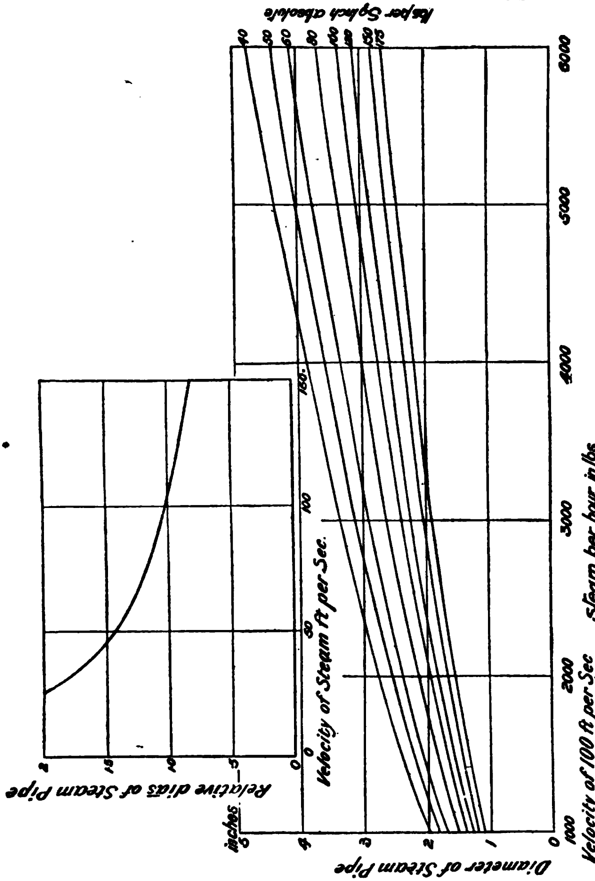


Fig. 35

The author is indebted to Messrs. James Simpson & Co. for the very convenient diagram (fig. 35) which gives

the diameter for any flow directly for a velocity of 100 feet per second. For other velocities a factor can be found from the subsidiary diagram which, multiplied by the diameter for 100 feet per second, gives the diameter for other velocities. To extend the use of the diagram, for twice any given diameter, the flow is four times as great; or if the diameter is found for one-quarter the required flow, the diameter required will be double that diameter.

41. *Clearance.*—Clearance distance must be left between the piston at the end of the stroke and the cylinder covers as a security against the displacement of the piston by the wear of its connections. The distance in inches left between the piston and cylinder cover at each end of the stroke may vary from $\frac{1}{8} + \frac{n}{15}$ in small engines to $\frac{1}{4} + \frac{n}{10}$ in large engines, where n is the number of joints subject to wear between piston and crank shaft.

The clearance space has an important influence on the action of the steam, and as the steam passages form part of the clearance space (regarded as space occupied by steam during expansion), they are reckoned with the cylinder clearance in estimating the clearance volume, or, as it is usually termed, simply, the clearance. The clearance volume is most conveniently given as a fraction or percentage of the volume described by the piston in a single stroke.

Clearance Volume

Simple engines	Percentage of Cylinder volume
Corliss valves	2 to 4
Double-beat valves	5 to 7
Long slide with short passages	1·8 to 2·7
Ordinary slide	6 to 12
Piston valves	7 to 15

Compound	HP	LP
Woolf with long slide and short passages	3 to 4	2 to 3
Ordinary slides . . .	7 to 12	5 to 10
Gridiron slides . . .	5	3

Triple expansion	HP	IP	LP
Slide valves .	10 to $12\frac{1}{2}$	5 to 10	5 to 7

Receiver space.—In compound engines when the cranks are at 90° the receiver volume between the high- and low-pressure cylinders is usually from one to two times the volume of the high-pressure cylinder. In slow engines the receiver is generally jacketed or formed into a reheater by having pipes in it supplied with boiler steam.

42. *Hydraulic press cylinders.*—When the thickness of the cylinder is not small compared with the radius, the following equation must be used (Part I. p. 56):—

$$t = \frac{d}{2}\left\{-1 + \sqrt{\frac{3f + 2p}{3f - 4p}}\right\}$$

where t is the thickness of metal in the cylinder; d its internal diameter; f the safe working stress on the material, and p the excess of internal over external pressure.

Steel . . .	$f = 15{,}000$
Wrought iron . . .	12,000
Cast iron . . .	5,000

In many cases p is fixed by the conditions of the case. Then if the total effort of the ram P is also fixed,

$$\frac{d}{2} = \sqrt{\frac{P}{p\pi}},$$

and then t is ascertained by the equation above. Hermann has shown that there is a value of p for which the external diameter of the press cylinder is least, and obviously this

makes the construction of the press cheap. Let D be the external diameter of the press. Then

$$D = d + 2t = d\sqrt{\frac{3f + 2p}{3f - 4p}}$$

Let $x = p/f$.

$$\frac{D^2}{d^2} = \frac{3f + 2p}{3f - 4p} = \frac{3 + 2x}{3 - 4x}$$

and putting $d^2 = 4P/\pi p$.

$$\frac{D}{2} = \sqrt{\frac{P}{\pi p}}\sqrt{\frac{3 + 2x}{3 - 4x}}$$

$$= \sqrt{\frac{P}{\pi f}}\sqrt{\frac{3 + 2x}{x(3 - 4x)}}.$$

This will be a minimum for $x = p/f = 0{\cdot}336$. In other words, with the above values of the working stress, the working pressure should be 5,040 lbs. per sq. in. for steel; 4,030 for wrought iron, and 1,700 for cast iron. In any case, if $x = 0{\cdot}336$,

$$D = 1{\cdot}5d \text{ nearly.}$$

CHAPTER III

INDICATOR DIAGRAMS

By using a steam-engine indicator on any actual engine a curve is drawn, the co-ordinates of which are the pressure and volume of steam in the cylinder at each point of the stroke. For a complete revolution the curve is a closed curve, the area of which is proportional to the work done by the steam on one side of the piston. In double-acting engines a corresponding diagram gives the work done on the other side of the piston. The mean vertical width of the diagram is called the mean effective steam pressure. In a multiple-expansion engine there will be a diagram (if single acting) or a pair of diagrams (if double acting) from each cylinder.

Simple Engines

43. *Indicator diagrams for single-expansion engine.*—Fig. 36 shows a pair of indicator diagrams as ordinarily taken in succession on one card from the front and back end of a simple engine. The atmospheric line is drawn by the indicator pencil, when the diagram is taken. Pressures measured from the atmospheric line to the diagram curves are pressures above (or below) atmospheric pressure, or are gauge pressures. A line below the atmospheric line at a distance p_a = to the barometric pressure[1] at the time is the line of zero pressure, and pressures measured from this line are absolute pressures. During the *outstroke* the *forward*

[1] If h is the height of the barometer in (mercury) inches, then $p_a = 0{\cdot}4912h$ is the barometric pressure in lbs. per sq. in.

pressure per sq. in. of piston is given by the ordinates, measured from *o o*, of the line *a b c*. During the return instroke, the *back pressure* against the piston is given by the ordinates of *c e f*. Similarly for the other diagram. The vertical scale of pressures per sq. in. is that of the indicator spring used. But a scale may be drawn such that ordinates measured by it are the total loads on the piston

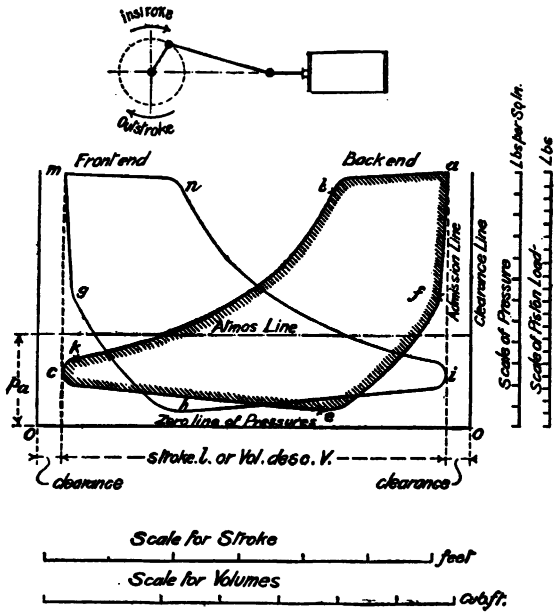


Fig. 36

in lbs. The length representing 1 lb. per sq. in. on the former represents A lbs. on the latter, if A is the piston area in sq. ins.

The horizontal length of the diagram represents either the piston stroke *l* or the volume $V = A\,l$ described by the piston in one stroke. If A is in square feet and *l* in feet, then the length which represents 1 foot on the stroke scale

represents $A\,l$ cubic feet on the volume scale. The cycle represented by the diagram consists of an admission period, $f\,b$; expansion period, $b\,k$; exhaust period, $k\,e$; and compression period, $e\,f$.

In every engine there is a clearance space at each end of the cylinder, comprising the space between piston and cylinder cover at the end of a stroke and the volume of the steam passage. The clearance may be expressed either as a volume $c\,V$ in cubic feet or as an equivalent length of stroke $c\,V/A$ feet. c is a fraction, values of which for different types of engines are given in § 41. If the clearances are set off at the ends of the diagrams and vertical lines drawn, we get zero lines from which to measure horizontally the volume of steam in the cylinder at any point of the stroke. The left-hand line relates to the front-end diagram, and the right-hand line to the back-end diagram.

The following points can be more or less exactly fixed on the diagram. The cut-off point b, at which expansion begins; the release point k, at which expansion ends and exhaust begins; the cushion point e, at which exhaust ends and compression begins; lastly, a point f, at which admission begins. For two periods of the stroke, expansion and compression, the cylinder is closed, and the volume of saturated steam in the cylinder expanding or being compressed is the horizontal distance from the curve to the clearance line. Water in the cylinder occupies no appreciable volume, and therefore is not shown by the diagram. During the exhaust and admission periods, the cylinder is open to the condenser (or atmosphere) and the steam chest.

44. *Horse-power from the diagrams.*—The area of a diagram represents the work done on one side of the piston in a revolution. If on the diagram a ins. represent 1 lb. per sq. in. and b ins. represent a cubic foot, then $a\,b/144$ sq. ins. represent 1 foot-pound. It is more convenient to proceed thus. The mean height of a diagram, most easily found by a planimeter, is termed the mean effective pressure.

Let p_m be the mean height or mean effective pressure of the two diagrams in lbs. per sq. in. and A the mean effective area of front and back of piston, after deducting the area of piston or back rod, in sq. ins. Then p_m A is the mean effective steam effort on the piston in lbs. If N is the number of revolutions per minute and s the stroke in feet, $2 N s$ is the piston speed in feet per minute. Then the indicated horse-power is

$$\text{I.H.P.} = \frac{2 p_m \text{A N} s}{33{,}000}$$

45. *Changing the scales of an indicator diagram.*—It is frequently necessary to redraw a diagram to given scales of

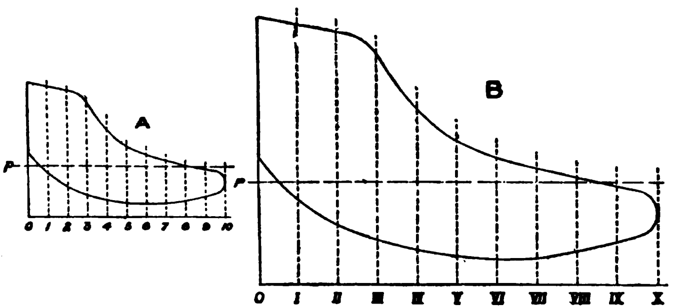


Fig. 37

pressure and volume. Divide the given diagram A (fig. 37) into ten strips of equal width by verticals. In B take O X equal to the volume described by piston in one stroke on the required scale of volumes and divide it into ten equal parts and draw verticals. Set off O P = atmospheric pressure on the required scale of pressures, and draw the atmospheric line. Then the heights at each vertical above and below the atmospheric line of the given diagram, measured on the scale of the diagram, may be set off on the corresponding ordinate of the required diagram to the required scale of pressures. The diagram can be sketched in through points

so found. Sometimes the mean heights from one or more pairs of diagrams may be taken to find a diagram representing the average of the pressures on the two sides of the piston.

46. *Steam effort on the piston.*—At any moment the effort transmitted to the crosshead due to the steam pressure is the difference between the forward pressure on one side of the piston and the back pressure on the other side. Thus during the outstroke the piston effort is the vertical intercept between *a b c* and *g h i*, fig. 36; similarly, during the instroke it is the vertical intercept between *m n i* and *c e f*.

If these intercepts, measured to the scale of piston loads

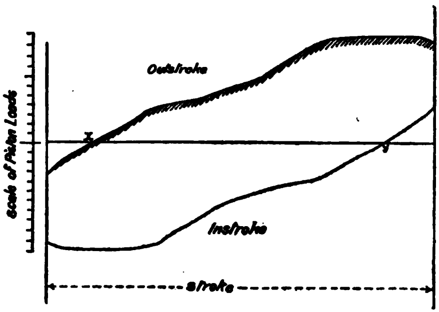


Fig. 38

in lbs., are set off on a line representing the stroke, upwards for the forward effort during the outstroke, downwards for the backward effort during the instroke, we get such a diagram as fig. 38. In consequence of compression the effort changes sign at points *x y* before the end of the stroke, and this change of direction of the effort, unless gradual, may give rise to a knock of the loose joints of the mechanism.

So far only the effort due to the steam pressure has been considered. It will be seen presently that these curves of effort are much modified by the inertia forces due to the weight of the reciprocating parts.

47. *Expansion and compression curves.*—Let p be the absolute pressure in lbs. per sq. in., and v the volume in cubic feet of a pound of gas or vapour. Then, if the expansion is isothermal, $p\,v =$ constant and the expansion curve is a common hyperbola. If the expansion is adiabatic, then $p\,v^{\gamma} =$ constant, where γ is a constant for any given fluid. For air $\gamma = 1{\cdot}41$. For steam the value of γ depends on the initial condition of dryness of the steam. If the dryness fraction of the steam is x, so that a lb. contains x lb. of steam and $(1 - x)$ lb. of water, the value of γ has been found by Zeuner to be,

$x =$	$1{\cdot}0$	$0{\cdot}9$	$0{\cdot}8$	$0{\cdot}7$
$\gamma =$	$1{\cdot}135$	$1{\cdot}125$	$1{\cdot}115$	$1{\cdot}105$
Approximately $\gamma =$	$\frac{17}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{11}{10}$

If steam initially dry and saturated expands so as to remain dry and saturated, $p\,v^{1{\cdot}065} =$ constant, or approximately $p\,v^{\frac{17}{16}} =$ constant. The expansion curve in this case is termed a saturation curve.

48. *Construction of expansion and compression curves.*—On a diagram the ordinates of which are pressure and volume, the curves of expansion (or compression) of a definite weight of gas or vapour are in a number of cases curves of the hyperbolic class defined by the equation $p\,v^{n} =$ constant, where the index n is a constant depending on the conditions of the expansion. The curves have the axes of pressure and volume for asymptotes. The actual expansion curve on a steam-engine indicator diagram approximates to a curve of this kind, although the weight of vapour in the cylinder varies during the period of expansion from condensation and re-evaporation.

The simplest of these curves is the rectangular or common hyperbola corresponding to the equation $p\,v =$ constant. The axes of pressure and volume $o\,y$ and $o\,x$ are asymptotes to the curve. Let any point a (fig. 39) be given corresponding to $p_1\,v_1$, and let it be required to find

another point. Take any point c on the horizontal through a. Join $o\,c$ cutting the vertical through a in d. Draw $d\,b$ horizontal, cutting the vertical from c in b. Then b is a point on the hyperbola. Similarly b_1 on the vertical through c_1 and the horizontal through d_1 is another point on the hyperbola. The ratio $v_2/v_1 = r$ is the ratio of expansion of volume. It is a property of the curve that the

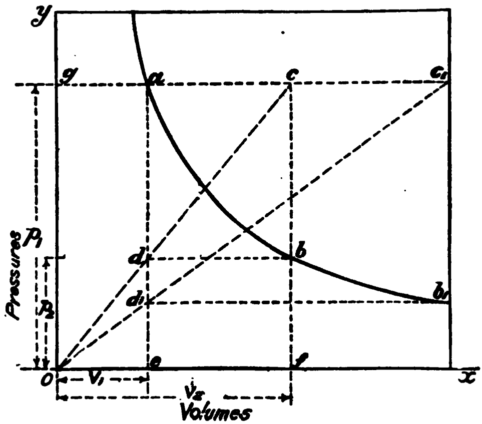


Fig. 39

area under any portion of the curve, such as the area $e\,a\,b\,f$, is given by the relation :—

$$\text{Area} = p_1\,v_1 \text{ hyp log } r = 2{\cdot}3\,p_1\,v_1 \text{ com log } r,$$

where p_1 is the absolute not the gauge pressure. The whole area $o\,g\,a\,b\,f$, including the admission area $o\,g\,a\,e$, is

$$\text{Area} = p_1\,v_1\,(1 + \text{hyp log } r)$$
$$= p_1\,v_2\,\frac{1 + \text{hyp log } r}{r}$$

$r =$	2	4	6	8	10	12	15
$\dfrac{1 + \text{hyp log } r}{r} =$	·847	·597	·465	·385	·330	·290	·247

When the equation to the curve is $p v^n =$ constant and n is not unity, the construction of the curve is a little more complicated. Since $p_1/p = v^n/v_1^n$, it follows that if a series of values of v be taken in geometrical progression the corresponding values of p will also be in geometrical progression. Thus let the values of v be v; $v_1 = k v$; $v_2 = k v_1 = k^2 v$; . . . . . Then the corresponding values of p are p; $p_1 = p/k^n$; $p_2 = p_1/k^n = p/k^{2n}$; . . . . To draw the expansion curve, in fig. 39, let m be a point the ordinates of which are p and v. Choose any value for k and calculate

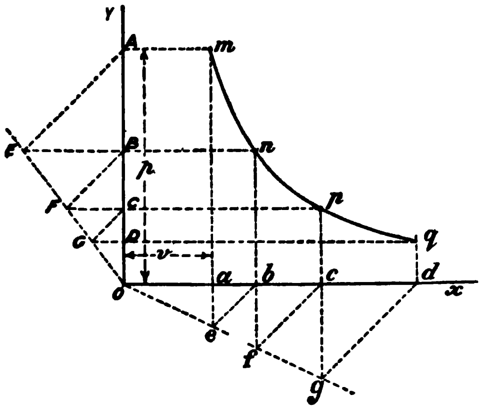


Fig. 40

values of $v_1 = k v$ and $p_1 = p/k^n$. Let n (fig. 40) be the point corresponding to p_1 and v_1. Then other points can be found by a graphic construction. Through b draw a line at 45° with O x, cutting the vertical through m in e, and draw the line O e g. At f on the vertical, through n, draw fc parallel to $e b$; through g on the vertical through c, draw gd parallel to $e b$; then O a, O b, O c, O d are the values of v, v_1, v_2, v_3 . . . in geometrical progression. Draw AE at 45° with OY, cutting the horizontal through n in E, and draw the line OE. Then, if BF is drawn parallel to AE and FC

horizontal, C G parallel to A E and G D horizontal, O A, O B, O C, O D will be the values of p, p_1, p_2, p_3 . . . corresponding to the values of v, v_1, v_2 . . . and m, n, p, q will be points on the expansion curve.

By this method it is easy to draw the adiabatic $p\,v^{1\cdot4} =$ constant, for air; or the approximate adiabatic for steam initially dry, $p\,v^{1\cdot135} =$ constant. Also the approximate saturation curve for steam $p\,v^{1\cdot0646} =$ constant. This last curve, however, is more easily constructed from a table giving the corresponding pressures and specific volumes of saturated steam.

The area $a\,m\,q\,d$ under any part of the curve, if the expansion is from O $a = \mathrm{v}_1$ to O $d = \mathrm{v}_2$, the corresponding absolute pressures being p_1, p_2, is—

$$\text{Area} = \int_{\mathrm{v}_1}^{\mathrm{v}_2} p\,d\,\mathrm{v}$$

But since $p\,\mathrm{v}^n = p_1\,\mathrm{v}_1{}^n$, then $p = p_1\,\mathrm{v}_1{}^n/\mathrm{v}^n$.

$$\text{Area} = p_1\,\mathrm{v}_1{}^n\,\frac{\mathrm{v}_2{}^{1-n} - \mathrm{v}_1{}^{1-n}}{1 - n}$$

and if $\mathrm{v}_2/\mathrm{v}_1 = r$, the ratio of expansion,

$$\text{Area} = p_1\,\mathrm{v}_1\,\frac{1 - r^{1-n}}{n - 1}$$

$$= \frac{p_1\,\mathrm{v}_1 - p_2\,\mathrm{v}_2}{n - 1}$$

The whole area of the diagram O A $m\,q\,d$, including the admission area O A $m\,a$, is—

$$\text{Area} = p_1\,\mathrm{v}_1\frac{n - r^{1-n}}{n - 1} = \frac{n\,p_1\,\mathrm{v}_1 - p_2\,\mathrm{v}_2}{n - 1}.$$

Weight of steam per I.H.P. *per hour.* Non-condensing simple engines under 100 I.H.P. use from 28 to 50 lbs. of steam per I.H.P. per hour, when working with full load. Similar engines compound use 20 to 28, or if under 20 H.P.

25 to 35. Simple condensing engines of 50 I.H.P. and upwards use 15 to 22. When compound 12 to 20 and when triple 11 to 13. Slow condensing engines for pumping use about 20 to 22 when simple, 14 to 20 when compound, and 12 to 18 when triple. The weight w per stroke can be deduced from these data when the dimensions of the engine are known.

49. *Standards of comparison for engines.*—It is convenient in considering the degree of efficiency reached in engines to have some ideal standard of comparison. Various standards have been used. First, assume that an ideal engine uses steam initially dry, that its expansion curve is adiabatic, and that it works without clearance, compression, or exhaust waste. Further, as in actual engines the expansion is always for practical reasons incomplete—that is, not carried down to the back pressure—let the ideal engine have a cylinder of the same volume V as that which is to be compared with it.

Let p_B be the initial absolute boiler pressure (or, better, if it is known, the pressure at the engine stop valve), v the volume in cubic feet of 1 lb. of steam at p_B; V the volume described by the piston per stroke; w the weight of steam in lbs. used per stroke; p_b the mean back pressure (or, better, the pressure due to the temperature in the exhaust pipe). p_b is about $15\frac{1}{2}$ to 16 lbs. per sq. in. for a non-condensing and $1\frac{1}{2}$ to 2 lbs. per sq. in. for a condensing engine.

From these data, taken from any actual engine, the indicator diagram of the ideal engine described above can be drawn.

In the ideal engine the volume of steam admitted per stroke is $w\,v$ cubic feet. Set off (fig. 41) OB $= p_B$; BC $= w\,v$. Then C is the cut-off point in the ideal engine, and $r = \mathrm{V}/w\,v$ is the expansion ratio in the ideal engine. If no heat is received or lost, the expansion curve CD is an adiabatic which can be drawn by the method already

explained. Lastly, draw the back pressure line E F, corresponding to the back pressure p_b. Then B C D E F is the indicator diagram of the ideal engine. Its area is the work W which would be done by the ideal engine per stroke, and this is given very approximately thus :—

$$\text{Area of diagram} = \text{OBCG} + \text{GCDE} - \text{OAEF}$$

$$= W = p_B\, wv + p_B\, wv\, \frac{1 - r^{1-n}}{n-1} - p_b\, V$$

$$= V \left\{ \frac{p_B}{r} \left(\frac{n - r^{1-n}}{n - 1} \right) - p_b \right\} \quad . \quad . \quad (1)$$

where if the pressures are taken in lbs. per square foot, and the volumes in cubic feet, the result is in foot-pounds. n is

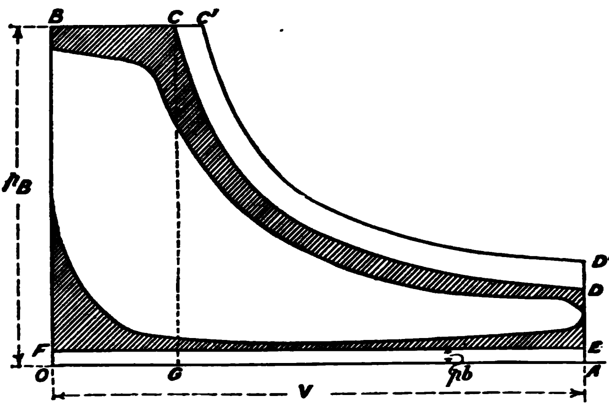


Fig. 41

the index of the adiabatic expansion curve, which for steam initially dry is approximately 1·135, or $1\frac{1}{8}$ nearly.

The actual diagram may be superposed on the ideal diagram by making the atmospheric and admission lines coincide. The superposition is most easily made by dividing the actual diagram length into ten parts and transferring the pressures above and below the atmo-

spheric line at each division to corresponding ordinates of the ideal diagram. It will be found that the actual diagram often occupies only 0·5 to 0·6 of the area of the ideal diagram, the difference (shaded) representing work lost from various causes, the chief being condensation in the cylinder. The loss due to the compression is not wholly a loss, because if the engine worked without compression the expansion curve would be lowered. The diagram has been drawn irrespectively of the jacket steam. If w lbs. of steam are used in the cylinder per stroke, and w_j lbs. in the jacket, then $(w + w_j)v$ is the total volume of steam used per stroke. Take $BC' = (w + w_j)v$ and

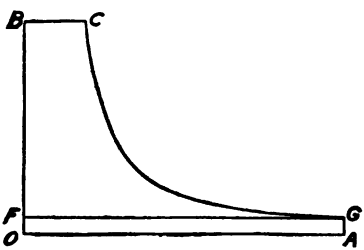


Fig. 42

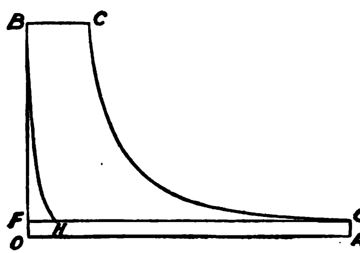


Fig. 43

draw the adiabatic $C'D'$. Then $OBC'D'EF$ is the diagram of an ideal engine using $w + w_j$ lbs. of steam per stroke.

50. *Other Standards. Rankine and Carnot ideal engines.*—As a standard of comparison for the thermal efficiency of actual steam engines, either a Carnot or a Rankine cycle has been assumed. The Rankine cycle is like that assumed above, except that the adiabatic expansion curve CG is supposed to be prolonged till the terminal pressure is equal to the back pressure (fig. 42). In the Carnot cycle, besides complete adiabatic expansion to the back pressure, it is assumed that there is a final adiabatic compression HB to the initial pressure (fig. 43).

Let T_B be the absolute steam temperature correspond-

ing to the initial pressure p_B, and T_b that corresponding to p_b. Let L_B be the latent heat or heat of vaporisation of 1 lb. of steam at T_B in thermal units. Then for a Rankine engine using w lbs. of steam per stroke the work per stroke in foot-pounds is

$$W = 778\, w\left\{\left(T_B - T_b\right)\left(1 + \frac{L_B}{T_B}\right) - T_b \text{ hyp log } \frac{T_B}{T_b}\right\} \quad \ldots (2)$$

In a Carnot engine using w lbs. of steam per stroke

$$W = 778\, w\, L_B \frac{T_B - T_b}{T_B} \quad . \quad . \quad . \quad . \quad (3).$$

The thermal efficiency of the Carnot engine is the greatest possible for the given temperature limits, and the simplicity of eq. 3 is one reason for using this engine as a standard.

Example.—Suppose the following data are assumed. $p_B = 150$ lbs. per sq. in.; $p_b = 2$ lbs. per sq. in.; $T_B = 819°$; $T_b = 587°$; $T_B - T_b = 232°$; v = specific volume at $p_B = 3{\cdot}011$ cubic feet; $V = 15{\cdot}055$ cubic feet; $r = 5$; $L_B = 861{\cdot}2$ Th. U. Then for an engine using 1 lb. of steam per stroke, the work done per stroke is

By Eq. 1,	154,794	ft.-lbs.
Eq. 2, Rankine cycle,	218,360	,,
Eq. 3, Carnot cycle,	189,800	,,

The quantity of heat expended per stroke would be for the Carnot engine L_B Th. U., and for the others $L_B + T_B - T_b$ Th. U. Hence the absolute thermal efficiencies would be for the ideal engine with incomplete expansion 0·1820, for the Rankine engine 0·2567, and for the Carnot engine 0·2833. In any actual engine the amount of work done per lb. of steam will be considerably less than in either of the ideal engines, as is to some extent shown by the difference of area of the actual and ideal diagrams in fig. 47.

51. *Construction of approximate indicator diagrams as a step in engine designing.*—The actual diagrams from an

engine give the means of determining the actual straining actions on the mechanism at all points of a revolution. Conversely, if in designing an engine an approximate indicator diagram is first drawn corresponding to the intended steam pressures and ratio of expansion, then the straining actions to be provided for in designing the mechanism can be very approximately determined. Further, there are questions of steam distribution, of variation of turning effort, &c., as to which useful information can be obtained by first constructing such approximate indicator diagrams.

The expansion and compression curves in actual engines do not differ in any material way from rectangular hyperbolas. This is not at all because the expansion or compression is really isothermal—it is very far from being isothermal—but the condensation and subsequent evaporation of steam on the cylinder walls makes the expansion curve more or less approximately hyperbolic. For the purpose of investigating the straining actions during the stroke, at all events, it is quite accurate enough to assume a hyperbolic expansion curve, a curve very easily drawn.

52. *Nominal and real ratio of expansion.*—If steam is cut off at $1/r$ of the stroke in a cylinder of volume V and clearance volume cV the nominal ratio of expansion is r. But in fact a volume $c\mathrm{V} + \mathrm{V}/r$ of steam is in the cylinder at cut-off and expands to $c\mathrm{V} + \mathrm{V}$. Consequently the real ratio of expansion is

$$\rho = \frac{1 + c}{c + 1/r} = \frac{r(1 + c)}{cr + 1}.$$

53. *Provisional determination of mean effective pressure. Diagram factor.*—In designing an engine the ratio of cut-off $1/r$ and the absolute boiler pressure (or, better, the stop valve pressure) p_B, are usually predetermined. Then the probable mean effective pressure must be calculated, and from this the cylinder dimensions are determined Let p_b

be the back pressure, which may be taken at 16 lbs. for a non-condensing engine and at 2 or $2\frac{1}{2}$ lbs. for a condensing engine. Let V be any length taken to represent the cylinder volume, and cV the clearance volume, c being a fraction known from experience with similar engines. Consider an engine working without compression. Its diagram is found thus: Take OA, AB (fig. 44) to represent cV and V. On any scale of pressures set up AC = OH = p_B, AG = p_b. Take CD = AB/r. Draw the hyperbolic expansion curve DE, with OH, OB as asymptotes. Lastly, draw the

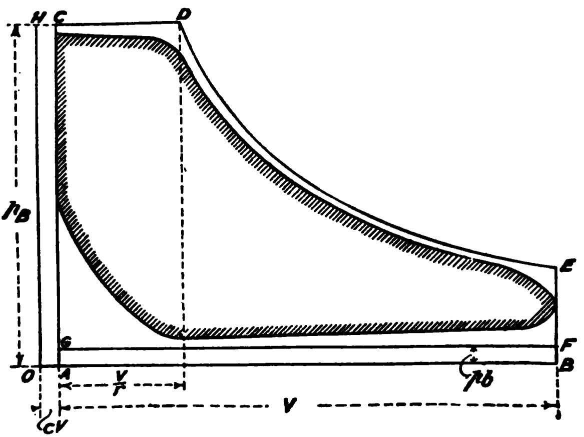


Fig. 44

back pressure line GF. Then CDEFG is the theoretical diagram for the conditions. The actual diagram will have some such form as the shaded figure, and its area will be less than that of the theoretical diagram in a ratio ϵ which may be termed the diagram factor, values of which will be given later.

Let ρ be the real ratio of expansion as above, the area OHDEB is

$$p_B \left(c + \frac{1}{r}\right) V \,(1 + \text{hyp log } \rho).$$

Hence the diagram area GCDEF is

$$p_B \left(c + \frac{1}{r}\right) V (1 + \text{hyp log } \rho) - p_B c V - p_b V$$

and the mean height of the diagram or mean effective pressure is

$$p_m = p_B \left\{ \left(c + \frac{1}{r}\right) (1 + \text{hyp log } \rho) - c \right\} - p_b.$$

The probable mean effective pressure in the actual engine will be ϵp_m. The diagram factor ϵ in simple engines varies from 0·7 in engines with much compression to 0·8 or 0·9 in engines with moderate compression. The mean effective pressure being determined, the piston area and stroke can be settled for any given I.H.P.

54. *Most economical ratio of expansion if cylinder condensation is neglected.*—The waste due to condensation in the cylinder increases generally with the ratio of expansion, and this is not easily calculated, it depends on so many conditions. But, even apart from cylinder condensation, there is a limit to the economical expansion depending on the back pressure p_b and engine friction, which may be taken at f lbs. per sq. in. of piston area. Suppose an engine uses 1 lb. of saturated steam (in excess of the steam initially condensed) of specific volume v cubic feet at an initial absolute pressure p lbs. per sq. in. If not expanded, the terminal volume of steam in the cylinder will be v; if the expansion ratio is $r = 2$, it will be $2v$, and so on. Then neglecting clearance, with accuracy enough, the effective work done per lb. of steam will be

$$144 \left[p v (1 + \log_e r) - p_b r v - f r v\right] \text{ ft.-lbs.}$$

In fig. 45 let the horizontal ordinates represent to one scale terminal volumes v, $2v$, $3v$ . . . per lb. of steam, and to another scale ratios of expansion 1, 2, 3 . . . and let vertical ordinates represent work per lb. of steam. Let values of $144\, p v\, (1 + \log\ r)$ for any given pressure p

and different values of r be set up, a curve like ab is obtained. The back pressure p_b may be taken at 16 to 16½ lbs. per sq. in. for a non-condensing, and 1½ to 2½ lbs. per sq. in. for a condensing engine. Values of $144 p_b r v$ set up give a straight line such as cd. Lastly, the engine friction reckoned per sq. in. of piston varies in different types of engine, but may be taken as from 2 to 5 lbs. Values of $144 f r v$ set up from cd give the straight line ef. Then for any value of r, the vertical intercept between ab and ef, found by drawing a tangent to ab parallel to ef, is the maximum work obtainable from a lb. of steam, and the value of r to which it corresponds is, under the

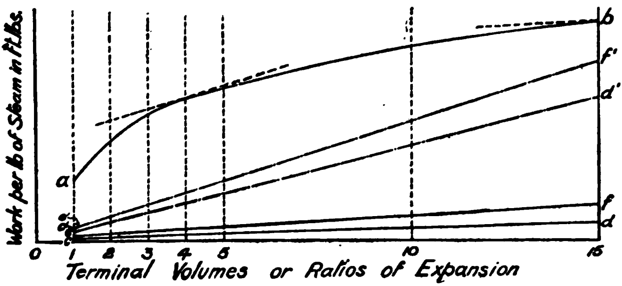


Fig. 45

restrictions assumed, the most economical ratio of expansion. The lines cd, ef are drawn with values suitable for a condensing engine; $c'd'$, $e'f'$ with those for a non-condensing engine. The best ratio of expansion for the values taken would be about 4 for a non-condensing and 15 for a condensing engine, the initial absolute pressure having been taken at 100 lbs. per sq. in.

The best actual ratios would be lower than these, especially for condensing engines, in consequence of cylinder condensation, which increases with increased expansion.

COMPOUND ENGINES

55. *Notation.*—In the discussion of compound engines the following notation will be used.

p_i = initial absolute pressure in HP cylinder.
p'_i = initial absolute pressure in LP cylinder.
p_r = absolute receiver pressure.
p_b = absolute back pressure in LP cylinder.
p_B = absolute boiler pressure.
V_1 = LP cylinder volume.
V_2 = HP cylinder volume.
$n = V_2/V_1$ the cylinder ratio.
$c_1 V_1$ = clearance volume in HP cylinder.
$c_2 V_2$ = clearance volume in LP „
r_1 = ratio of expansion in HP „
r_2 = ratio of expansion in LP „
R = total nominal ratio of expansion $= n\, r_1$.
U = volume of receiver space. (R in §§ 66, 67.)

56. *Compound engine diagrams.*—Fig. 46 shows two pairs of diagrams as they would be taken from the HP and LP cylinders of a compound engine. These can be dealt with like simple engine diagrams, each with reference to its own cylinder. As taken, the HP and LP diagrams will be to different pressure and volume scales. For some purposes it is convenient to redraw them as a combined diagram to the same scales. Choose scales for pressure and volume, and set off, as shown in fig. 47, the HP volumes $c_1\, V_1$ and V_1 and LP volumes $c_2\, V_2$ and V_2. Usually it is convenient that the ordinates of the combined diagram should be the means of the corresponding ordinates of the front and back diagrams of each cylinder. Divide the HP diagram length into ten equal parts and the length V_1 into the same number of equal parts, measuring always from the atmospheric line set off at each division of the combined diagram the mean of the two corresponding pressures of the actual diagrams for the out- and in-stroke. Dividing the

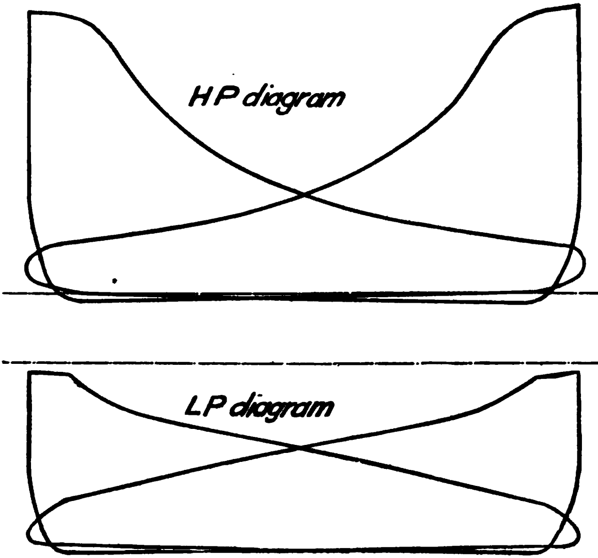

HP diagram
LP diagram

Fig. 46

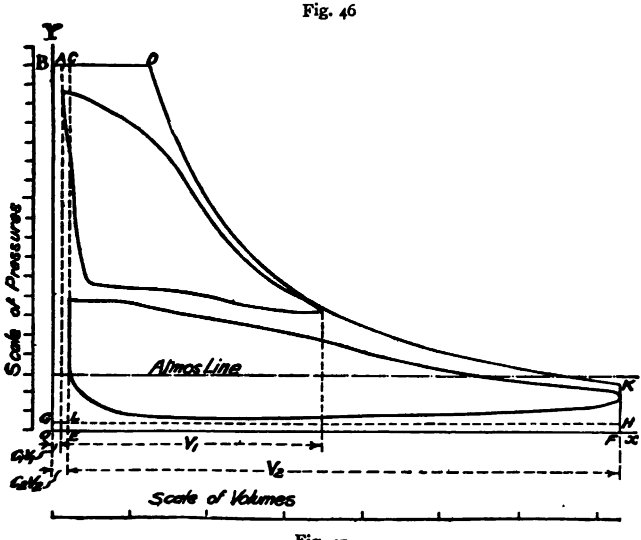

Y
B
A
C
D
Scale of Pressures
Atmos Line
K
G
L
H
O
E
F
X
V_1
V_2
Scale of Volumes

Fig. 47

LP diagrams and the length v_2 into ten equal parts, proceed in the same way. The points so found being connected by smooth curves give the combined diagram.

57. *Nominal and real ratios of expansion.*—Let the cylinder ratio be $v_2/v_1 = n$, and the HP and LP nominal ratios of expansion r_1 and r_2. Then, neglecting clearance, v_1/r_1 cubic feet of steam admitted to the HP cylinder expand to v_2 cubic feet in the LP, and the total nominal ratio of expansion in the two cylinders is

$$R = v_2 r_1 / v_1 = n r_1.$$

Taking clearance into account, $c_1 v_1 + v_1/r$ cubic feet admitted to the HP expand to $c_2 v_2 + v_2$ cubic feet in the LP, and the real total ratio of expansion is

$$\rho = \frac{(c_2 + 1)\, v_2}{c_1 v_1 + v_1/r_1} = \frac{(1 + c_2)\, n\, r_1}{c_1 r_1 + 1}.$$

The same equation applies to a triple-expansion engine if for c_2 is substituted c_3, the corresponding quantity for the third cylinder of the triple.

58. *Theoretical indicator diagram of compound engine.*—Neglecting compression and loss of pressure between the cylinders, an approximate theoretical diagram is constructed thus: Take E C the intended boiler pressure p_B (or, better, the stop valve pressure); take B D so that O F/B D $= \rho$ the intended total ratio of expansion, or take C D/v_1 the intended ratio of cut-off in the HP cylinder. Draw the hyperbola D K with O x, O y as asymptotes, and the back pressure line G H. Then L C D K H is the theoretical diagram.

The area O B D K F is

$$p_B \frac{(c_2 + 1)\, v_2}{\rho} (1 + \text{hyp log } \rho)$$

the area L C D K H is

$$p_B \frac{(c_2 + 1)\, v_2}{\rho} (1 + \text{hyp log } \rho) - c_2 v_2 p_B - v_2 p_b,$$

and the mean height of the diagram, or theoretical mean effective pressure, reckoned on the LP piston, is

$$p_m = p_B \left\{ \frac{c_2 + 1}{\rho} (1 + \text{hyp log } \rho) - c_2 \right\} - p_b.$$

The probable actual mean effective pressure, reduced to the LP piston, will be ϵp_m, where ϵ is a diagram factor found by examining diagrams from similar engines.

The diagram factor has been found to have the values in the Table on page 82.

59. *Terminal drop of pressure.*—In most actual engines the terminal expansion pressure in the HP is greater than the receiver pressure into which the cylinder exhausts, and there is a sudden drop of pressure and unbalanced expansion at the end of the HP stroke, just as there is a drop of pressure at the end of the stroke of a simple engine. In certain conditions of load the HP terminal pressure may be less than the receiver pressure, and then the diagram has a loop. Obviously cylinder drop involves the loss of part of the area of the diagram, and some engineers take great pains to arrange the cylinder and expansion ratios so that the terminal pressure drop shall be small in normal conditions of load. The importance of this point seems to have been overrated. It is not clear that, balancing all considerations, the condition of greatest economy involves the condition that the pressure drop shall be small. To secure that condition, also, some other conditions of importance must be compromised.

60. *Ratio of the cylinders and ratios of expansion when the work done in each cylinder is the same.*—Clearance neglected. Receiver assumed very large. The compound engine problem is complicated, but it will tend to clear up some points if a preliminary calculation is made on the arbitrary assumptions just stated. With a very large receiver the back pressure in the HP cylinder and the admission pressure in the LP cylinder may be taken to

II. G

Mean Effective Pressures by Calculation and Experiment. Diagram Factor

Type of engine	Authority	Indicated horse-power	Boiler pressure p_B	Clearances c_1	Clearances c_2	Ratio of cylinder volumes	Nominal expansion r_1	Real total expansion R	p_b	Calculated p_m	By experiment $e p_m$	Diagram factor e
Simple non-condensing	Willans	16	50·7	·07	—	—	1·66	1·59	16	30·3	19·6	·59
Simple non-condensing	,,	36	136·7	·07	—	—	4·63	3·75	16	66·0	38·6	·59
Simple condensing	,,	33	85·1	·08	—	—	3·45	2·92	2	56·0	41·2	·73
,, ,,	Mair (L)	123	56·7	·018	—	—	4·3	4·1	2	31·1	22·7	·73
,, ,,	,, (N)	120	59·4	·024	—	—	3·2	3·0	2	37·7	28·7	·76
Compound non-condensing	Willans	25	95·1	·10	·07	2·0	1·66	3·04	16	47·8	29·1	·61
Compound non-condensing	,,	40	179·5	·10	·07	2·0	3·2	5·2	16	69·1	46·0	·67
Compound condensing	,,	15	139·7	·08	·08	2·8	3·57	8·4	2	43·2	35·2	·82
Woolf condensing	Mair (Q)	213	67·2	·021	·02	4·0	3·0	11·5	2	17·1	13·9	·81
,, ,,	,, (R)	215	67·8	·021	·02	4·0	2·6	10·0	2	19·5	14·1	·72
,, ,,	,, (S)	133	78·0	·034	·034	4·3	3·8	15·0	2	15·3	11·1	·72
,, ,,	,, (T)	172	76·0	·034	·034	4·3	3·1	12·9	2	17·0	14·4	·85
Marine compressing	Kennedy (Colchester)	1,022	95·5	·094	·06	3·7	1·6	5·5	2	42·3	24·8	·59
Worthington compressing	Mair	119	74·0	·053	·023	4·0	2·4	8·7	2	23·8	20·0	·84
Worthington compressing	,,	108	95·1	·053	·023	4·0	4·2	14·0	2	21·1	20·1	·98
Worthington compressing	,,	131	115·7	·053	·023	4·0	4·2	14·0	2	26·1	24·6	·94
Marine triple	Kennedy (Meteor)	1,994	160·1	·124	·08	5·9	1·84	9·52	2	44·2	29·9	·68
,, ,,	,, (Iona)	645	179·6	·124	·08	6·9	3·3	17·5	2	26·4	21·1	·80
Pumping triple	Carpenter (Milwaukee)	574	136·1	·014	·02	7·1	2·96	20·6	2	22·5	21·8	·93

be the practically constant receiver pressure p_r. The compound diagram, with hyperbolic expansion, is then as shown in fig. 48.

The cylinder ratio to be determined is $V_2/V_1 = n$. The volume V_1/r_1 of steam admitted expands to V_2, so that the total ratio of expansion is $R = V_2 r_1/V_1 = n r_1$.

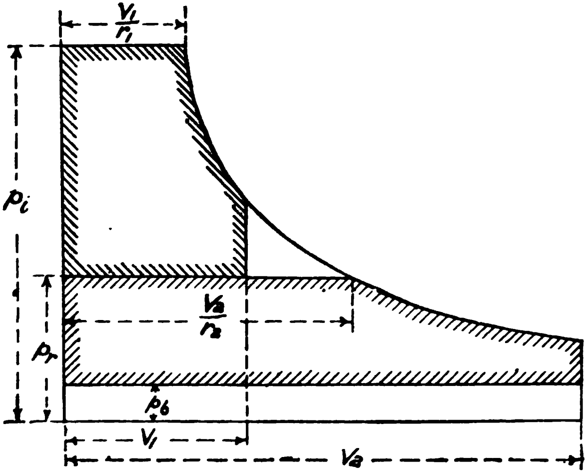


Fig. 48

The area of the HP diagram is

$$p_i \frac{V_1}{r_1} (1 + \text{hyp log } r_1) - p_r V_1,$$

and that of the LP diagram is

$$p_r \frac{V_2}{r_2} (1 + \text{hyp log } r_2) - p_b V_2.$$

Let $p_r = p_i/n$. Then since the expansion is hyperbolic, $r_1 = p_i/p_r = n$. The total ratio of expansion is $R = n r_1 =$

n^2. Approximately enough the ratio of expansion in the LP is $r_2 = p_r/p_b = p_i/np_b = n$. Then

$$\text{Area of HP diagram} = p_i \text{ hyp log } n \frac{V_1}{n}$$

$$\left.\begin{aligned} &= p_i \text{ hyp log } n \frac{V_2}{n^2} \\ \text{Area of LP diagram} &= p_i \text{ hyp log } n \frac{V_2}{n^2} \end{aligned}\right\}$$

That is, the work in each of the two cylinders is the same. The condition $r_1 = r_2 = n$ secures this. The cylinder ratio in compound engines is commonly from 3 to 4. Then for equal work in the cylinders, on the assumptions made above, the cut-off in each cylinder should be at a third to a quarter of the stroke, and the total ratio of expansion should be 9 to 16. The actual expansions are less, because there is more or less drop of pressure between the HP and LP cylinders.

61. *Receiver pressure when the initial load on the two pistons is the same.*—Retaining the same assumptions as above, and remembering that if the pistons have the same stroke their areas are proportional to V_1, V_2, the initial loads will be equal if

$$(p_i - p_r)\, V_1 = (p_r - p_b)\, V_2$$

$$p_r = \frac{p_i + np_b}{n + 1}$$

which is valid for any given value of n. If the work in the two cylinders is to be equal also, $p_r = p_i/n$, and then,

$$n = \sqrt{\left(\frac{p_i}{p_b}\right)}$$

62. *Effect of varying the ratios of expansion in compound engines.*—If the ratio of expansion in the HP is diminished the total work in the two cylinders is increased. If the LP cut-off is kept constant the work is more increased in the LP than in the HP. With a fixed cut-off in the HP, decreasing

the ratio of expansion in the LP decreases the work in the LP and increases that in the HP.

63. *Theoretical diagram for a compound Woolf engine when the cylinder volumes and ratios of expansion are given.* Cranks at 0° or 180°. Clearance and intermediate reservoir space between HP and LP cylinders treated as negligible. It is more troublesome to draw the diagram for a compound engine than that for a simple engine, and it is convenient therefore to consider first two simplified cases. Later a general graphic method will be given by which a better approximation to the compound diagram can be obtained.

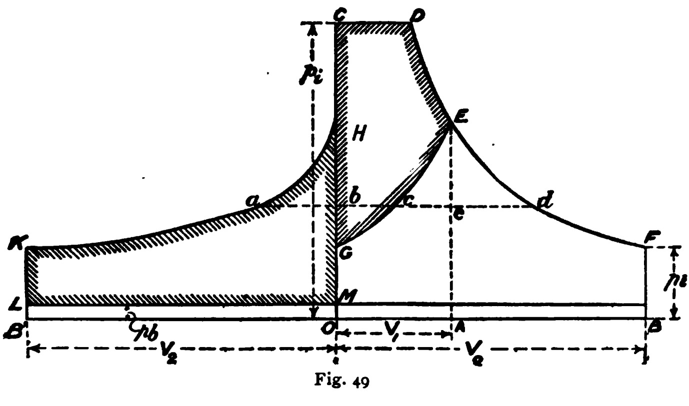


Fig. 49

In a Woolf engine the HP and LP pistons describe equal spaces in any given time, and the steam leaving the HP enters directly the LP cylinder, both cylinders being in communication throughout the return stroke.

Let O A $= V_1 =$ HP cylinder volume ; O B $=$ O B$' = V_2 =$ LP cylinder volume ; $r_1 =$ O A/C D $=$ ratio of expansion in HP cylinder ; R $=$ M L/C D, the total ratio of expansion. The steam, which at cut-off in the HP occupies the space C D $= V_1/r_1$ comes finally to fill the volume V_2 of the LP cylinder. Hence

$$V_2/R = V_1/r_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Let p_i = initial steam pressure O C; p_t = terminal pressure at LP release. Then, expansion being hyperbolic, $p_t = p_i/R$. Draw the hyperbolic expansion curve D E F. Then E on the vertical through A is the HP release, and the pressure at this point is the initial pressure O H in the LP cylinder. The back pressure line for the HP is some curve E G, the terminal pressure O G being the pressure B F $= p_t$, and this is also the terminal pressure B′ K in the LP cylinder. The expansion curve in the LP is some curve H K. It only remains to find the form of the curves E G and H K. Draw any horizontal line $a\ b\ c\ d$ corresponding to a pressure O $b = p$. When the steam has expanded to the pressure p its volume will be $b\ d$. Of this volume the part $b\ c$ still remains in the HP and the rest has passed into the LP. Hence $b\ a = c\ d$, and $a\ c = b\ d$. Let $c\ e = x$. When the HP piston has described in the return stroke a volume $c\ e = x$ the LP piston will have described in its forward stroke a volume $V_2 x/V_1$. Hence when the pressure is p the total volume of the steam is $a\ c = b\ c + b\ a = V_1 - x + V_2 x/V_1 = V_1 - x + R\,x/r_1$. But in hyperbolic expansion $p_i \,.\, CD = p \,.\, a\,c$,

$$p_i \frac{V_1}{r_1} = p\left\{V_1 - x + \frac{R\,x}{r_1}\right\}$$

$$p = p_i \frac{V_1}{r_1 V_1 - r_1 x + R\,x}$$

$$x = \frac{p_i V_1 - p\,r_1 V_1}{p\,(R - r_1)} \quad . \quad . \quad . \quad (2)$$

x being found, the point c is determined and therefore a. Other points for other pressures can be found similarly.

64. *Theoretical indicator diagram for receiver compound engine with cranks at right angles. Case I Cut-off in* LP *after half-stroke.*—Clearance neglected, and also the effect of the obliquity of connecting rod.

Let fig. 50 represent the position of the LP crank O L and the HP crank O H at the moment when the steam is

cut off in the LP cylinder. The LP piston has travelled approximately a distance AC from the inner dead point and $AB/AC = r_2$ is the expansion ratio in the LP cylinder. The HP piston has travelled a distance BD, nearly, in the return stroke, and up to that moment the fraction AD/AB

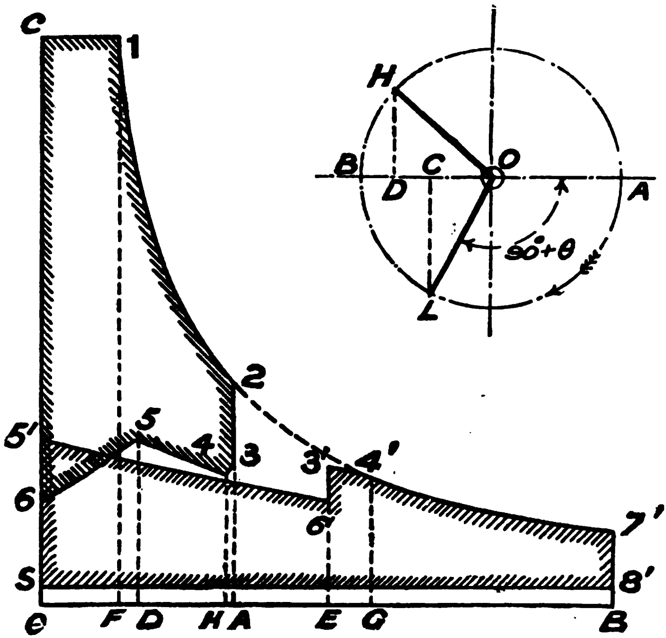


Fig. 50

of the cylinder end, which is exhausting, has been in communication through the receiver with the LP cylinder. It is necessary to express this fraction in terms of the expansion ratio in the LP. If $AOL = 90° + \theta$, then $OLC = BOH = \theta$. But $1/r_2 = AC/AB = (1 + \sin\theta)/2$. Hence $\sin\theta = (2 - r_2)/r_2$ and $\cos\theta = \{2\sqrt{r_2 - 1}\}/r_2$.

But $AD/AB = (1 + \cos\theta)/2 = \frac{1}{2} + \frac{1}{r_2}\sqrt{(r_2 - 1)} = k$ for simplicity,

$r_2 =$	2	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{1}{4}$	1
$k =$	1·0	0·995	0·971	0·90	0·5

If the obliquity of connecting rod is neglected, when one piston is at the dead point the other is at mid-stroke. Let the front and back of the HP cylinder be denoted by A and B, and the front and back of LP by C and D. Then the simultaneous operations in the four cylinder ends are shown in fig. 51, where *a* stands for admission, *exp* for expansion, and *exh* for exhaust. The stroke ends are shown by full lines.

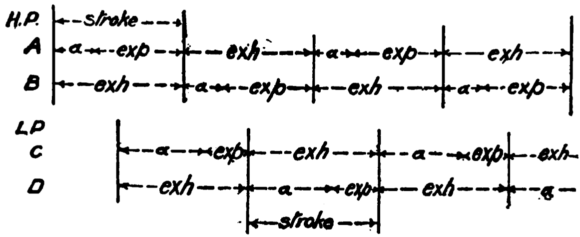


Fig. 51

In the diagram (fig. 50) points in the HP and LP diagrams which correspond are denoted by the same numerals. Let $OA = V_1$ be the volume of HP cylinder; $OB = V_2$ the volume of LP cylinder; $U =$ the volume of receiver space between the cylinders. Also let $V_2 = nV_1$, and $U = \gamma V_1$. Bisect OA and OB in D and E so that D and E correspond to HP and LP mid-stroke. The ratio of expansion in the HP is $r_1 = AO/OF$, and that in the LP is $r_2 = OB/OG$. The total ratio of expansion is $R = OB/OD = V_2 r_1/V_1 = n r_1$. OC is the initial pressure p_1, and OS the LP back pressure p_b.

Consider the diagram of the end A of the HP cylinder. Steam admitted at the pressure OC is cut off at 1 and

expands to the end of the stroke at 2. It is then released into the receiver, then in communication with the end C of the LP cylinder. There is a sudden fall of pressure to 3, a pressure intermediate between that at 2 and that in the receiver. As the HP piston makes its return stroke the steam exhausts from A and expands in C, and the pressure falls till the point of cut-off for the LP at 4. The end C of the LP being now closed to the receiver, the steam exhausting from A is compressed in the receiver, and the pressure rises till the HP mid-stroke is reached at 5. At this moment the end D of the LP begins to take steam from the receiver. Then the steam exhausting from A is expanding in D, and the pressure falls to the end of the HP stroke at 6.

Next consider the diagram for the end D of the LP cylinder. At the beginning of the stroke 5′ the pressure in the receiver is that at mid-stroke 5 of the HP diagram. During the first part of the stroke the steam exhausts from A and expands in D, and the pressure falls to 6′ at the LP mid-stroke, the terminal pressure at 6′ being the same as at the end of the exhaust from A at 6. At this point the end B of the HP cylinder begins to exhaust, and the pressure suddenly rises to 3′, the same pressure as at 3 on the HP diagram. For a time steam exhausts from B and expands in D, the pressure falling till cut-off in D is reached at 4′, the pressure then being the same as at 4 on the HP diagram. Finally, expansion continues in the LP only to the terminal pressure 7′ on the hyperbola drawn from 1. Exhaust from the LP is given by the back pressure line S 8′.

The pressures at the critical points may be determined on the assumption that $p v =$ constant. Pressures will be distinguished by subscript numerals corresponding to those on the diagram, but accents will be omitted. The total ratio of expansion being R, the terminal pressure B 7′ is the initial pressure O C divided by R. That is, $p_7 = p_1 / R$. The

terminal pressure in the HP cylinder $p_2 = p_1/r_1 = p_1\, n/R$. As the curve 4′ 7′ is an hyperbola, $p_4 = p_7\, r_2$. It has been found above that the volume OH in the HP at cut-off in the LP is $k\,V_1$. At the point 4, therefore, there is in the HP and receiver a volume of steam $k\,V_1 + U = (k + \gamma)\,V_1$. At mid-stroke 5 of the HP this volume will be reduced to $\frac{1}{2}V_1 + U = (\gamma + \frac{1}{2})\,V_1$. Consequently $p_5 = p_4\,(k + \gamma)/(\gamma + \frac{1}{2})$, which is also the initial pressure in the LP cylinder at 5′. At mid-stroke 6′ of the LP this steam has entirely exhausted from the HP and occupies the space $U + \frac{1}{2}V_2 = (\gamma + 0{\cdot}5\,n)\,V_1$. Hence $p_6 = p_5\,(\gamma + \frac{1}{2})/(\gamma + 0{\cdot}5\,n)$. At this point communication opens with one end of the HP containing a volume V_1 at a pressure p_2, which mixes with the volume $U + \frac{1}{2}V_2$ in the receiver and LP cylinder. The resulting pressure is

$$p_3 = \frac{p_6\,(\gamma + 0{\cdot}5\,n) + p_2}{\gamma + 0{\cdot}5\,n + 1}.$$

The pressures at all the critical points of the two diagrams are then determined. It is not necessary to find the exact forms of the curves.

It will be seen that in engines of this type there is often a considerable drop of pressure at the end of the HP stroke, and that consequently there is a considerable gap between the cylinder diagrams and the hyperbolic expansion curve 127′. There is waste expansion not effective in driving the pistons. It is an object in designing an engine of this type to reduce this pressure drop by suitable choice of the cylinder ratio n, the reservoir capacity, and the point of cut-off in the LP cylinder. At the same time a certain amount of pressure drop does not appear to be prejudicial on the whole to the engine economy.

65. *Case II. Cut-off in low-pressure cylinder before half-stroke.*—In this case, when the HP cylinder opens to the receiver, the communication between receiver and LP cylinder is already closed. Let fig. 52 represent the

position of the cranks at cut-off in the LP, when the LP crank has turned through θ from the inner dead point.

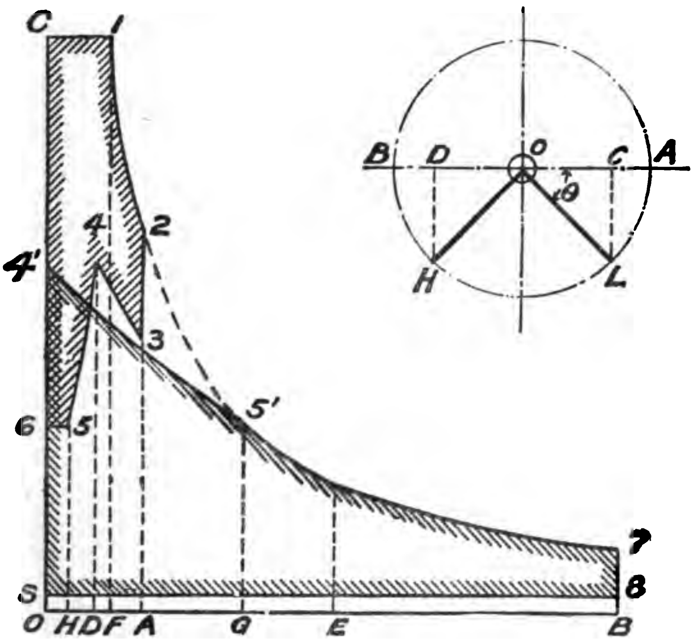


Fig. 52

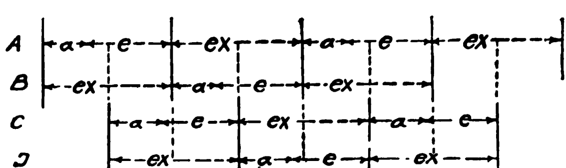


Fig. 53

Then $AB/AC = r_2$; $1/r_2 = \frac{1}{2}(1 - \cos\theta)$. Hence $\cos\theta = (r_2 - 2)/r_2$, and $\sin\theta = 2\sqrt{(r_2 - 1)}/r_2$. The

fraction of the stroke through which the HP piston has advanced at cut-off in the LP is $k = \text{AD}/\text{AB} = \frac{1}{2}(1 + \sin\theta) = \frac{1}{2} + \sqrt{(r_2 - 1)}/r_2$.

$r_2 =$	5	4	3	2
$k =$	0·9	0·93	0·97	1·00

Using the same notation as before, $\text{OA}/\text{OF} = r_1$; $\text{OB}/\text{OG} = r_2$; $\text{OB}/\text{OF} = \text{R}$; $\text{OB}/\text{OA} = \text{V}_2/\text{V}_1 = n$; U = receiver volume $= \gamma \text{V}_1$. Then $p_2 = p_1/r_1 = p_1 n/\text{R}$. The terminal pressure in the LP is $p_7 = p_1/\text{R}$. At cut-off the LP contains all the steam it receives, so that the pressure at 5′, the cut-off point, is $p_5 = p_7 r_2$. But this is the pressure in the HP at 5 when the HP piston has traversed AH = a fraction k of its return stroke, and there is still in the exhausting end of the cylinder the volume $\text{OH} = (1 - k)\text{V}_1$. The total volume of steam at the point 5 is therefore $\text{U} + (1 - k)\text{V}_1 = (\gamma + 1 - k)\text{V}_1$. At the end of the stroke, when exhaust is complete, this volume is reduced to $\text{U} = y\text{V}_1$, and the pressure therefore rises to $p_6 = p_5 \gamma/(\gamma + 1 - k)$. At this moment the end B of the HP begins to exhaust. It contains a volume V_1 at the pressure $p_.$, and the receiver contains a volume U at the pressure p_6. The pressure suddenly changes to $p_3 = (p_2 \text{V}_1 + p_6 \text{U})/(\text{V}_1 + \text{U}) = (p_2 + p_6 \gamma)/(1 + \gamma)$. As the piston B exhausts into the receiver no steam is admitted to the LP till mid-stroke, when the steam volume in HP and receiver is reduced to $\text{U} + \frac{1}{2}\text{V}_1 = (\gamma + \frac{1}{2})\text{V}_1$. Hence the mid-stroke back pressure in the HP is $p_4 = p_3(\gamma + 1)/(\gamma + \frac{1}{2})$, and this is the initial pressure at which steam is admitted to the LP. The pressures at all the critical points have now been fixed.

66. *More accurate construction of the indicator diagrams for a compound engine, taking clearance and compression into account. Case I. Woolf engine.*—Supposing the sizes of cylinders and points of cut-off and compression provisionally settled, then it is possible to construct much more accurate diagrams with a view of determining how far

the engine complies with the required conditions, and especially how far there is any waste expansion or fall of pressure between the cylinders.

In fig. 54, set off *a b*, the intermediate space R between the cylinders ; *a o*, *b* 5′, the high- and low-pressure clearance volumes, C_a and C_b ; and 05, 5′0′ the volumes described by the high- and low-pressure pistons V_a and V_b. Draw the crank-pin semi-circles on 05, 5′0′.

On the vertical to the left take any distance to represent a revolution and divide it and the crank-pin circles into any number (conveniently 10) of equal parts. Number them in order, starting from the dead point. We can now draw the curves of sines or of piston displacement, numbered 0, ·1, ·2, . . ., points of which lie on the intersection of horizontal lines through the points of the revolution line and verticals through corresponding points of the crank-pin circles. This line has this property, that the horizontal abscissa from the vertical through *a* gives the whole volume of steam in cylinder and clearance space at any given point of the revolution. The two lines of sines are so placed that for those parts of the revolution, during which both cylinders are in communication, the horizontal intercept between the two lines of sines is the total volume of steam in the two cylinders and receiver. Now select and mark on the lines of sines the cut-off and cushion-points for both cylinders ; they are the points on the verticals through 1 and 5 of the high-pressure diagram, and through 11 and 14 of the low-pressure diagram. In the figure the high-pressure diagram is drawn for an initial pressure of 100 lbs. per sq. in., and the admission line 0 1 is horizontal. 1 2 is an hyperbolic expansion line for the assumed point of cut-off. The volumes are measured to the vertical through *a* in drawing this curve, because the steam is in the high-pressure cylinder and clearance. At 2 the high-pressure cylinder opens to the receiver and there is a drop of pressure (2 3), which will be determined presently. The pressure at 3 is the initial

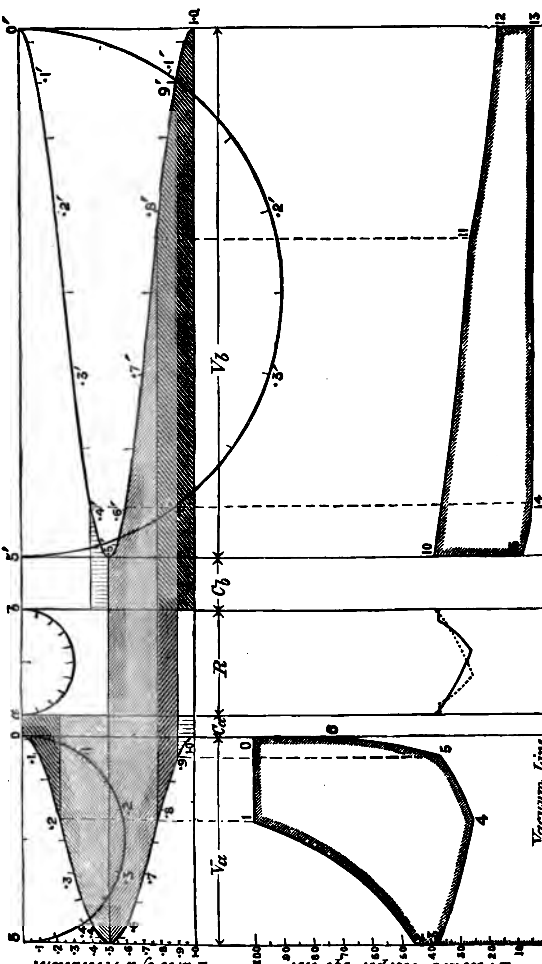


Fig. 54.—Woolf Engine. Cranks at 0° or 180°

pressure at 10 in the low-pressure cylinder. During 3–4 and 10–11 the steam expands in the high-pressure cylinder receiver and low-pressure cylinder, and its volume is the horizontal intercept between the two lines of sines. Thus the horizontal ·7–·7′ is the volume at those points of the revolution in the two cylinders. Calculating the pressure from the volume, it is set up on verticals through ·7 and ·7′ to give points on 3–4 and 10–11. At the point 4, corresponding to 11, the low-pressure slide-valve closes, and during 4–5 the steam is compressed in the high-pressure cylinder and receiver. The volumes for this part of the revolution are found by measuring from the curve of sines to the vertical through *b*. At 5 the cushion-point is reached and the steam is compressed in the high-pressure cylinder only, the volumes being now measured to the vertical through *a* in determining the pressures for the compression line 5–6. In fact 5–6 is an hyperbola, of which the vertical through *a* is an asymptote.

To complete the low-pressure diagram, draw the back-pressure line 13–14 with an assumed value of the condenser pressure. 11–12 and 14–15 are hyperbolas, having the vertical through *b* as an asymptote; that is, the volumes are measured from the low-pressure curve of sines to the vertical through *b*.

A curve of pressures in the receiver has also been drawn in the same way. The shading of the upper part of the diagram indicates what volumes of steam are in action at any given moment. The horizontal breadth of the shaded part is the steam volume.

Let p and v denote the pressure and volume of steam undergoing any operation of expansion and compression, then $p\,\mathrm{v} =$ constant is the equation determining the indicator diagram curve. For any fraction of the stroke the values of p and v may be distinguished by subscript figures corresponding to those on the indicator diagrams in fig. 54.

c_a and c_b are the high-pressure and low-pressure cylinder clearance volumes. R is the volume of the receiver.

At the end of admission the value of the product pv for the high-pressure cylinder is $p_1\ (v_1 + c_a)$, and since the expansion is treated as hyperbolic,

$$p_2\ (v_2 + c_a) = p_1\ (v_1 + c_a) \quad . \quad . \quad . \quad (1)$$

which determines the pressure at the end of expansion. Release takes place at 2, and the steam in the cylinder mixes with that in the receiver. But the receiver was cut off in the previous stroke at 5, and hence the product pv for the steam in the receiver is p_5 R. Hence, during the period 3–4 the value of pv for the steam acting in the two cylinders conjointly is

$$p_1\ (v_1 + c_a) + p_5\ R = p_3\ (v_3 + c_a + R + c_b). \quad . \quad (2)$$

an equation which determines p_3 when we have found the value of p_5.

The small quantity of steam $p_{15}\ c_b$ in the clearance space of the low-pressure cylinder is neglected for simplicity. The steam now expands in the diminishing high-pressure and increasing low-pressure cylinder volume, plus the clearances and receiver. Then

$$p_4\ (v_4 + c_a + R + c_b + v_{11}) = p_1\ (v_1 + c_a) + p_5\ R\ (3).$$

The low-pressure clearance and cylinder is now cut off, and during 4–5 the value of pv for the steam in the high-pressure cylinder and receiver is $p_4\ (v_4 + c_a + R)$. Hence

$$p_5\ (v_5 + c_a + R) = p_4\ (v_4 + c_a + R) \quad . \quad . \quad . \quad (4)$$

$$p_4 = p_5\ \frac{v_5 + c_a + R}{v_4 + c_a + R};$$

putting this in 3, we get an equation which determines p_5. Then from (2) we can determine p_3. Thus all the points on the high-pressure diagram are fixed.

In the low-pressure cylinder during 10–11 the pressures

are the same as on 3–4 for corresponding parts of a revolution, that is, for points on the same level on the two curves of sines. From 11–12 the value of $p\,v$ is $p_{11}\,(v_{11} + c_b)$, the volumes during expansion being measured from the low-pressure clearance line. At 12 there is release to the condenser. 13–14 is the condenser back-pressure line, and 14–15 an hyperbola for $p\,v = p_{14}\,(v_{14} + c_b) = p_{15}\,c_b$.

Solving the equations, we get

$$p_2 = \frac{p_1\,(v_1 + c_a)}{v_2 + c_a}$$

$$p_3 = \frac{p_1\,(v_1 + c_a) + R\,p_5}{v_3 + c_a + R + c_b}$$

$$p_4 = \frac{p_5\,(v_5 + c_a + R)}{v_4 + c_a + R}$$

$$p_5 = \frac{p_1\,(v_1 + c_a)\,(v_4 + c_a + R)}{(v_5 + c_a)\,(v_4 + c + R + c_b + v_{11}) + R\,(c_b + v_{11})}.$$

67. *Case II. Indicator diagram for a receiver engine. Cranks at* 90°.—Begin as in the previous case, setting off horizontally the high- and low-pressure cylinder volumes v_a and v_b, the clearance volumes c_a and c_b, and the receiver volume R. Take on the vertical through ·5 a length to represent a revolution and divide it into ten parts. Draw the crank-pin circles and divide them similarly, remembering that the points ·5 of the high-pressure crank and ·5′ of the low-pressure crank are at right angles; that is, at half a revolution, if the high-pressure crank is horizontal, the low-pressure crank is vertical. The numbering of corresponding points is then easy.

The high-pressure diagram (fig. 55) is drawn for an initial pressure of 140 lbs. per sq. in. The admission line 0–1 is horizontal; 1–2 is an hyperbolic expansion curve, the volumes being measured to the vertical through *a*. At 2 there is a drop as the receiver opens, which will be determined presently. During 3–4 the volume in the high-pressure cylinder is diminishing, and it is open to the

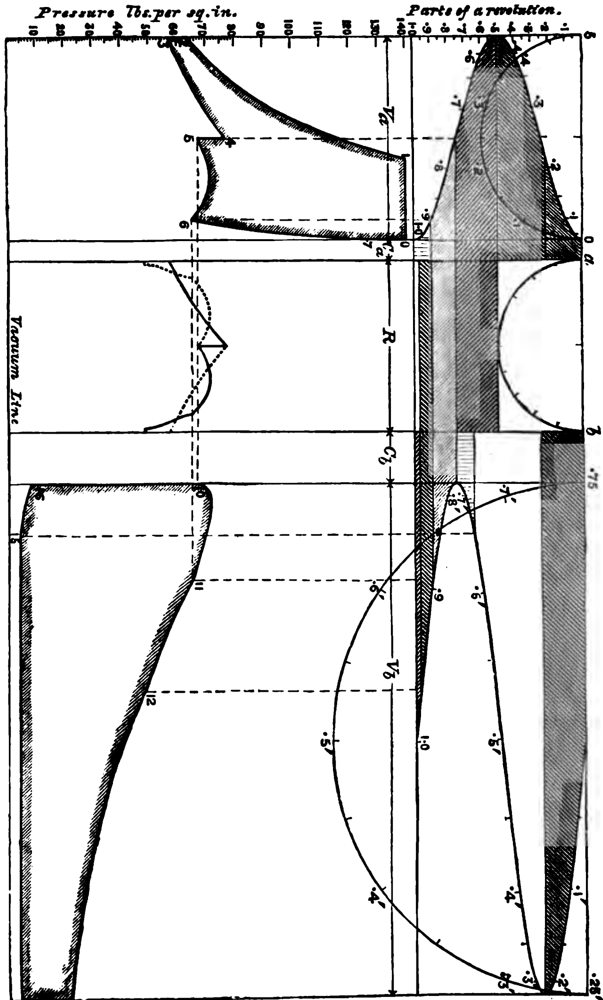


Fig. 55.—Receiver Engines. Cranks at right angles

receiver. The volumes are measured from the high-pressure curve of sines to the vertical through *b*. At 4, mid-stroke, there is a second fall of pressure as the low-pressure clearance opens, and this will be determined by calculation. At 5 the low-pressure cylinder opens to steam at mid-stroke of the high-pressure cylinder. The volumes for the curves 5–6 and 10–11 are the horizontal intercepts between the two curves of sines, the cylinders being in communication. 6 is the chosen cushion-point for the high-pressure cylinder, and 6–7 is an hyperbolic compression curve with the vertical through *a* as an asymptote.

For the low-pressure diagram, during 11–12, the cylinder is in communication with the receiver. The steam expands, the volumes being measured from the low-pressure curve of sines to the vertical through *a*. Now draw the back-pressure line 14–15. Then 12–13 and 15–16 are hyperbolic expansion and compression lines having the vertical through *b* as an asymptote, and the volumes are measured from the curve of sines to this line.

As before, a curve of receiver pressure has been drawn, and the shading of the upper part of the figure indicates what volumes of steam are acting in either cylinder at any moment.

To determine the points on the diagram we have the following equations. During 1–2, expansion in high-pressure cylinder,

$$p_1 (V_1 + C_a) = p_2 (V_2 + C_a) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

During 3–4, compression in high-pressure cylinder and receiver,

$$p_3 (V_3 + C_a + R) = p_4 (V_4 + C + R) \quad . \quad . \quad . \quad (6)$$

At 4, midstroke of the HP cylinder, the LP cylinder opens, and there is a drop from the steam filling the clearance space of LP cylinder, in which the product $p\,V$ has the value $p_{16}\,C_b$. Then

$$p_{16} C_b + p_4 (V_4 + C_a + R) = p_5 (V_4 + C_a + R + C_b) \; (7)$$

During 5–6 and 10–11

$$p_5 (v_4 + c_a + R + c_b) = p_6 (v_6 + c_a + R + c_b + v_{11}) \quad (8)$$

At 6 compression begins in the HP cylinder, and 6–7 is an hyperbola with the vertical through a as asymptote. The HP diagram can now be completed; for if the LP back-pressure line 14–15 is drawn, and 15–16 as an hyperbola with the vertical through b as an asymptote, the quantity $p_{16}\, c_b$ is determined.

To complete the LP diagram. During 11–12 the steam expands in receiver and LP cylinder,

$$p_6 (R + c_b + v_{11}) = p_{12} (v_{12} + c_b + R) \quad . \quad . \quad (9)$$

At 12 cut-off in LP cylinder occurs, thence the steam expands in LP cylinder only.

$$p_{12} (v_{12} + c_b) = p_{13} (v_{13} + c_b) \quad . \quad . \quad . \quad . \quad . \quad (10)$$

For the drop of pressure in the HP cylinder we have the equation, since p_{12} is the pressure in receiver when expansion into the LP cylinder ceases,

$$p_2 (v_2 + c_a) + p_{12} R = p_3 (v_2 + c_a + R) \quad . \quad . \quad (11)$$

The simultaneous equations are a little troublesome, but only require patience for solution.

$$p_2 = \frac{p_1 (v_1 + c_a)}{v_2 + c_a}$$

Let
$$M = v_6 + c_a + R + c_b + v_{11}$$
$$N = v_{12} + c_b + R$$

$$p_3 = \frac{p_1 (v_1 + c_a)\, N M}{(v_2 + c_a + R)\, M N - R (v_3 + c_a + R)(R + c_b + v_{11})}$$

$$p_4 = \frac{p_3 (v_3 + c_a + R)}{v_4 + c_a + R}.$$

Neglecting $p_{16}\ C_b$,

$$p_5 = \frac{p_4 (V_4 + C_a + R)}{V_4 + C_a + R + C_b}$$

$$p_6 = \frac{p_5 (V_4 + C_a + R + C_b)}{V_6 + C_a + R + C_b + V_{11}}$$

$$p_{12} = \frac{p_6 (R + C_b + V_{11})}{V_{12} + C_b + R}$$

$$p_{13} = \frac{p_{12} (V_{12} + C_b)}{V_{13} + C_b}$$

For a triple engine a similar process may be used, two pairs of diagrams being drawn, one for the HP and IP cylinders, and one for the IP and LP cylinders.

CHAPTER IV

ON SOME KINEMATICAL AND DYNAMICAL PROBLEMS IN ENGINE DESIGNING

68. According to the most modern views of the nature of a machine, it consists of a series of pairs of elements linked together into one or more kinematic chains. The cylindrical journal and its pedestal form at once the commonest and most typical example of a pair of elements. If the pedestal is fixed, the journal can rotate and can have no other motion. A pair of elements in which the relative motion is thus fixed is called a 'closed pair,' and in machinery the only pairs which are useful are pairs which are closed either geometrically or virtually by constraints which prevent any relative motion other than the definite relative motion required. Suppose a series of pairs taken and linked, one element of one pair to one element of another pair. We get a kinematic chain. Such a chain is termed a 'closed chain,' if any one link being fixed all the other links have one definite motion, and one only, relatively to the fixed or frame-link. Obviously, only closed chains are useful in machinery, because definite motions are to be given to the working tools or other driven parts of the machine.

The simplest of all kinematic chains consists of four cylinder pairs (or journals and bearings) linked together, as shown in fig. 56, the fixed link being shaded. As drawn, *b* rotates and *c* oscillates relatively to the frame-link; *d* has a more complex motion. Links like *b* or *c* are termed 'cranks' or 'levers,' and links like *d*, the bearings of

which do not rest on the frame but on other moving pieces, and which have in general a more complex motion than pieces

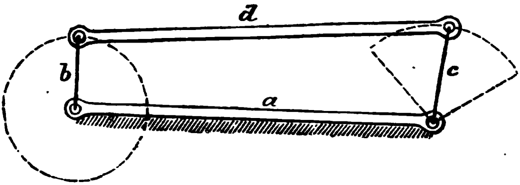


Fig. 56

resting on the frame, are called 'connecting' or 'coupling rods.' The chain shown in fig. 56 may be recognised as being identical with the half-beam *c*, the crank *b*, connecting rod *d*, and column and entablature *a* of a beam engine.

Such a simp'e closed chain is capable of various transformations. The relative size of the parts may be changed sometimes with important results. If the fixed link is changed, the chain is said to be *inverted*, and then, although the relative motion of each pair of links is unchanged, the general motion of the whole mechanism is often quite different. Thus, if the short link *b*, fig. 56, is fixed, we get two completely rotating cranks linked together (see *b*, fig. 57). The mechanism is identical with that of the drag-link coupling used to connect two shafts not exactly in line. If the link *c*, fig. 56, is fixed, we get two levers, having only a limited range of oscillation (see *c*, fig. 57). Some forms of Watt's parallel motion are identical with this chain (*d*, fig. 57).

Suppose that now the chain of fig 56 is changed by substituting a slide block and slide bars for one pair of cylindrical elements. Then we get a chain (fig. 58) exceedingly common in machinery, and easily recognisable as identical with the direct-acting engine mechanism. Here a rotating crank *b*, gives straight-line motion to the slide-block *c* and crosshead, or *vice versâ*. There are three

cylinder pairs and one sliding pair. If this chain is inverted, we get, by fixing the link *d*, a mechanism identical with

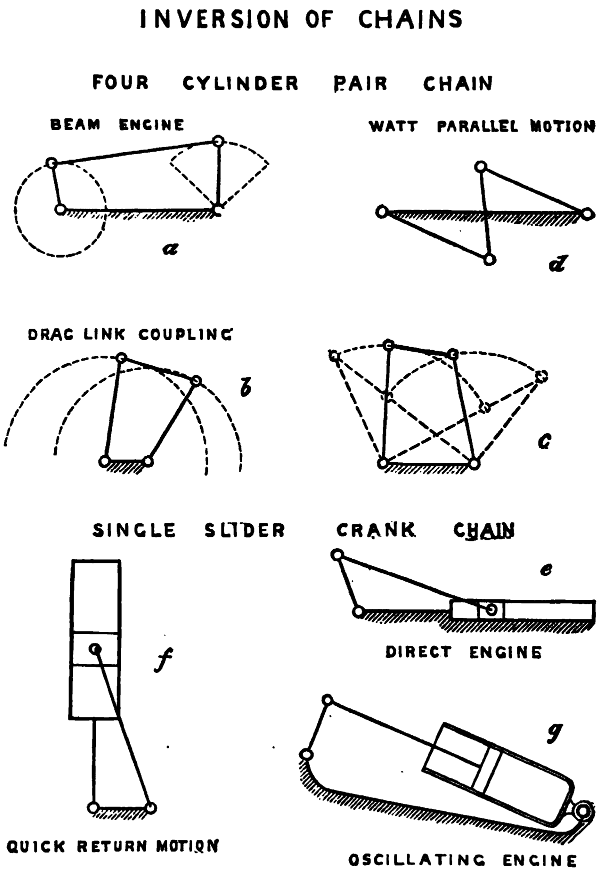


Fig. 57

that of the oscillating steam engine (fig. 57, *g*); by fixing *b*, a mechanism used at one time to get a slow forward and

quick return motion for planing machines (fig. 57, *f*); and by fixing *c*, a mechanism which has been utilised in the pendulum steam pump. This chain is known as the 'single-slider crank chain.'

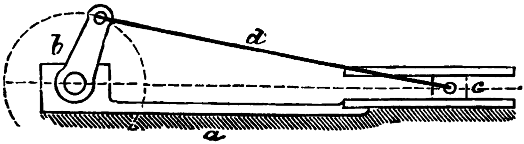


Fig. 58

By replacing two of the cylinder pairs in fig. 56 by sliding pairs, we get the double-slider crank chain shown in fig. 59, recognisable as identical with some forms of steam pump, and this again takes other forms by inversion.[1]

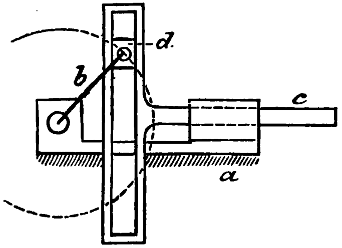


Fig. 59

69. *Displacement, velocity, and acceleration curves of linkwork. General case.*—In what follows it will be assumed that, as is usually the case, the motion of the pieces is in, or parallel to, one plane. Let fig. 60 represent a four-cylinder pair kinematic chain, O C being a revolving crank and A D an oscillating lever. For any position O C_1 of the crank the position of D A is found by setting off C 1, D 1′ equal to the connecting-rod length. Let the points 0, 1, 2, . . . on the crank-pin circle correspond to 0′, 1′, 2′ . . . on the lever-pin circle. Then 0′ 1′, 1′ 2′ . . . are lever-pin displacements corresponding to the crank-pin displacements

[1] A much fuller account of Reuleaux's analysis of machines will be found in Cotterill's 'Applied Mechanics' and Kennedy's 'Mechanics of Machinery.'

or travels o 1, 1 2, . . . Take any line o″ 6″, and set off the distances o″ 1″, 1″ 2″ . . . equal to the arcs o 1, 1 2, . . . which are crank-pin travels. At 1 set up an ordinate equal to the lever-pin travel o′ 1′, at 2 an ordinate equal to o′ 1′ + 1′ 2′, and so on. Joining the points, the curve o″ B 6″ is a curve of lever-pin travel on a crank-pin travel base. In the case that the crank pin moves uniformly, which has often to be considered, the crank-pin travel is

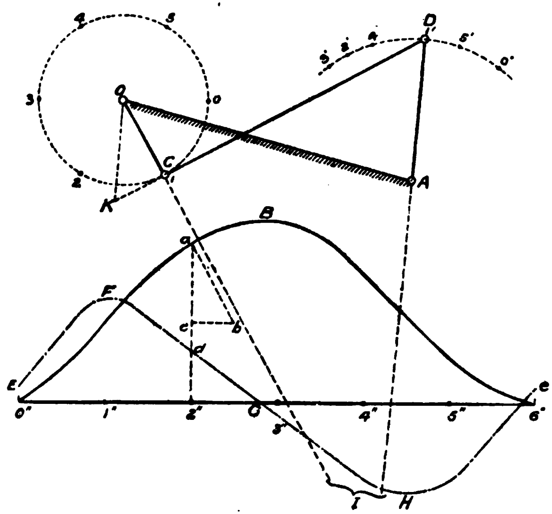


Fig 60

proportional to the time, so that the base o″ 6″ may equally well be taken to represent either the circumference o 3 o or the time of one revolution of the crank.

The points C, D move at any instant tangentially to the arcs described. Produce O C, D A, which are perpendicular to these directions of motion. The join I is the instantaneous axis of C D. If V is C's velocity, and *v* is D's velocity,

$$v/V = DI/CI.$$

Draw O K parallel to D A, meeting D C in K. Then

$$v/\text{V} = \text{D I}/\text{C I} = \text{O K}/\text{O C}.$$

If O C represents V to any scale, O K represents v to the same scale. If V is constant, the construction repeated for different positions of C gives successive values of v to the same scale. If these are set up at 0″, 1″, 2″ . . . the curve E F G H of lever-pin velocities on a crank-pin travel or time base is obtained. The *area* of a strip of the velocity curve between two ordinates is proportional to the increment of the *ordinate* of the travel curve in the time. Conversely, the subnormal of the travel curve at any point is proportional to the lever-pin velocity at that point. Draw $a\,b$ normal to the curve, and take $a\,c = \text{O C} = \text{V}$. Then $c\,b = 2''\,d = v$.

In fig. 61, let x = crank-pin travel at any instant, when s is the lever-pin travel, and v the lever-pin velocity, corresponding to a velocity V of the crank pin. The lever-pin travel curve corresponds to some equation $s = f(x)$, and the lever-pin velocity curve to some equation $v = \text{F}\,(x)$. If $d\,s$ and $d\,x$ are small corresponding increments of s and x, $d\,s/d\,x = v/\text{V}$; and if $a\,b$ is normal to the

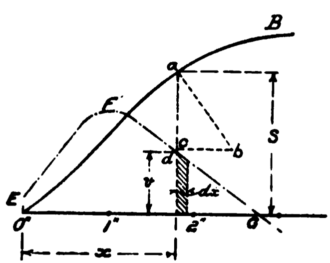


Fig. 61

curve, $d\,s/d\,x = \tan b\,a\,c$. If $a\,c$ = V on any scale, $c\,b = a\,c \tan b\,a\,c = \text{V}\,d\,s/d\,x = v$.

Next consider the strip of the velocity curve between ordinates x and $x + d\,x$. But $d\,s/d\,x = v/\text{V}$, that is, the increment $d\,s$ of the ordinate of the travel curve is proportional to $v\,d\,x$, the area of the strip of the velocity curve. The whole area of the velocity curve (reckoning parts below 0″ 6″ as negative) between two ordinates, is propor-

tional to the difference of the ordinates of the travel curve at the same points on o″ 6″.

It can be shown that the curve of acceleration of D when C moves uniformly, is related to the velocity curve in the same way that the velocity curve is related to the travel curve (I. p. 41).

70. *Case of the double-slider crank chain.*—In this mechanism, fig. 62, the crank pin moves tangentially to the

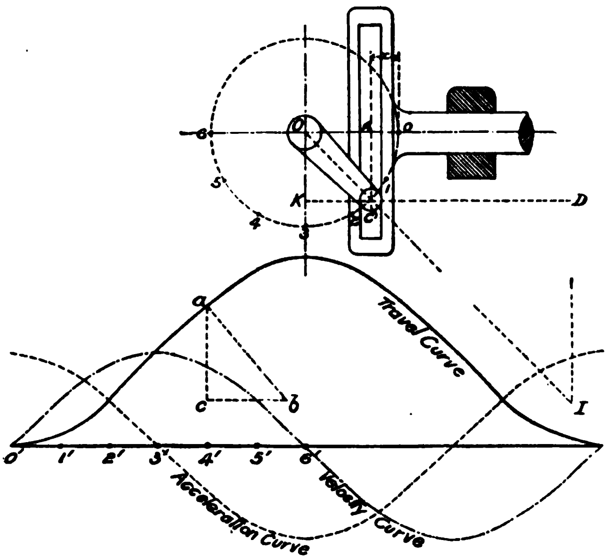


Fig. 62

crank-pin circle, while the slotted crosshead moves parallel to O A 6. Draw C A perpendicular to the line of stroke. Then when the crank pin moves from O to C, the crosshead travels the distance O A. Divide the crank-pin circle by equidistant points 0, 1, 2, 3, . . . and set off on any line, 0′ 1′, 1′ 2′ . . . equal to the arcs 0 1, 1 2, . . . Dropping perpendiculars from 0, 1, 2 . . . on the line of stroke,

set up the crosshead travels so found at o′, 1′, 2′ . . . This gives the crosshead travel curve. The instantaneous axis of the crosshead is a point I at an infinite distance along O C. Draw C D parallel to the line of stroke, and D I perpendicular to it at infinity. Then v and V being the crosshead and crank-pin velocities, as before, $v/\text{V} = \text{DI}/\text{CI} = \text{OK}/\text{OC}$. When V is constant and $=$ O C to any scale, values of O K for the crank-pin positions 0, 1, 2 . . . set up at o′, 1′, 2′ . . . give the curve of crosshead velocities to the same scale. As above, if $a\,b$ is the normal to the curve of travel and $a\,c =$ O C, then $c\,b =$ the corresponding ordinate of the velocity curve.

It can be shown that distances O A, measured from the crank centre O, represent the acceleration a of the crosshead to the scale on which O C represents the radial acceleration V^2/OC of the crank pin. If distances O A are found for the points 0, 1, 2 . . . and set up at o′, 1′, 2′, . . . the acceleration curve is obtained.

Let A O C $= \theta$, and let ω be the constant angular velocity of the crank, V the velocity of crank pin, r the crank radius. Then $\text{V} = \omega r$ and $\omega = d\theta/dt$. If x is the crank-pin travel, $x = r\ (1 - \cos\theta)$. But $v = dx/dt = (dx/d\theta)\ (d\theta/dt) = \omega r \sin\theta = \text{V}\sin\theta =$ A C on the scale on which O C $=$ V. The acceleration of the crosshead is $a = dv/dt = (dv/d\theta)\ (d\theta/dt) =$

$$\frac{d}{d\theta}\,(\omega r \sin\theta)\,\frac{d\theta}{dt} = \omega^2 r \cos\theta$$

$=$ O A on the scale on which O C $= \omega^2 r$.

71. *Different ways of plotting the curves.*—The variable quantities s or v or a may be plotted as ordinates, (a) with the crank-pin travels as abscissæ, or (b) with the crosshead travels as abscissæ. In the former case for one revolution the base of the curve is a line equal to the circumference of crank-pin circle; in the latter a line equal to the stroke. Again, the variable for any position of the crank may be

laid off on the crank, either from the centre or circumference of the crank-pin circle. Then the curve obtained is a polar curve of the values of the variable. Lastly, the variable quantities may be taken as ordinates, the abscissæ being the time elapsed from the beginning of a revolution. When v is constant the crank-pin travel is proportional to the time; and the curve, which to one scale of abscissæ is a curve with crank-pin travel base, is on another scale a curve to a time base. If, however, v is not constant, then the two curves differ.

72. *Ordinary crank and connecting rod. Piston travel for different crank angles. Effect of obliquity of connecting rod.*[1]—Let $r = \text{OC}$ be the crank length, and $l = \text{CD}$ the

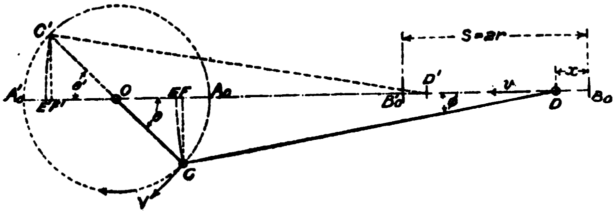


Fig. 63

connecting-rod length of an ordinary direct-acting engine. Then $\nu = l/r$ is a ratio commonly having a value between 4 and 6. Set off $A_0 B_0 = A'_0 B'_0 = l$, then $B_0 B'_0$ is the path of the crosshead, and A_0, A'_0 are the dead points of the mechanism. The crosshead, piston, and reciprocating parts have a stroke $s = 2r = A_0 A'_0 = B_0 B'_0$. Suppose the engine rotates in the direction of the arrow. During the *outstroke* the crank pin moves from A_0 to A'_0 and the crosshead from B_0 to B'_0. During the *instroke* the crank pin moves from A'_0 to A_0 and the crosshead from B'_0 to B_0. If in a time t the crank moves through an angle $A_0 O C = \theta$, the reciprocating

[1] A very interesting study of the inertia forces in an actual engine is that of Prof. V. Dwelshauvers-Dery, 'Etude expérimentale dynamique d'une Machine à Vapeur,' Gauthier Villars et fils.

parts move a distance $x = B_0D$, found by striking an arc with centre C and radius l. From centre D with radius $l = DC$ describe the arc EC and draw CF perpendicular to the line of stroke. Then the point E divides $A_0A'_0$ in the same ratio that D divides $B_0B'_0$ and $x = B_0D = A_0E$. With ordinary values of ν the distance EF is not very large, and may be regarded as the error in the motion of the reciprocating parts due to the obliquity of the connecting rod. As an approximation the distance A_0F is often taken as the piston travel instead of A_0E, and in that case $x = A_0F = r(1 - \cos\theta)$ nearly.

More exactly, if in the outstroke the piston travels from B_0 to D in time t,

$$x = B_0D = A_0E = A_0F + FE$$
$$= r(1 - \cos\theta) + l(1 - \cos\phi).$$

But $OC/CD = \sin\phi/\sin\theta = r/l$

$$\sin\phi = (r\sin\theta)/l = (\sin\theta)/\nu$$
$$\cos\phi = \sqrt{(1 - \sin^2\theta/\nu^2)}$$
$$x = r(1 - \cos\theta) + l\{1 - \sqrt{(1 - \sin^2\theta/\nu^2)}\}.$$

But $\sqrt{(1 - \sin^2\theta/\nu^2)} = 1 - \frac{1}{2}(\sin\theta/\nu)^2 - \frac{1}{8}(\sin\theta/\nu)^4 \ldots$

Disregarding terms beyond the second

$$x = r(1 - \cos\theta) + \tfrac{1}{2}l(\sin\theta/\nu)^2 \text{ very nearly.}$$

Similarly, for the instroke,[1]

$$x' = A'_0E' = A'_0F' - E'F'$$
$$= r(1 - \cos\theta') - \tfrac{1}{2}l(\sin\theta'/\nu)^2 \text{ very nearly.}$$

When $\nu = l/r$ is very large, x and x' approach the value $r(1 - \cos\theta)$ for equal angles θ measured from the dead points. The crank angle may be found from the piston travel by the relations—

[1] The same expression is valid for outstroke and instroke, if θ is reckoned from the position OA_0. By reckoning from OA_0 for the outstroke and from OA'_0 for the instroke, some risk of error of sign is avoided.

Outstroke:

$$\text{Cos}\,\theta = -\nu + \sqrt{\left(1 + \frac{2(r-x)\nu}{r} + \nu^2\right)};$$

Instroke:

$$\text{Cos}\,\theta' = -\nu + \sqrt{\left(1 - \frac{2(r-x)\nu}{r} + \nu^2\right)}.$$

With centre B_0 and radius $B_0 A_0$, fig. 64, also with centre B'_0 and radius $B'_0 A'_0$ describe arcs of circles. Draw CG, C′G′ parallel to the line of stroke. Then $x = A_0 E = CG$ and $x' = A'_0 E' = G'C'$. This construction for the piston travel is convenient in some cases, as in finding the crank position in valve diagrams. The arcs through A_0, A'_0, being drawn,

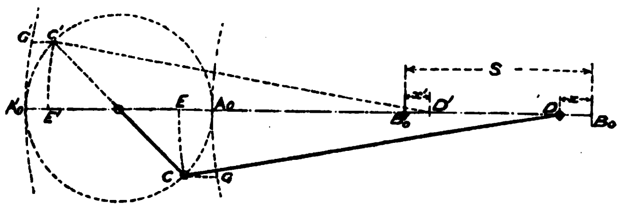


Fig. 64

the piston travel for any crank position OC or OC′ is easily found as the intercept CG or C′G′.

73. *Relation of crank pin and crosshead velocity.*—The crosshead D moves along the line of stroke with the velocity $v = dx/dt$. The crank pin C moves in the direction of a tangent to the crank-pin circle with the velocity V. The crank turns with the angular velocity $\omega = V/r = d\theta/dt$. In many problems ω may be taken constant. Then, if the crank makes N revolutions per minute, $\omega = 2\pi N/60 = \pi N/30$ in radians per second. C and D are connected points. In fig. 65 draw DI, CI perpendicular to the direction of motion of these points. Then I is the instantaneous axis of the link CD, and $v/V = DI/CI$. Draw OK perpendicular to OD, and produce DC to meet it in K. From similar triangles $v/V = DI/CI = OK/OC$, and OK represents the

crosshead velocity to the scale on which O C $= r$ represents the crank-pin velocity. Another construction is sometimes convenient. If C $c =$ V to any scale, and $c\,d$ is parallel to C D, then D $d = v$. Let G be any point in C D, join I G and draw O g parallel to it, so that g divides C K in the same ratio that G divides C D. If u is the velocity of G, $u/\mathrm{V} = \mathrm{IG}/\mathrm{IC} = \mathrm{O}g/\mathrm{OC}$, and the direction of u is perpendicular to I G.

Remembering that $\omega = \mathrm{V}/r$, we have

$$\mathrm{V} = \omega r; \; v = \omega \times \mathrm{OK}; \; u = \omega \times \mathrm{O}g.$$

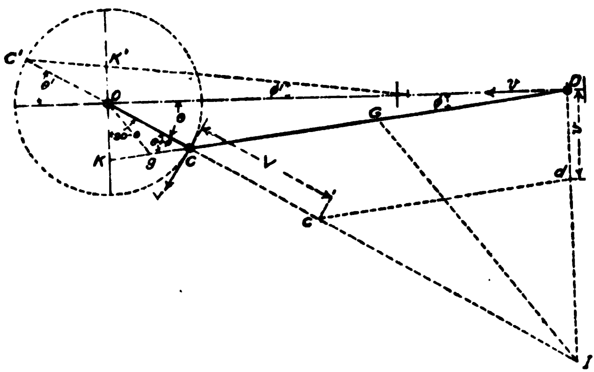


Fig. 65

It is convenient, also, to express the velocities in terms of the crank angle θ, and inclination of the connecting rod ϕ.

But $$\frac{\mathrm{OK}}{\mathrm{OC}} = \frac{\sin \mathrm{OCK}}{\sin \mathrm{OKC}} = \frac{\sin(\theta + \phi)}{\cos \phi}.$$

For the outstroke

$$v/\mathrm{V} = \sin(\theta + \phi)/\cos\phi;$$

For the instroke

$$v'/\mathrm{V} = \sin(\theta' - \phi')/\cos\phi'.$$

Hence, $$\left.\begin{aligned} v &= \mathrm{V}\,(\sin\theta + \cos\theta\tan\phi) \quad . \\ v' &= \mathrm{V}\,(\sin\theta' - \cos\theta'\tan\phi) \quad . \end{aligned}\right\} \quad (1)$$

which give the crosshead velocities for the outstroke and instroke.

But
$$\nu = l/r = \sin\theta/\sin\phi$$
$$\sin\phi = (\sin\theta)/\nu$$
$$\tan\phi = \sin\phi/\{\sqrt{(1 - \sin^2\phi)}\}$$
$$= \sin\theta/\{(\nu^2 - \sin^2\theta)\}$$
$$\therefore\ v = \mathrm{v}\left\{\sin\theta\left(1 + \frac{\cos\theta}{\sqrt{(\nu^2 - \sin^2\theta)}}\right)\right\} \quad . \quad (2)$$

But since in practical cases ϕ is always a small angle, $\tan\phi = \sin\phi = (\sin\theta)/\nu$ nearly.

Then for the outstroke, very approximately,
$$v = \mathrm{v}\ \{\sin\theta + (\cos\theta\sin\theta)/\nu\}$$
$$= \mathrm{v}\ \{\sin\theta + (\sin 2\theta)/2\nu\} \quad . \quad . \quad (3)$$
and for the instroke
$$v' = \mathrm{v}\ \{\sin\theta' - (\sin 2\theta')/2\nu\} \quad . \quad . \quad (3a)$$

If ν is very large, or the connecting rod is treated as if it were of great length, then simply $v = \mathrm{v}\sin\theta$ and $v' = \mathrm{v}\sin\theta'$. These may be called the values of v and v' when the obliquity of the connecting rod is neglected.

74. *Angular velocity of the connecting rod.*—The crank rotates about O, and its angular velocity is $\omega = \mathrm{v}/r = \mathrm{v}/\mathrm{OC}$. The connecting rod rotates about I, and its angular velocity is $\Omega = \mathrm{v}/\mathrm{CI}$. Hence $\Omega/\omega = \mathrm{CO}/\mathrm{CI} = \mathrm{KC}/\mathrm{CD} = \mathrm{KC}/l$. In the cases in which ω is constant, the angular velocity of the connecting rod is proportional to KC.

$$\mathrm{KC} = \mathrm{OC}\,\frac{\sin(90 - \theta)}{\sin(90 - \phi)} = \mathrm{OC}\,\frac{\cos\theta}{\cos\phi}.$$

Since $\nu = l/r$, and ϕ is small,

$$\mathrm{KC} = \frac{r\cos\theta}{1 - \frac{1}{2}\,\frac{\sin^2\theta}{\nu^2}}\ \text{nearly}$$

$$\Omega = \omega\,\frac{\nu\cos\theta}{\nu^2 - \frac{1}{2}\sin^2\theta} \quad . \quad . \quad . \quad (4)$$

75. *Ratio of efforts at crosshead and crank pin.*—If friction and the inertia of intervening parts are neglected, then the ratio of the effective forces or efforts at two points of a mechanism is the reciprocal of the velocity ratio. Let p be the effort in the line of stroke at the crosshead due to the steam pressure on the piston, and t the effort at the crank-pin tangential to the crank-pin circle. These are conveniently reckoned per sq. in. of piston. Then

$$\frac{t}{p} = \frac{v}{\mathrm{V}} \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

It is only in slow-moving machines that the inertia of the heavy parts can safely be neglected. Generally the steam effort p_s at the crosshead is known, and from this the inertia force of connecting rod, piston rod, and piston must be deducted to find p, before proceeding to calculate the crank-pin effort, t. The friction is commonly small enough to be negligible. The inertia forces of the parts having a simple motion of translation can be found, by methods given later, comparatively easily. But the motion of the connecting rod is more complicated, and its exact inertia effect is not so easily found. So long as the problem is only the determination of the crank-pin effort, it is generally approximate enough to treat the connecting-rod weight as part of the total reciprocating weight. If in this way p is found from p_s by deducting (or adding) the inertia force due to the reciprocating masses, t may be found from it by Eq. 1. Then, substituting for v/V,

$$\begin{aligned} t &= p\ \{\sin\theta + (\sin 2\theta)/2\nu\} \text{ outstroke} \\ &= p\ \{\sin\theta' - (\sin 2\theta')/2\nu\} \text{ instroke} \end{aligned} \qquad (2)$$

In the graphic construction $t/p = \mathrm{D}\,d/\mathrm{C}\,c$, which is convenient in constructing diagrams of crank-pin effort or twisting moments, when p is taken from indicator diagrams corrected for inertia.

76. *Graphic representation of piston or crosshead velocity.*—Let o *d*, fig. 66, be the crank length *r*, and *d c* the connecting-rod length *l*, as before. Take *d f* = v, the crank-pin velocity, draw *f g* parallel to the connecting rod to meet a perpendicular to the line of stroke in *g*. Then *c g* is the crosshead or piston velocity *v*. If the ordinate, *c g*, is found for several positions of the crank, the locus of *g* is an oval curve, which becomes an ellipse if the connecting rod is indefinitely long. This is the curve of piston velocity with *abscissa* = piston displacement, and *ordinate* = piston velocity. It is sometimes convenient to set off *c g* = *v* on the crank at o *e*. Then the locus of *e* is a kind of lemniscate curve which becomes a pair of circles for the case of a very long connecting rod. This is the polar curve of piston velocities, the vector of the curve in the direction of the crank being the piston velocity. The maximum piston velocity occurs very nearly when the crank and connecting rod are at right angles.

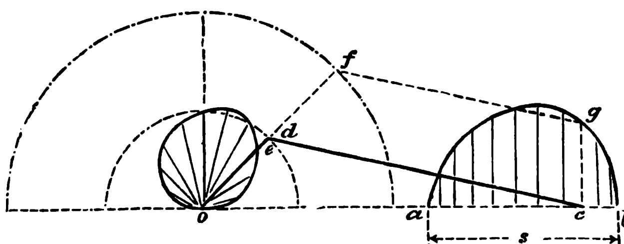


Fig. 66

From what has been said as to the relation of the force ratio and velocity ratio, it will be seen that the curves in fig. 66, which are curves of crosshead velocity when the crank pin has a constant velocity, are also curves of tangential effort at the crank pin, when the piston effort is constant, as it is for instance very nearly in pumps lifting against a constant head.

77. *Acceleration of reciprocating parts in the line of stroke.*—Using the notation above, at the time t from the beginning of the outstroke when the crank has turned through θ, the acceleration is, when v is constant,[1]

$$a = \frac{dv}{dt} = \frac{dv}{d\theta} \cdot \frac{d\theta}{dt} = \omega \frac{dv}{d\theta} = \frac{\mathrm{v}}{r} \frac{dv}{d\theta} \qquad (3)$$

Using Eq. 1, for the outstroke,

$$a = \frac{\mathrm{v}^2}{r} \left\{ \cos\theta + \frac{\sin^2\theta}{\nu\cos\phi} - \frac{\cos^2\theta}{\nu\cos^3\phi} \right\}$$

$$= \frac{\mathrm{v}^2}{r} \left\{ \cos\theta + \frac{\nu^2 \cos 2\theta - \sin^4\theta}{(\nu^2 - \sin^2\theta)^{3/2}} \right\} \qquad (4)$$

But using Eq. 3, we get the much simpler approximate expression,

$$\left.\begin{aligned} a &= \frac{\mathrm{v}^2}{r} \left\{ \cos\theta + (\cos 2\theta)/\nu \right\} \text{ for the outstroke.} \\ &= \frac{\mathrm{v}^2}{r} \left\{ \cos\theta' - (\cos 2\theta')/\nu \right\} \text{ for the instroke.} \end{aligned}\right\} \qquad (5)$$

For an indefinitely long connecting rod $l/r = \nu = \infty$ and then $a = (\mathrm{v}^2 \cos\theta)/r$, or $(\mathrm{v}^2 \cos\theta')/r$ for the out and in strokes.

The outstroke begins, $\theta = 0$, with the acceleration $\frac{\mathrm{v}^2}{r}\left(1 + \frac{1}{\nu}\right)$; and ends, $\theta = \pi$, with the retardation $-\frac{\mathrm{v}^2}{r}\left(1 - \frac{1}{\nu}\right)$. Similarly the instroke begins, $\theta' = 0$, with the acceleration $\frac{\mathrm{v}^2}{r}\left(1 - \frac{1}{\nu}\right)$; and ends with the retardation $-\frac{\mathrm{v}^2}{r}\left(1 + \frac{1}{\nu}\right)$. These values are exact, because $\phi = 0$ for these values of θ. It is sometimes convenient to

[1] If v is not constant and β is the acceleration of the crank pin, then $a = \beta \frac{v}{\mathrm{v}} + \frac{\mathrm{v}}{r}\frac{dv}{d\theta}$.

substitute $\omega^2 r$ or $(\pi^2 N^2 r)/900$ for V^2/r. N is the number of revolutions per minute.

Fig. 67 represents the inertia forces, p_i, during an out and in stroke when $\nu = l/r = 5$. The forces are drawn downwards during acceleration when the force required to accelerate the mass diminishes the effective effort, and upwards during retardation when the energy stored is given back and increases the effective effort. The numbers in the figure must be multiplied by $w V^2/gr$ to give the

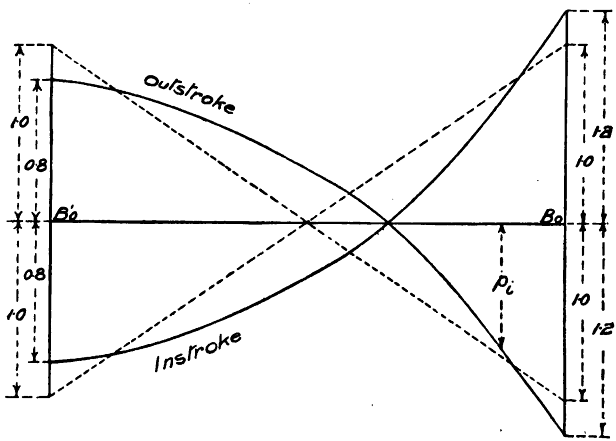


Fig. 67

inertia forces. The units are the foot, the second, and the lb. The dotted lines give the inertia forces for an indefinitely long connecting rod, when $\nu = \infty$. The unsymmetry of the curves is greater as ν is smaller.

78. *Forces in the line of stroke due to the inertia of reciprocating parts.*—The steam efforts transmitted to the crank pin are commonly reckoned per sq. in. of piston area. It is convenient to deal with the inertia forces in the same way. Let w be the weight in lbs. of the reciprocating

parts (piston, rod, crosshead, and slide block) reckoned per sq. in. of piston area. For the purpose of designing, the following values may be assumed provisionally for w, and they include the connecting-rod weight.

	Weight of reciprocating parts in lbs. per sq. in. of piston w
Simple expansion engines	2·0 to 3·0
Compound engines, L.P. cylinder	2·1 „ 2·5
„ H.P. cylinder	5·0 „ 7·0
Triple engines, L.P. cylinder	1·0 „ 1·5
„ I.P. cylinder	1·7 „ 2·1
„ H.P. cylinder	2·8 „ 4·2

Let a be the acceleration in feet per sec. per sec.; p_i the force required to accelerate the reciprocating parts; p_s the steam effort on the piston, both reckoned in lbs. per sq. in. Let w = the weight of reciprocating parts in lbs. per sq. in. of piston. Then

$$p_i = \frac{w}{g} a,$$

and the effective effort at the crosshead is

$$p = p_s - p_i.$$

It must be remembered that p_i is + during acceleration and − during retardation. During nearly half the outstroke and a little more than half the instroke the inertia diminishes the crosshead effort. During the remainder it increases it. In engines with large expansion and diminishing steam pressure as the piston moves forward, the inertia forces tend sometimes in an important degree to render more constant the crosshead effort.

The following Table gives values of the translational acceleration a and the force required to produce acceleration p_i for various crank angles, and for two values of l/r or v.

Angle from Dead Point	Acceleration $a = \frac{v^2}{r} \times$	Force due to inertia $p_i = \frac{w}{g} \cdot \frac{v^2}{r}$ $\nu = 5$	$\nu = \infty$
Outstroke $\theta = 0°$	$1 + \frac{1}{\nu}$	$-1{\cdot}2$	$-1{\cdot}0$
45	$\cdot707 + \frac{0{\cdot}25}{(\nu^2 - 0{\cdot}5)^{3/2}}$	$-0{\cdot}709$	$-0{\cdot}707$
90	$-\frac{1}{\sqrt{(\nu^2 - 1)}} = \frac{1}{\nu}$ nearly	$0{\cdot}204$	$0{\cdot}0$
135	$-\left\{\cdot707 - \frac{0{\cdot}25}{(\nu^2 - 0{\cdot}5)^{3/2}}\right\}$	$0{\cdot}705$	$0{\cdot}707$
180	$-\left(1 - \frac{1}{\nu}\right)$	$0{\cdot}8$	$1{\cdot}0$
Instroke $\theta' = 0°$	$1 - \frac{1}{\nu}$	$-0{\cdot}8$	$-1{\cdot}0$
45	$\cdot707 - \frac{0{\cdot}25}{(\nu^2 - 0{\cdot}5)^{3/2}}$	$-0{\cdot}705$	$-0{\cdot}707$
90	$\frac{1}{\sqrt{(\nu^2 - 1)}} = \frac{1}{\nu}$ nearly	$-0{\cdot}204$	$0{\cdot}0$
135	$-\left\{\cdot707 + \frac{0{\cdot}25}{(\nu^2 - 0{\cdot}5)^{3/2}}\right\}$	$0{\cdot}709$	$0{\cdot}707$
180	$-\left(1 + \frac{1}{\nu}\right)$	$1{\cdot}2$	$1{\cdot}0$

79. *Graphic construction of the curve of acceleration of reciprocating parts. Method of Rittershaus.* ('Civilingenieur,' xxv. s. 461).—The simplest graphic construction for determining the translational acceleration and force due to inertia of reciprocating parts is that of Rittershaus. In fig. 68, produce the connecting rod D C to meet a perpendicular at O to the line of stroke in K. Draw K M perpendicular to K D and join C M. Draw K N parallel to C M. Then it can be shown that the acceleration of D is represented by the difference M N — N O, to the scale [1] on which O C represents

[1] If m ins. represent 1-foot length on the scale of the drawing, it represents ω^2 feet per sec. per sec. on the scale of accelerations.

the radial acceleration $\omega^2 r = V^2/r$ of C. Take MP = MN. Then the acceleration to this scale of D is

$$a = OP = MN - NO.$$

Join PC. Then OPC is a diagram of accelerations just as OCK is a diagram of velocities (§ 73). If G is any point in the connecting rod, and $Pg/PC = DG/DC$, then Og represents in magnitude and direction the acceleration at G. The point g may be found by drawing Gg parallel to the line of stroke.

Let $OC = r$; $CD = l$; $OD = x$; $OK = h$; $KC = k$; $DOC = \theta$; $ODC = \phi$. Let the crank revolve uniformly with velocity $\omega = V/r$. The angular velocity of the connecting rod is given by the relation $\Omega/\omega = KC/CD$. Hence $\Omega = \omega k/l$.

The velocity of the crosshead is

$$v = \frac{dx}{dt} = V\frac{OK}{OC} = V\frac{h}{r} = \omega x \tan\phi \quad . \quad . \quad (1)$$

But the acceleration at D is,

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\omega x \tan\phi\right)$$

$$= \omega\left\{\tan\phi\frac{dx}{dt} + \frac{x}{\cos^2\phi}\frac{d\phi}{dt}\right\} \quad . \quad (2)$$

The angular velocity of the connecting rod is (§ 74)

$$\Omega = -\frac{d\phi}{dt} = -\frac{\omega k}{l} \quad . \quad . \quad (3)$$

Inserting this value and that of dx/dt in Eq. 2,

$$a = \omega^2\left\{x\tan^2\phi - \frac{x}{\cos^2\phi}\frac{k}{l}\right\}$$

$$= \omega^2\left\{\frac{h^2}{x} - \frac{x}{\cos^2\phi}\frac{k}{l}\right\} . \quad . \quad . \quad (4)$$

But by the construction—

$$\cos\phi = KD/MD = OD/KD = x/KD$$

$$MD = KD/\cos\phi = x/\cos^2\phi$$

$$\text{NM}/\text{MD} = \text{KC}/\text{CD} = k/l$$
$$\text{MN} = (k/l)/\text{MD} = (kx)/l\cos^2\phi$$
$$\text{MO}/\text{OK} = \text{OK}/\text{OD}$$
$$\text{MO} = h^2/x$$
$$\text{MN} - \text{MO} = -\left(\frac{h^2}{x} - \frac{kx}{l\cos^2\phi}\right)$$

and if $\text{MP} = \text{MN}$

$$a = -\omega^2(\text{MN} - \text{MO}) = -\omega^2 \,.\, \text{OP}.$$

So that OP represents the acceleration of D on the scale on which OC represents the radial acceleration $\omega^2 r$ of C. For the inner and outer dead points the construction fails. For these points,

$$x = l \mp r;\ k/l = \pm r/l;\ h = 0;\ \cos\phi = 1.$$

Hence, as found before by analysis (§ 77) the accelerations are

$$\text{Inner dead point, } a_0 = \pm\omega^2 r\left(1 + \frac{r}{l}\right);$$
$$\text{Outer dead point, } a'_0 = \mp\omega^2 r\left(1 - \frac{r}{l}\right);$$

the upper signs being taken for the outstroke, and the lower for the instroke.

It is often easier to make the construction thus:—Draw KR parallel to OD to meet OC produced in R; draw RS perpendicular to KR to meet CD in S. Finally draw SP perpendicular to CD to meet OD in P. Then it can be shown that OP thus found is equal to MN − MO. For:

$$\frac{\text{MN}}{\text{MD}} = \frac{\text{KC}}{\text{CD}} = \frac{k}{l}$$
$$\text{MD} = \frac{\text{KD}}{\cos\phi} = \frac{l + k}{\cos\phi}$$
$$\cos\phi = \frac{\text{OD}}{\text{KD}} = \frac{x}{l + k}$$
$$\text{MD} = \frac{(l + k)^2}{x} \quad . \quad . \quad . \quad (1)$$

$$MN = \frac{(l+k)^2}{x} \cdot \frac{k}{l} \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

$$\frac{MP}{MD} = \frac{KS}{KD}$$

$$MP = \frac{MD \times KS}{KD} = \frac{KS}{\cos\phi} = \frac{KC + CS}{\cos\phi} \quad . \qquad (3)$$

But, $\frac{KC}{CD} = \frac{CR}{CO} = \frac{CS}{CK}$

$$CS = \frac{CK^2}{CD} = \frac{k^2}{l} \quad . \; . \quad . \quad . \quad . \quad . \qquad (4)$$

Putting $KC = k$, and this value of CS in (3),

$$MP = \frac{k + \frac{k^2}{l}}{\cos\phi} = \frac{k\,(l+k)}{l\cos\phi} = \frac{k\,(l+k)^2}{l\,x}.$$

Comparing this with Eq. 2,

$$MP = MN$$

and $OP = MN - MO.$

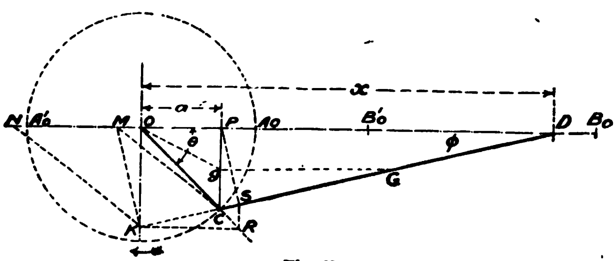


Fig. 68

80. *Construction of Klein and Massau for the curve of acceleration and inertia of reciprocating parts.*—Let fig. 69 represent one position of the mechanism, $OC = r$ being the crank, and $CD = l$ the connecting rod, and ω the constant angular velocity of the crank in radians. Produce DC to meet a perpendicular at O to the line of stroke in K. Then if $V = \omega r$ is the velocity of C, the velocity of D is $v = \omega \times OK$.

With centre at centre of connecting rod, and radius half the connecting-rod length, describe the arc E C F. With centre C, and radius C K, describe the arc E K F, cutting E C F in E and F. Join E F cutting the line of stroke in H. Then H D represents the acceleration of D in magnitude and direction on the scale on which C O represents the radial acceleration of C; that is, the scale on which r on the scale of feet represents $\omega^2 r$ feet per sec. per sec. The point H is the same point as P in fig. 68. If H O is set up as an ordinate at D L, and the same construction is made for other positions of the mechanism, points are obtained on the curves

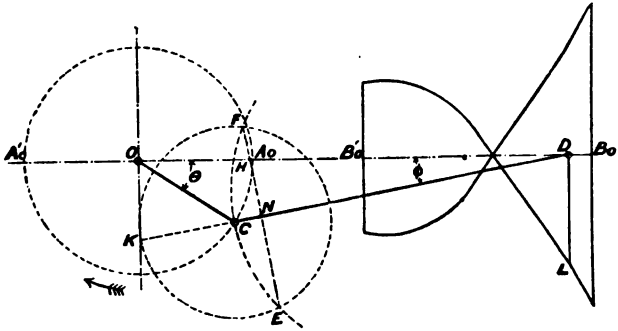


Fig. 69

of acceleration of D, or of the reciprocating parts of the engine. The acceleration of D is $\omega^2 r$ (H O / C O) $= \omega^2 \times$ H O, where H O is measured on the scale of feet.

Let w be the weight of the reciprocating parts reckoned in lbs. per sq. in. of piston. Then the inertia force in lbs. per sq. in. of piston is

$$p_1 = \frac{w}{g} \cdot \frac{V^2}{r} \frac{HO}{CO} = \frac{w}{g} \omega^2 r \frac{HO}{CO}$$

$$= \frac{w}{g} \omega^2 \times HO$$

The motion of the connecting rod is equivalent to a trans-

lation parallel to O D in common with the point D, and a rotation about D with the angular velocity (§ 74) $\Omega = \omega$ (K C / C D). Considered as a point of the crank pin, C has the total radial acceleration ω^2 O C, represented on the scale stated above by O C. But C considered as a point of the connecting rod has the same total acceleration, which may be considered as made up of a component $\omega^2 \times$ O H along O D, due to the motion in common with D; a component $\omega^2 \times$ H N perpendicular to C D due to rotation about D; and a radial component $\omega^2 \times$ C N along C D due to rotation about D.

81. *Extension of Klein's construction by Prof. Dunkerley.* In fig. 70, the acceleration of any point G of the connecting

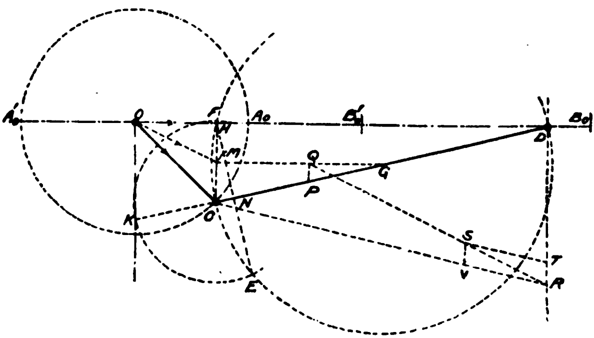


Fig. 70

rod is found by drawing G M parallel to D O, cutting C H in M and joining O M. Then O M represents the acceleration of G to the scale on which O C represents $\omega^2 r$. For the acceleration of G, due to the motion in common with D, is O H as before; but the radial and tangential components due to rotation about D are less than at C in the ratio D G / D C. The triangle O H C is the same triangle of accelerations as O P C, fig. 68.

82. *Crank angle when the crosshead velocity is greatest.*— It is easily seen that O K, fig. 65, will have nearly its greatest

value, and therefore the crosshead velocity will be nearly at its maximum, when the connecting rod is at right angles to the crank. The determination of the exact value of θ, which makes v a maximum, is however a problem of some difficulty. A solution was first found by Prof. Hill ('Proc. Inst. Civil Engineers,' cxxiv. p. 390). An easier solution may be found by assuming the construction of Rittershaus for the acceleration, for the crosshead velocity will be a maximum when the acceleration is zero.[1] The importance of the problem has perhaps been exaggerated, and it is sufficient to give the result of the investigation. The required value of θ is given by the equation,

$$\sin^6\theta - \nu^2\sin^4\theta - \nu^4\sin^2\theta + \nu^4 = 0 \quad . \quad . \quad (1)$$

Let $x = \sin^2\theta$,

$$x^3 - \nu^2 x^2 - \nu^4 x + \nu^4 = 0 \quad . \quad . \quad . \quad (2)$$

Let $y = \nu^2/3 - x$; and let $k = 4\,\nu^2/3$, and from tables find an angle β such that

$$\sin 3\beta = \frac{27}{16}\left(\frac{11}{27} - \frac{1}{\nu^2}\right).$$

Then the roots of Eq. 2 are

$$\begin{aligned} y &= k\sin\beta \\ &= k\sin(60 - \beta) \\ &= -k\sin(60 + \beta); \end{aligned}$$

of these roots the first only gives possible values of $\sin\theta$. This root gives

$$\sin^2\theta = x = \frac{\nu^2}{3} - k\sin\beta, \quad . \quad . \quad (3)$$

an equation from which the required value of θ can be found.

[1] Unwin: 'Proc. Inst. Civil Engineers,' cxxv.

The Author found the following expression, which is simple and very nearly exact,

$$\sin^2\theta = \frac{\nu^2}{\nu^2+1} + \frac{2\nu^2+1}{(\nu^2+1)(\nu^4+4\nu^2)} \quad . \quad (4)$$

Values of θ for $\nu =$

	2	3	4	5
Eq. 2 .	67° 41′ 59″	73° 10′ 30″	76° 43′ 24″	79° 6′ 32″
Eq. 3 .	67° 43′ 10″	73° 10′ 31″	76° 43′ 15″	79° 5′ 58″

83. *Advantages and disadvantages of the inertia of the reciprocating parts.*—In every reciprocating engine the resultant force at the crank pin changes from a push to a pull and back again to a push in every revolution. Now, as there must always be some slack in the crank pin and crosshead brasses, there is a liability to a knock of more or less violence if this change occurs suddenly or at much velocity. Probably the liability to an injurious knock is least if the change occurs at or before the dead points, where the motion in the direction of the line of stroke is limited to the slack of the brasses. A knock may be produced even at this point, if the initial pressure of the steam is very great, and compression is supposed to be useful in preventing a knock of this kind. Probably, however, a serious knock is generally produced in another way. If there is little compression and if the inertia of the reciprocating parts is great, the direction of the effort at the crank pin may not change till some time after the beginning of the stroke; or, in other words, for a sensible part of the stroke the crank-pin effort acts opposite to the piston effort. This occurs if the inertia line rises above the piston-effort line at the beginning of the stroke. Then a knock occurs from the reciprocating parts catching up with the crank pin. The heavier the reciprocating parts, the greater the speed of the engine and the less the compression, the more likely is it that there will be a negative crank effort for part of the beginning of the

stroke. Hence, taking it for granted that the weight of the reciprocating parts cannot practically be much modified, the arising of this negative crank effort fixes a limit of speed at which the engine can be run quietly. In order to get rid of the tendency to knock, some high-speed engines have been built with hollow piston rods, and in other cases single-acting engines have been adopted, so arranged that in all conditions there is a thrust between the connecting rod and crank.

On the other hand, the action of the inertia of the reciprocating masses is advantageous in equalising the crank-pin effort in engines having much expansion. The work

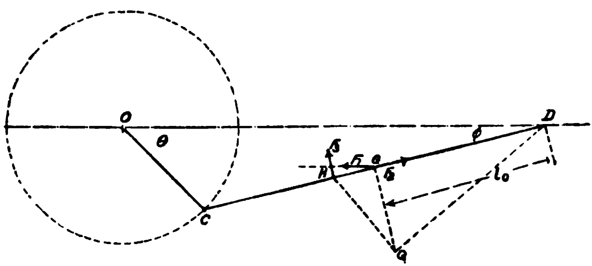


Fig. 71

expended in accelerating the reciprocating masses in the first half of the stroke when the steam pressure is high is given back in the second half of the stroke when the steam pressure has fallen from expansion. By suitably choosing the weight of the reciprocating masses for any given ratio of expansion a very uniform crank-pin effort can be secured. Generally, however, it is more convenient to depend on a fly-wheel to equalise the twisting moment than to alter the weight of the piston or crosshead.

84. *Inertia forces acting on the connecting rod.*[1]—The motion of the connecting rod is equivalent to a translation parallel to the line of stroke, and a rotation about the

[1] See Fleeming Jenkin, 'Trans. Roy. Soc. Edinburgh,' vol. xxviii. p. 711.

crosshead pin. The inertia forces, when the crank has the constant angular velocity ω, consist of three components :—(1) The force producing the change of velocity of translation F_1, which acts at the centre of gravity G, of the rod (fig. 71), parallel to the line of stroke. (2) A force F_2 equal and opposite to the centrifugal force of the rod due to its rotation, which acts at G along the rod towards D. (3) The force F_3 which produces change of angular velocity which acts perpendicular to the rod at the centre of percussion H. Let W be the weight of the rod in lbs. ; l_0 the distance D G of the centre of gravity from D ; a the acceleration of D in the line of stroke which has been found in the preceding articles. The angular velocity of the rod is $d\phi/dt$ and its angular acceleration is $d_2\phi/dt^2$.

$$\left.\begin{aligned} F_1 &= \frac{W a}{g} \\ F_2 &= \frac{W}{g} l_0 \left(\frac{d\phi}{dt}\right)^2 \\ F_3 &= \frac{W}{g} l_0 \frac{d_2\phi}{dt^2} \end{aligned}\right\} \quad . \quad (1)$$

Let $\nu = l/r$, the ratio of rod length to crank radius ; $\sin\phi = (\sin\theta)/\nu$; also $d\theta/dt = \omega$.

The acceleration of D in the line of stroke has already been shown to be

$$a = \omega^2 r \{\cos\theta + \cos 2\theta/\nu\}$$

$$\frac{d\phi}{dt} = \frac{\cos\theta}{\sqrt{(\nu^2 - \sin^2\theta)}} \frac{d\theta}{dt} = \frac{\omega\cos\theta}{\sqrt{(\nu^2 - \sin^2\theta)}} \quad . \quad (2)$$

When the rotation of the crank is uniform $d_2\theta/dt^2 = 0$, then,

$$\frac{d_2\phi}{dt^2} = -\frac{(\nu^2 - 1)\sin\theta}{(\nu^2 - \sin^2\theta)^{3/2}} \left(\frac{d\theta}{dt}\right)^2$$

$$= -\frac{\omega^2(\nu^2 - 1)\sin\theta}{(\nu^2 - \sin^2\theta)^{3/2}} \quad . \quad . \quad (3)$$

If I is the moment of inertia of the rod about its centre

of gravity G, the square of its radius of gyration about the same axis is $\rho^2 = I/w$, and the centre of percussion can be found by taking $GH = \rho^2/l_0$. Or if $GQ = \rho$ and QH is perpendicular to DQ, H is the centre of percussion.

Example.—The following data are taken from the connecting rod shown in fig. 72; length $= l =$ 62 ins.; crank radius $= r =$ 11 ins.; $\nu = l/r =$ 5·63; velocity of crank pin $= V =$ 9·6 feet per sec.; angular velocity $= \omega = V/r$ = 10·5 radians per sec.; radial acceleration of crank pin $= V^2/r =$ 101 feet per sec. per sec.; weight of rod = W = 145 lbs. Dividing the rod into segments and taking moments about the crosshead, the distance $l_0 = DG$ of the

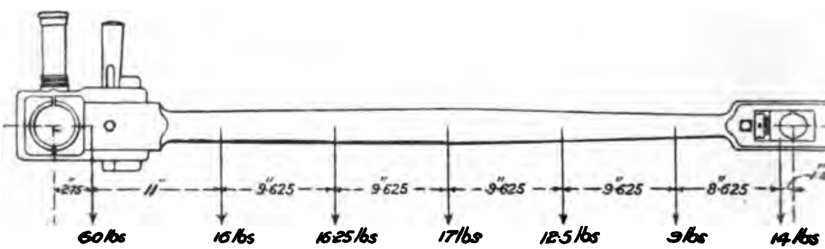


Fig. 72

centre of gravity from G is 39·25 ins. An approximate calculation gives the moment of inertia about an axis at D = 310815 in. lb. units, and about an axis through G = 87915 in. lb. units. Hence the square of the radius of gyration about G is $= \rho^2 = 87915/145 = 607{\cdot}3$, and the radius of gyration is $= \rho = 24{\cdot}64$ ins. The distance GH to the centre of percussion is $607{\cdot}3/36{\cdot}25 = 15{\cdot}4$ ins.

Suppose the crank and connecting rod at right angles, then $\tan\theta = 5{\cdot}63$; and $\theta = 80°$ nearly.

$$F_1 = \quad 4{\cdot}07 \text{ lbs.}$$
$$F_2 = 287 \quad \text{"}$$
$$F_3 = \quad 4{\cdot}87 \text{ "}$$

When the crank and connecting rod are in line,

$$F_1 = 534 \text{ lbs.}$$
$$F_2 = 0$$
$$F_3 = 27{\cdot}4 \text{ ,,}$$

85. *Another method of finding the acceleration and inertia forces at any point of a connecting rod.*—The following method is convenient, especially when the problem is to find the straining action on the connecting rod due to inertia.

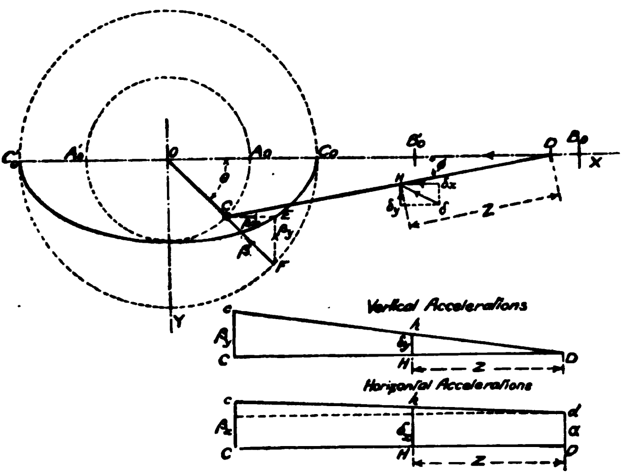


Fig. 73

The acceleration of D along D O (fig. 73) when v is the constant velocity of the crank pin is,

$$a = \frac{v^2}{r}\left\{\cos\theta + (\cos 2\theta)/\nu\right\}$$

and this is parallel to O X.

The radial acceleration of the crank pin C is,

$$\beta = v^2/r$$

Resolve this parallel to O X, O Y, the components are

$$\beta_x = \beta \cos\theta = (v^2 \cos\theta)/r$$
$$\beta_y = \beta \sin\theta = (v^2 \sin\theta)/r.$$

Graphically, if C F is taken to any scale of accelerations $= \beta$, then C E $= \beta_x$ and E F $= \beta_y$. Further, the point E is on the ellipse $C_0 E C'_0$. Hence the components of the acceleration at C for any position of the crank are easily found.

Consider a point H distant z from D. To find the accelerations parallel to the axes at H, draw the subsidiary figures. On a base D C, equal in length to the connecting rod set up, C $c = \beta_y$ the vertical acceleration at C. There is no vertical acceleration at D Consequently, if D H = z, the ordinate H $h = \delta_y$ is the vertical acceleration at H. Also on a line D C, equal to the connecting-rod length set up, C $c = \beta_x$ the horizontal acceleration at C, and D $d = a$ the horizontal acceleration at D. Then, if D H = z, the ordinate H $h = \delta_x$ is the horizontal acceleration at H. Transfer δ_x and δ_y to H in the original figure, and combine them ; we get the resultant acceleration δ at H. It is convenient to remember that $\delta_y = \beta_y z/l$ and $\delta_x = a + (\beta_x - a)\ z/l = a(1 - z/l) + \beta_x z/l$.

Now suppose the connecting rod cut up into a series of short lengths, each of which may be treated as concentrated at its mass centre. Let H be the mass centre of one such length, the weight of which is w. The force required to accelerate this part of the rod is $F = w\,\delta/g$, and it acts in the direction found by the graphic construction. If w is in lbs., and δ is measured in feet per sec. per sec., the accelerating force is in lbs. The resistance to acceleration which is to be combined with the effort transmitted to find the effective effort acts in the opposite direction to the acceleration.

The resultant inertia forces parallel to the axes O X, O Y are

$$F_x = \Sigma\, w\, \delta_x/g$$
$$F_y = \Sigma\, w\, \delta_y/g.$$

and if σ_x σ_y are the horizontal and vertical distances at which these resultants act from D, the moments about D are

$$M_x = \sigma_x \Sigma w \delta_x/g = \Sigma w \delta_x x/g$$
$$M_y = \sigma_y \Sigma w \delta_y/g = \Sigma w \delta_y y/g,$$

where $x = z \cos \phi$ and $y = z \sin \phi$. Values of $\sin \phi$ and $\cos \phi$ in terms of θ have been given. Hence from these equations the distances σ_x σ_y can be found, and therefore the positions and magnitudes of the resultant inertia forces.

By resolving the separate values of F into forces parallel and perpendicular to the rod, the direct and bending straining actions can be found.

86. *Steam effort on piston.*—Let fig. 74 represent two indicator diagrams, from the front and back ends of a cylinder as ordinarily taken on one card. When an engine is to be designed, the theoretical diagrams found as de-

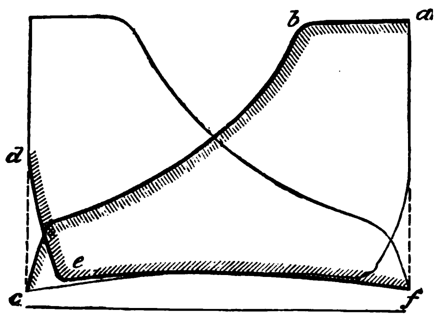


Fig. 74

scribed above must be taken instead of the actual diagrams. During the outstroke the forward pressure on the back of the piston is given by the vertical ordinates of the curve *a b c* of the back-end diagram, measured from the line of zero pressure. At the same time, the back pressure on the

front of the piston is given by the ordinates of the curve *fed* of the front-end diagram. The vertical intercept p_s at any point between the shaded curves is the effective pressure or piston effort due to the steam, which is transmitted to the crosshead during the outstroke. Similarly, the piston effort during the instroke is given by the vertical intercepts between the forward-pressure curve of the front-end diagram and the back-pressure curve of the back-end diagram. The intercepts give values of p_s per sq. in. of piston to the scale of the indicator spring used, and the total effort is $P_s = p_s A$ lbs., where A is the piston area.

If the areas of the front and back of the piston are equal, as when the piston rod and back rod are of equal size, the statement above is exact. But commonly the front area A_f and the back area A_b are not the same. Then a correction is required. Let A be the mean piston area and p_s the steam pressure per sq. in. of a piston of area A. Then the ordinates of the front diagram should be multiplied by A/A_f, and those of the back diagram by A/A_b, before measuring the intercept. But this correction is not usually of much importance.

Case of vertical engines.—In vertical engines there is a correction not unimportant. If the crank shaft is below the piston (inverted engines), the weight W of the reciprocating parts increases the effective effort during the down or out stroke, and diminishes it during the up or in stroke. Let $p_w = W/A$ be the weight of reciprocating parts, reckoned per sq. in. of piston. Then the effective piston effort is $p'_s = p_s \pm p_w$. The correction is most easily made by drawing the curve giving values of p_s, and then shifting the base line upwards for the upstroke and downwards for the downstroke a distance p_w. The heights of the curve to the new base lines are the values of p'_s. The weight of the moving parts above the crank pin is from 2 to 4 lbs. per sq. in. of piston in most engines, but in some cases reaches

6 lbs. The action of the weight in vertical engines is sufficient to materially alter the distribution of crank-pin

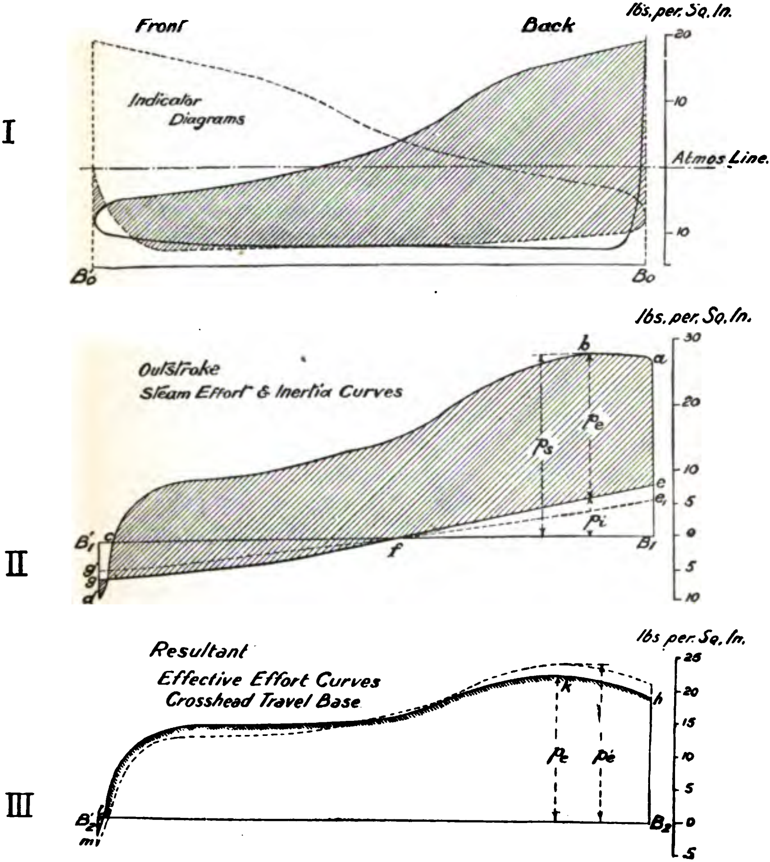


Fig. 75

effort, and is sometimes compensated for by an increased steam admission below the piston. It should be remembered that, if the effort at the crosshead is being considered

w should not include the connecting-rod weight. If the effort at the crank pin is considered, w should include the connecting-rod weight.

87. *Determination of resultant effort at crosshead and crank pin parallel to the line of stroke.*—In fig. 75, I, on the base $B_0 B'_0$, representing the stroke, the front and back diagrams of an engine are given. The effective piston effort p_s is then the intercept between the forward-pressure line of one diagram and the back-pressure line of the other, that is for the outstroke the vertical height of the shaded figure. In fig. 75, II, the values of p_s have been set up from the base $B_1 B'_1$, so as to give the curve of piston effort *a b c d*. In consequence of compression a part, *c d*, of this curve will generally have negative ordinates. On the same diagram has been plotted the translational inertia curve *e f g* of the reciprocating parts, the ordinates of which are the values of p_i found by the equations or constructions above. These are set off upwards when the parts are accelerated, and downwards when the parts are retarded. Then the effective effort parallel to the line of stroke at any point is the vertical intercept $p_e = p_s - p_i$ between the two curves or height of the shaded figure. Strictly there are two inertia curves, one *e f g* for the inertia force at the crank pin (connecting-rod weight included); the other *e′ f g′* for the inertia force at the crosshead (connecting-rod weight excluded). The values of the effective effort p_e set up on the base $B_2 B'_2$, give the curve of effective effort at the crank pin, fig. 75, III. The dotted curve is the curve of effective effort at the crosshead p'_e. In this case, also, if there is much compression, the effort becomes negative before the end of the stroke, so that for a portion of the stroke the engine, instead of driving, has to be driven. The fly-wheel in general supplies the driving effort for this part of the stroke. The curves have been drawn in fig. 75 for the outstroke. There are corresponding, but not quite, similar curves for the instroke.

88. *Thrust or tension in the connecting rod.*—The total effort at the crosshead is p'_e A lbs. If ϕ is the angle of the connecting rod and line of stroke, p'_e A is balanced by a tension or thrust p'_e A sec ϕ along the connecting rod, and a pressure p'_e A tan ϕ normal to the slide bars.

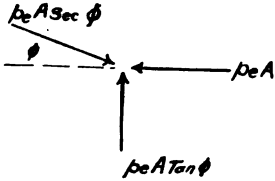


Fig. 76

At the crank-pin end of the connecting rod the force acting along the rod is p_e A sec ϕ, being different from that at the crosshead by the inertia of the connecting rod. Reckoned per sq. in. of piston, the effective forces at crosshead and crank pin are p'_e sec ϕ and p_e sec ϕ, acting along the connecting rod.

89. *Tangential effort and radial thrust at crank pin in the case of the double-slider crank chain.*—In the case shown

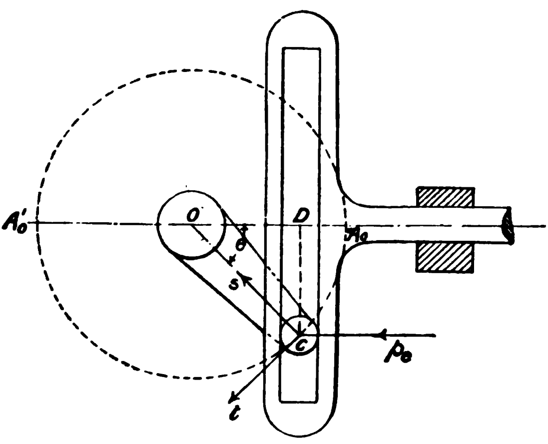


Fig. 77

in fig. 77, which is virtually the case of a crank driven by an infinitely long connecting rod, the stroke of the slotted

crosshead is $A_0 A'_0$, and the movement of the crosshead from A_0, when the crank has turned through θ, is $A_0 D = r(1 - \cos\theta)$. If $A_0 A'_0$ is the stroke, the position D of the crosshead for any crank position is found by dropping C D perpendicular to $A_0 A'_0$. If p_e is the crosshead effort per sq. in. of piston parallel to the line of stroke, then the tangential effort is $t = p_e \sin\theta$, and the radial thrust on the crank is $s = p_e \cos\theta$. If A is the piston area, the total crosshead effort is p_e A, and the total crank-pin effort is t A.

If p_{me} is the mean crosshead effort and t_m the mean crank-pin effort, these are connected by the relation

$$4 p_{me} r = t_m 2 \pi r$$

$$t_m / p_{me} = 4/2\pi$$

Since the crank radius is constant the twisting moment $t r$ on the crank shaft is proportional to t, and curves of crank-pin effort are also curves of crank-shaft twisting moment. Suppose the crosshead effort p_e constant, a condition approximately satisfied in some practical cases, as when the crosshead drives a plunger-pump against a constant head. Then, if values of t are set off along the crank in each position, a polar diagram of crank-pin efforts or of twisting moment is obtained, and this, in the conditions assumed, is a pair of circles, fig. 78. The dotted circles are polar curves of radial thrust.

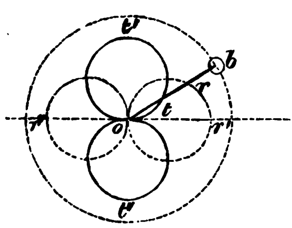


Fig. 78

The case of the double-slider crank mechanism occurs comparatively seldom in practice. It is a simpler case than that of the single-slider crank chain, and in approximate estimates is sometimes substituted for it. The results just given may be said to be true of the

single-slider crank chain when the obliquity of the connecting rod is neglected.

90. *Approximate construction of curves of crank-pin effort, to a base representing the stroke.*— In fig. 79, let A B C be the effective effort at the crank pin, parallel to the line of stroke, reckoned per sq. in. of piston, for the outstroke; that is, ordinates of the curve are values of p_e. Divide the stroke s into, say, ten equal parts, numbering them from right to left for the outstroke, and from left to right for the instroke. With centre 5 and radius r describe a crank-pin circle. Then I, II, III, . . . are approximate crank-pin positions corresponding to the crosshead positions 1, 2, 3, . . .

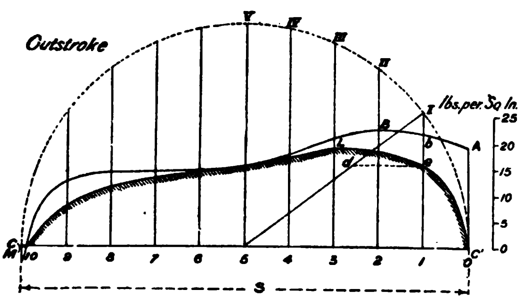


Fig. 79

5 I is the crank when the piston is at 1, and for that position $p_e = 1\,b$. Set off $5\,d = 1\,b$, and draw $d\,e$ horizontal. Then $1\,e = 5\,d \times \sin\theta = p_e \sin\theta = t$. If this construction is made for all the verticals, and points corresponding to e are connected, the approximate curves of values of the tangential effort t at the crank pin are obtained for the out and in stroke.

91. *Effort at crank pin when the obliquity of connecting rod is not neglected.*—The effective effort at the crosshead is p'_e found above. This produces a thrust (or tension) in the connecting rod $q' = p'_e \sec\phi$ and a normal pressure on the slide bars $p'_e \tan\phi$. At the crank-pin end the thrust q is

less than q' in consequence of the inertia of the connecting rod. If p_e is the effort in the line of stroke found by including the connecting-rod weight in that of the reciprocating parts, then with accuracy enough for any practical purpose $q = p_e \sec \phi$. Resolving this along the crank and at a tangent to the crank-pin circle,

$$\text{Thrust along crank} = p_e \frac{\cos(\theta + \phi)}{\cos \phi}$$

$$\text{Tangential effort} = t = p_e \frac{\sin(\theta + \phi)}{\cos \phi},$$

But, as shown above, ($\nu = l/r$)

$$\sin \phi = \sin \theta / \nu$$

$$\cos \phi = \surd \{1 - (\sin^2 \theta)/\nu^2\} = 1 - \tfrac{1}{2}(\sin \theta/\nu)^2 \text{ nearly}$$

$$t = p_e \sin \theta \frac{\nu^2 + \nu \cos \theta - \frac{1}{2} \sin^2 \theta}{\nu^2 - \frac{1}{2} \sin^2 \theta},$$

where t is in lbs. per sq. in. of piston area. Graphically t is more easily found. In fig. 80 produce the crank and draw a perpendicular to the line of stroke at D. Take C$f = p_e$, and draw fe parallel to the connecting rod. Then $t =$ De.

Finding a series of values of t, say at ten points of the stroke, and connecting the points corresponding to e, the shaded curve of crank-pin tangential efforts with the piston travel as base is determined. There are two other ways of exhibiting the variation of crank-pin effort. If Cg is taken equal to De, and the process repeated for other crank positions, and the points corresponding to g are connected, the polar curve of crank-pin efforts is obtained. The radial intercept between the crank-pin circle and the curve is the value of t for any position of the crank. Lastly, if a line is taken equal to the circumference of the crank-pin circle, abscissæ equal to the arcs A_0C measured from A_0 to the crank pin and ordinates equal to t, the curve of crank-pin effort to a crank-pin travel base is obtained as in the case

above, where the obliquity of the connecting rod was neglected. In the figure only the curves for the out-stroke are drawn.

92. *Combination of effort curves for two or more engines.*—Suppose there are two engines at right angles. Then it is convenient to draw the piston-effort curves, such as those in fig. 80, in positions at right angles, and so to obtain the polar curves for each engine in a convenient position for adding the vectors corresponding to each engine to obtain the combined polar curve. This has been done for a compound engine with cranks at right angles in fig. 81.

If the engine is a compound one, a further reduction must be made. Either the high-pressure diagram is drawn to natural scale, and for the low pressure the pressures are reduced to equivalent pressures on the high-pressure piston, or else the low-pressure diagram being drawn to natural scale, the high-pressure diagram pressures must be reduced

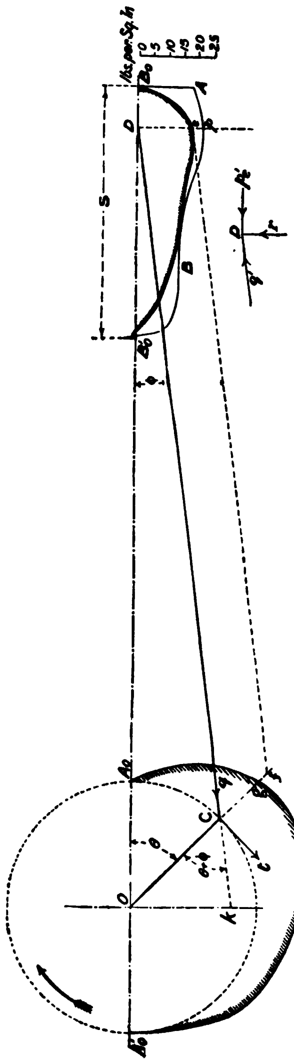


Fig. 80

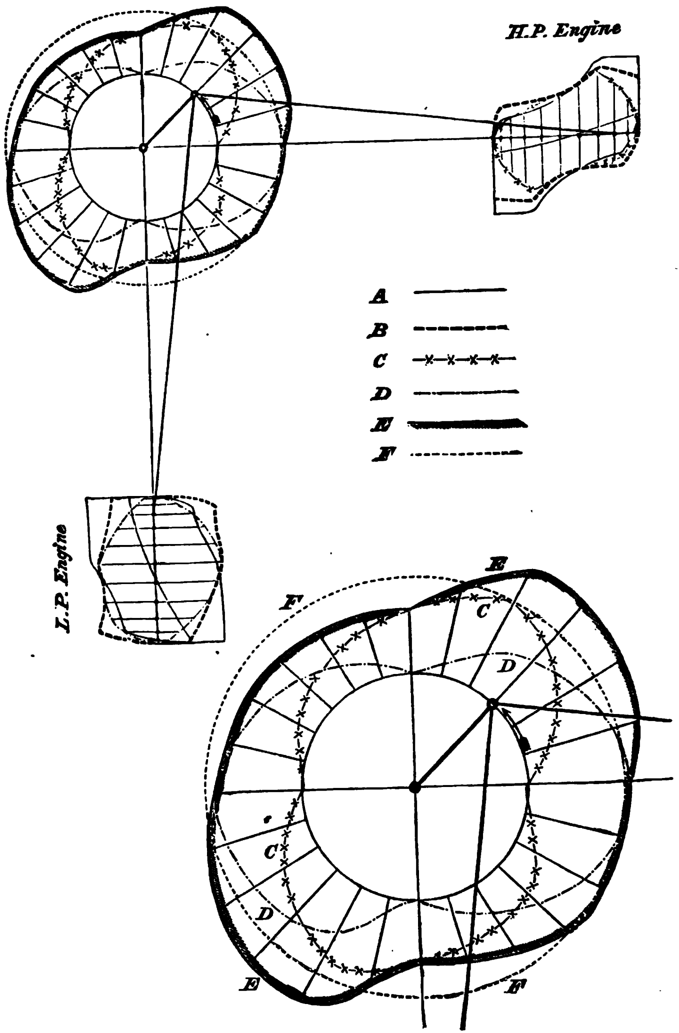


Fig. 81

to equivalent pressures on the low-pressure piston. If the low-pressure piston area is n times the high pressure, then either the low-pressure diagram ordinates must be increased in the ratio $n : 1$ or the high pressure reduced in the ratio $1 : n$. Then an ordinate on either diagram represents the piston effort in lbs. per unit of area of the piston to which the pressures are reduced. In the figure the curves A with a simple line are the steam pressure and inertia curves as in fig. 80. The resultant piston efforts are given by the dotted curves B. The curves C are the true piston effort curves corrected for inertia for the H.P. cylinder and D those for the L.P. cylinder. Adding the ordinates of these in the polar diagram, the curve of total effort E due to both engines is obtained. The dotted circle F is a curve of mean effort.

CHAPTER V

BALANCING

93. In an engine running at uniform speed the forces due to the steam pressure and resistance overcome are internally balanced by the stresses in the frame, except that when the motive and resisting forces do not act in one plane, there may be an unbalanced couple neutralised by the reaction of the foundation, and this couple is not ordinarily of much moment. But it is otherwise with the inertia forces due to the periodical variation of velocity and direction of the moving pieces, unless special arrangements are made. The provision of means to neutralise partially or wholly the disturbing inertia forces is termed balancing. The magnitude of the inertia forces increases as the square of the speed, and hence the importance of balancing becomes very great in machines running at high speed. In fixed engines the unbalanced inertia forces cause rocking of the foundations and set up vibrations in the surrounding ground and buildings which sometimes are felt over considerable distances. In the case of locomotives in which the engine rests on springs, want of balance induces vertical, horizontal, sinuous, and rocking oscillations, which are prejudicial and may be dangerous. Lastly, in marine engines, if the ship or some part of it responds at certain speeds to periodical impulses, due to want of balance in the engines, vibrations are set up which are extremely inconvenient. The balancing of locomotives which can only be approximately effected, has long been pretty well understood, but within

the last fifteen years a very considerable amount of attention has been given to the balancing of marine engines, and there is now a large literature relating to that subject. In this chapter only the general principles of balancing can be dealt with, and these will be explained as simply as possible.

94. *Forces acting on an engine frame and foundation due to the piston load.*—In the case of a single engine, the forces due to the steam pressure and resistance may be taken to act in one plane—namely, the plane containing the centre line of the cylinder and the circle of rotation of the middle point of the crank pin. Let p be the effective steam pressure in lbs. per sq. in., a the area of the piston in sq. ins.

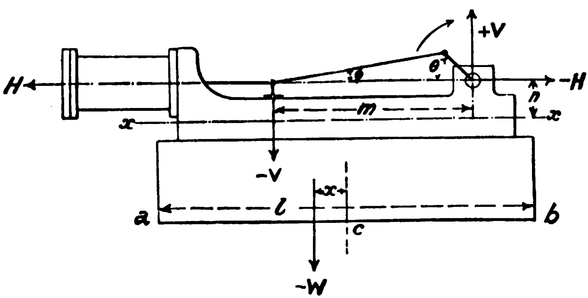


Fig. 82

Then $p\,a$ is the piston load in lbs. In the position shown in fig. 82, there is a horizontal force $\text{H} = p\,a$ acting on the cylinder cover, balanced by an equal force $-\text{H}$ at the main bearing. These forces produce stresses in the engine frame which, apart from the attachments to the foundation, would be subjected to a tension H and a bending moment $-\text{H}\,n$, where n is the distance from the line of stroke to the neutral surface of the frame. In the instroke there is a thrust instead of a tension, and the bending moment changes sign. In addition there is vertical pressure $-\text{V} = -p\,a \tan\phi$ at the slide bars, and an equal force at the crank shaft, forming a couple $\text{V}\,m$ equal and opposite to the

turning moment on the crank shaft. In some engines this is balanced by an equal couple in the same plane, but more commonly the resisting couple is in another plane and acts on another foundation. Then the couple $\mathrm{V}\,m$ alters the distribution of pressure on the foundation $a\,b$.

Let $-\mathrm{W}$ be the weight of engine and foundation in lbs. acting at x feet to the left of the centre c of $a\,b$. The action of $-\mathrm{W}$ is equivalent to that of $-\mathrm{W}$ at c and a couple $\mathrm{W}\,x$. Let l be the length, b the breadth of the base $a\,b$ in feet. The stress in the plane $a\,b$ is a thrust $\mathrm{W}/b\,l$ lbs. per sq. foot uniformly distributed, and a uniformly varying stress which at the edges a, b of the base has the intensity $\pm\, 6\,(\mathrm{W}\,x + \mathrm{V}\,m)/b\,l^2$. The resultant stress at a and b is therefore

$$\frac{\mathrm{W}}{b\,l} \pm \frac{6}{b\,l^2}(\mathrm{W}\,x + \mathrm{V}\,m) \text{ lbs. per sq. foot.}$$

If W acts to the right of c, $\mathrm{W}\,x$ is negative; and $\mathrm{V}\,m$ is negative in the instroke. The effect of the variation of $\mathrm{V}\,m$, which is zero at the ends of the stroke, is to produce a tendency to rock the engine on its foundations. With the exception of this moment, which is not of great importance, the forces due to the steam pressure are internally balanced.

95. *Forces acting on the frame and foundation due to unbalanced revolving masses.*—Let W_1 be the unbalanced revolving mass attached to the crank shaft in lbs., its mass centre being at R_1 feet radius. Let v be the linear velocity of the mass centre in feet per sec., ω the angular velocity of the crank shaft in radians per sec., and N the rotations of the crank shaft per minute. The unbalanced mass consists chiefly of the crank arms, crank pin, and, roughly, one-half to two-thirds of the connecting rod.[1] Then the radial centrifugal force is

$$\mathrm{C} = \frac{\mathrm{W}_1}{g}\,\omega^2\,\mathrm{R}_1 = \frac{\mathrm{W}_1}{g}\,\frac{v^2}{\mathrm{R}_1} = \frac{\mathrm{W}_1}{g}\,\frac{\pi^2\,\mathrm{N}^2\,\mathrm{R}_1}{900} \text{ lbs.} \qquad (1)$$

[1] Let W be the weight of the connecting rod, a and b the distances of the mass centre of the rod from the crank pin and crosshead pin respectively. Then the best assumption is that $\mathrm{W}\,b/(a+b)$ revolves with the crank pin, and $\mathrm{W}\,a/(a+b)$ reciprocates with the crosshead.

This unbalanced centrifugal force may be resolved into $C \cos \theta$ acting horizontally, and $C \sin \theta$ acting vertically (fig. 83); $C \cos \theta$ is positive during the first half of the outstroke and last half of the instroke, negative in the last half of the outstroke and first half of the instroke; $C \sin \theta$ is positive in the outstroke and negative in the instroke; $C \cos \theta$ exerts a moment $C h \cos \theta$ and $C \sin \theta$ a moment $C y \sin \theta$ on the base $a b$, tending to rock the foundation.

96. *Forces acting on the frame and foundation due to the inertia of reciprocating pieces.*—Let W_2 be the mass in lbs. of the reciprocating pieces, including the piston, piston rod,

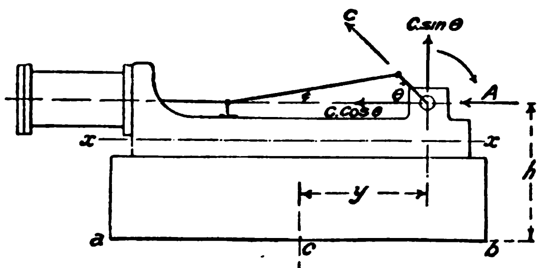


Fig. 83

crosshead, and, roughly, one-third of the connecting rod. Let R_2 be the crank radius in feet, L the connecting-rod length, and let $L/R_2 = n$. Then if the effect of the obliquity of the connecting rod is neglected—that is, if the reciprocating mass is taken to have simple harmonic motion—the force A required for acceleration is equal to the horizontal component of the centrifugal force of a mass W_2 revolving at R_2. That is,

$$A = (W_2 \, R_2 \, \omega^2 \cos \theta) / g \text{ lbs.} \quad . \quad . \quad . \quad (1)$$

which is negative when $\cos \theta$ is negative. As this force balances part of the piston load, the force acting on the cylinder (in the position shown in fig. 83) is greater than the horizontal force at the main bearing by this amount, or

the horizontal force at the main bearing is H — A. This unbalanced force produces a moment $A h$ on the base of the foundation, and is to be included with the moments of the other couples in estimating the distribution of stress.

If the effect of the obliquity of the connecting rod is taken into account,

$$A = \frac{W_2}{g} R_2 \omega^2 \left(\cos \theta + \frac{\cos 2\theta}{n}\right) \text{ lbs.} \quad . \quad (2)$$

At the ends of the stroke, this has the values

$$A_s = \frac{W_2}{g} R_2 \omega^2 \left(1 \pm \frac{1}{n}\right) \quad . \quad . \quad . \quad . \quad (3)$$

Hence, if $A'_s = \frac{W_2}{g} R_2 \omega^2$ is the force required to accelerate the reciprocating mass at the ends of the stroke when the obliquity of the connecting rod is neglected, this becomes $\frac{5}{4} A'_s$ and $\frac{3}{4} A'_s$ for a connecting rod four cranks in length when the obliquity is taken into the reckoning. The difference is by no means unimportant. It is convenient to regard A as consisting of two parts, corresponding to the two terms in the bracket, the first being termed the primary and the other the secondary disturbing force. In some cases of balancing only the primary forces are balanced. But the secondary are not unimportant, not only from their magnitude, but because their period is only half that of the primary forces.

97. *Graphic representation of the disturbing forces.*—In fig. 84, values of θ are set off along $a\,b$, the length of which corresponds to one revolution. Values of $\cos \theta$ set up as ordinates give the curve I, and these are proportional to the horizontal disturbing forces due either to an unbalanced revolving or unbalanced reciprocating mass. Backward acting forces are drawn upwards, and forward acting forces downwards. Values of $\sin \theta$ set up as ordinates give curve II, and these are proportional to the vertical disturbing forces due to a revolving mass, upward forces being drawn

upwards, and downward forces downward. Lastly, the secondary horizontal forces, that is values of $(\cos 2\theta)/n$ give the curve III. The dotted curve IV corresponds

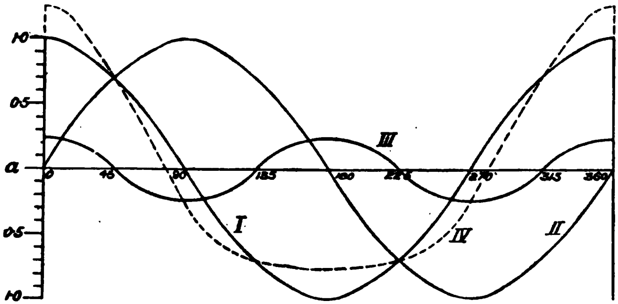


Fig. 84

to $\cos\theta + (\cos 2\theta)/n$, and its ordinates are proportional to the disturbing forces of a reciprocating mass when the obliquity of the connecting rod is taken into account.

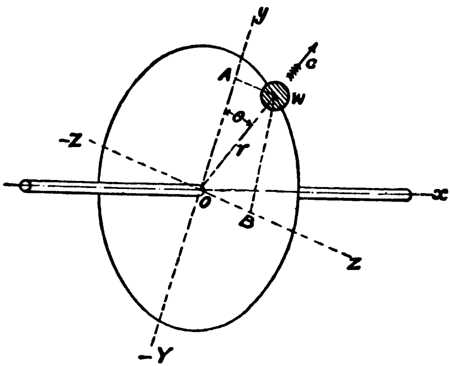


Fig. 85

98. *Equivalent masses.*—In fig. 85, let O X be an axis of rotation, and O Y, O Z co-ordinate axes perpendicular to each other and to O X. For simplicity consider O X, O Z horizontal

and O Y vertical. A mass W lbs., revolves at radius r feet and angular velocity ω radians per sec. or N revolutions per minute. The radial centrifugal force of W is

$$C = \frac{W}{g} \cdot \omega^2 r = \frac{W}{g} \frac{\pi^2 N^2 r}{900} \text{ lbs.} \qquad (1)$$

But a mass $U = W r$ lbs. at 1 foot radius would exert the same centrifugal force. Hence U at unit radius may be called the equivalent of W at r. Since all masses attached to a rotating axis revolve at the same angular speed their centrifugal forces are proportional to their equivalent masses at 1 foot radius. It is convenient in balancing problems to begin by substituting equivalent masses for the actual masses.

Mass equivalent to two masses in the same plane.—Let W_1, W_2, fig. 86, be two masses at radii r_1, r_2. Then $U_1 = W_1 r_1$

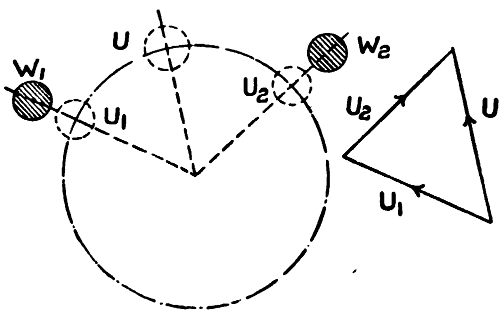


Fig. 86

and $U_2 = W_2 r_2$ are their equivalent masses at unit radius. Take lines in the triangle of forces on any scale equal to U_1, U_2, and parallel to the radii drawn to them. Then U on the same scale placed at unit distance on a radius parallel to U is equivalent to W_1 and W_2.

Resolution of an equivalent mass into two equivalent masses in planes at right angles.—Let the radius of W (fig. 85) make an angle θ with the vertical O Y. Then its

centrifugal force C can be resolved into C $\cos\theta$ along O Y and C $\sin\theta$ along O Z. From the mass centre of W drop perpendiculars on O Y, O Z, and let O A $= y$, O B $= z$.

The centrifugal forces in vertical and horizontal planes are,

$$C \cos \theta = \frac{W}{g} \omega^2 r \cos \theta = \frac{W}{g} \omega^2 y$$

$$C \sin \theta = \frac{W}{g} \omega^2 r \sin \theta = \frac{W}{g} \omega^2 z.$$

But masses W y lbs. and W z lbs. at 1 foot radius would exert the same forces. Hence Wy and Wz at unit radius are masses equivalent to W at r as regards action vertically or horizontally. $U' = Wy$ and $U'' = Wz$ will be termed masses equivalent to W when considering action in the plane O X, O Y, and the plane O Z, O X. Values of U', U'', when treated as forces will be regarded as + when y and z are measured from O towards Y and Z, and negative when they are measured in the reverse direction.

Balancing of Revolving Masses

99. Revolving masses can be perfectly balanced for all constant speeds of rotation by suitable revolving balance

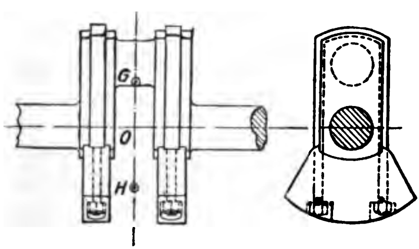


Fig. 87

weights, and it is desirable that this balance should be effected whatever is done as regards the reciprocating masses. In some cases the balancing of the latter, so far

as it is secured at all, is obtained by treating them or part of them as revolving masses. Fig. 87 shows an ordinary crank shaft. The disturbing mass, consisting of crank arms and crank pin, being symmetrically disposed has its mass centre at G on the mid-plane of the engine. This can be balanced by weights attached to the crank arms, also symmetrically placed so as to have their mass centre at H. Let W_1, W_2 be the disturbing and balancing masses in lbs., r_1, r_2 the radii OG and OH. The equivalent masses at unit radius are $U_1, = W_1 r_1$ and $U_2 = W_2 r_2$. Then the centrifugal forces are equal and opposite in all positions and at all speeds, if

$$U_1 = -U_2.$$

If three masses are attached to an axis so that they rotate in one plane (fig. 88), they balance if the centrifugal force of one is equal and opposite to the resultant of the centrifugal forces of the other two. Let U_1, U_2, U_3 be the equivalent masses at unit radius. There will be balance, if a triangle with its sides parallel to the radii drawn to U_1, U_2, U_3, and their lengths equal on any scale to U_1, U_2, U_3, closes. Or if U_1, U_2 are given, the closing side, U_3, is the equivalent of the required balance weight. The actual balance weight must be a mass W_3 at radius r_3 such that $W_3 r_3 = U_3$.

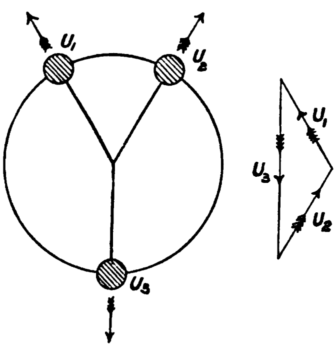


Fig. 88

100 *Balancing of masses not in one plane of rotation.*—Three masses attached to a shaft so that they are in one plane passing through the axis, but rotate in different planes (fig. 89), will be in balance if two conditions are

satisfied :—(1) There must be no resultant force tending to translate the axis parallel to itself in the axial (shaded) plane; (2) There must be no resultant couple tending to rotate the axis in the axial (shaded) plane.

Let $U_1, -U_2, U_3$, be the equivalent masses.

Then there will be no resultant force, if

$$U_1 - U_2 + U_3 = 0. \quad \quad (1)$$

Also, taking moments about a or c, there will be no resultant couple, if,

$$\left.\begin{array}{l} -U_2 m + U_3(m+n) = 0 \\ -U_2 n + U_1(m+n) = 0 \end{array}\right\} \quad \quad (2)$$

or if,

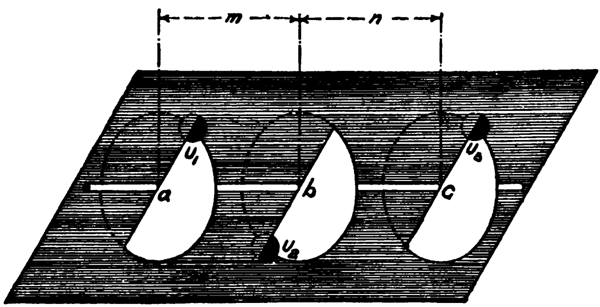


Fig. 89

Two equations to determine any two of the quantities U_1, U_2, U_3, m and n, if the other three are given.

Thus if U_2, n and m are given, U_1 and U_3 can be found. Then any actual balance weights W_1, W_3 may be chosen at any radii r_1, r_3, provided that

$$W_1 r_1 = U_1 \text{ and } W_3 r_3 = U_3.$$

101. *Balancing four masses not in the same axial plane and not in the same planes of rotation.*—Suppose the masses at the cranks of a pair of engines, with cranks not opposite but at any angle θ, are to be balanced. Unless these are separately balanced in their planes of rotation, they may be

balanced by two balance weights fore and aft of the cranks. Let the fore-engine crank B, fig. 90, be vertical, then the aft crank makes an angle θ with the vertical. If $U_1 = W_1 r_1$ is the equivalent mass at B, this is also the equivalent mass for action in a vertical plane, and there is no action in a horizontal plane. If $U_2 = W_2 r_2$ is the equivalent mass at C, then $U'_2 = W_2 r_2 \cos\theta$ is the equivalent mass for vertical action, and $U''_2 = W_2 r_2 \sin\theta$ that for horizontal action. Let U'_3 U'_4 be the masses equivalent to the balance weights as regards vertical action, and U''_3 U''_4 those for horizontal

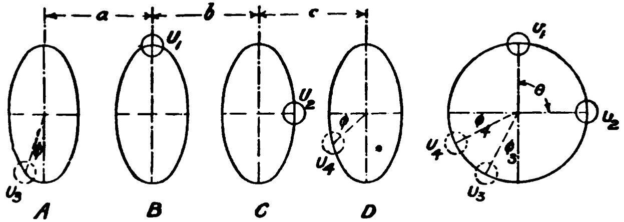


Fig. 90

action. Let a, b, c, be the distances between the planes of rotation. For balance of vertical forces

$$U_1 + U'_2 + U'_3 + U'_4 = 0 \quad . \quad . \quad (3)$$

for balance of horizontal forces,

$$U''_2 + U''_3 + U''_4 = 0 \quad . \quad . \quad . \quad (4)$$

and for balance of couples, taking moments about the planes A and D,

$$\left.\begin{array}{l}\text{Vertical couples,}\\ \quad + U'_3(a+b+c) + U_1(b+c) + U'_2 c = 0\\ \quad U_1 a + U'_2(a+b) + U'_4(a+b+c) = 0\\ \text{Horizontal couples,}\\ \quad + U''_3(a+b+c) + U''_2 c = 0\\ \quad U''_2(a+b) + U''_4(a+b+c) = 0\end{array}\right\} \quad (5)$$

Six equations which determine U'_3 U''_3 U'_4 U''_4 a and c. The forward balance-weight equivalent mass is found by

combining U'_3 U''_3 on vertical and horizontal radii. The equivalent mass is $U_3 = \sqrt{(U'^2_3 + U''^2_3)}$ and it is on a radius making an angle ϕ_3 with the vertical such that $\tan\phi_3 = U_3''/U'_3$. Similarly the equivalent mass for the aft balance weight is $U_4 = \sqrt{(U'^2_4 + U''^2_4)}$ and its radius makes an angle ϕ_4 with the horizontal such that $\tan\phi_4 = U'_4/U''_4$.

Example. Balancing the revolving masses on the axle of an inside cylinder locomotive.—Conditions of symmetry

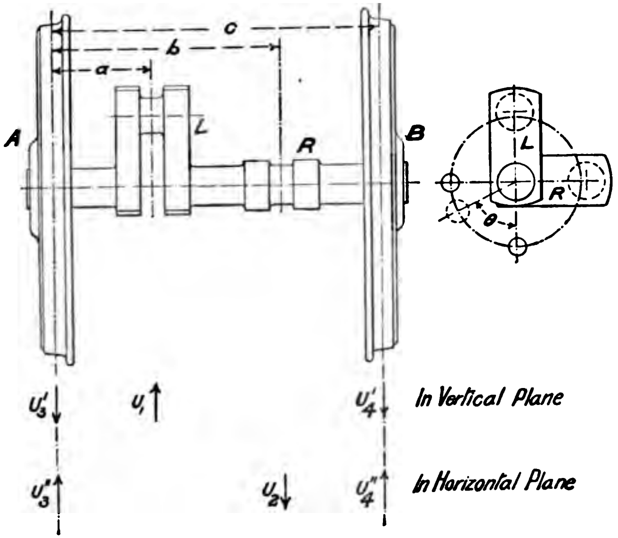


Fig. 91

simplify this problem. Let L and R, fig. 91, be the cranks at right angles, and A and B the planes in which the balance weights are to be placed. The positions of these are fixed by the construction of the engine. Let $a = 1 \cdot 5$ feet; $b = 3 \cdot 417$ feet; $c = 4 \cdot 925$ feet.[1] The unbalanced masses at the

[1] The data are taken from Prof. Dalby's paper, 'Proc. Inst. Mech. Eng.,' 1902.

crank pins are $W_1 = W_2 = 1{,}011$ lbs., and they act at $1 \cdot 083$ feet radius.

If in the right-hand figure the cranks are taken to be turning clockwise, the crank R is at the outer dead point. Then the horizontal forces are drawn so that upward-acting forces point to the cylinders.

At the left crank L, which is vertical, the equivalent mass for vertical action is $U_1 = 1{,}011 \times 1 \cdot 083 = 1{,}095$ lbs., and there is no horizontal action. Similarly the equivalent mass at the right crank is 1,095 lbs. for horizontal action, and there is no vertical action. Let U'_3, U''_3 be the vertical and horizontal equivalents of the left balance weight, and U'_4, U''_4 those of the right balance weight.

In the vertical plane,

$$+ U'_3 + 1{,}095 + U'_4 = 0 \quad . \quad . \qquad (1)$$

In the horizontal plane,

$$+ U''_3 + 1{,}095 + U''_4 = 0 \quad . \quad . \qquad (2)$$

Taking moments about A,

$$(1{,}095 \times 1 \cdot 5) + (U'_4 \times 4 \cdot 925) = 0 \quad . \qquad (3)$$

$$(1{,}095 \times 3 \cdot 417) + U''_4 \times 4 \cdot 925 = 0 \qquad (4)$$

From (4), $U''_4 = -760 \cdot 8$ lbs.

From (2), $U''_3 = -334 \cdot 2$ „

From (3), $U'_4 = -334 \cdot 2$ „

From (1), $U'_3 = -760 \cdot 8$ „

As these are negative they are to be placed on radii drawn negatively from the axis. Combining, $U_4 = \sqrt{(U'^2_4 + U''^2_4)} = \sqrt{(334 \cdot 2^2 + 760 \cdot 8^2)} = -831$ lbs., and from symmetry U_3 has the same value. Also the radius to U_4 makes an angle θ with the vertical, such that $\tan \theta = U''_4 / U'_4 = 760 \cdot 8 / 334 \cdot 2 = 2 \cdot 277$. Hence $\theta = 66°$. The radius of U_3 makes the same angle with the horizontal. If the mass centres of the balance weights can be placed at 3 feet radius they will each be $831/3 = 277$ lbs.

Balancing Reciprocating Masses

102. Let W be the reciprocating mass in lbs., r the crank radius, l the connecting-rod length, both in feet, and let $n = l/r$. Let ω be the angular velocity of the crank in radians per sec., N its revolutions per minute. Then if θ is the angle through which the crank has turned from its inner dead point, the inertia of the reciprocating mass acting in the line of stroke is,

$$A = \frac{W}{g} \omega^2 r \left\{ \cos \theta + \frac{\cos 2\theta}{n} \right\}$$

$$= \frac{W}{g} \frac{\pi^2 N^2 r}{900} \left\{ \cos \theta + \frac{\cos 2\theta}{n} \right\} \text{ lbs.} \qquad (1)$$

It has already been stated that A is conveniently taken to consist of two parts corresponding to the two terms in the bracket, that proportional to $\cos \theta$ being termed the primary disturbing force, and that corresponding to $(\cos 2\theta)/n$ being termed the secondary disturbing force. The secondary disturbing force is greater the less the value of n, and vanishes for an infinitely long connecting rod. The secondary force varies in a period half as long as the primary force.

The value of the primary disturbing force due to a reciprocating mass is the same as the component in the line of stroke of the centrifugal force of an equal revolving mass. Hence, as regards that direction, the reciprocating mass can be balanced by a revolving mass calculated just as if the reciprocating mass were a revolving one. In locomotives part of the reciprocating mass is thus balanced. But the reciprocating mass produces no action at right angles to the line of stroke, while the revolving balance weight does. Hence, if a revolving balance weight is used to balance the inertia forces of a reciprocating mass in the line of stroke, it at the same time introduces unbalanced inertia forces of the same amount as those balanced, but acting at right angles to the line of stroke.

Hence, to properly balance a reciprocating mass another reciprocating mass commonly termed a bob weight must be used. In 1892 Mr. Yarrow carried out important experiments on the waves produced by a torpedo-boat having an unbalanced engine,[1] and the results led to a system of balancing by bob weights placed fore and aft of the engine. On this system the primary forces due to the reciprocating masses can be perfectly balanced without introducing unbalanced forces at right angles. As however the bob weights driven by eccentrics must have a short stroke, they are heavy, and consequently this system has not been generally adopted. Later it was shown that in a four-crank engine, by adjusting the disturbing reciprocating weights and the crank angles, the same result would be obtained. This system is known as the Yarrow-Schlick-Tweedie system of balancing. Properly carried out this system secures balance in the line of stroke of primary and secondary forces and primary couples, but leaves secondary couples unbalanced.

103. *Arrangements of engine to secure balance of reciprocating masses.*—In single-crank engines the reciprocating parts can be balanced in the line of stroke for primary inertia forces by a revolving balance weight combined with that required to balance the revolving parts, but only at the expense of introducing unbalanced forces at right angles to the line of stroke. By using a bob weight (fig. 92, A) attached to a lever to balance the reciprocating mass, and a revolving balance weight to balance the revolving parts (including part of the connecting rod), almost perfect balance of the forces can be obtained, but an unbalanced couple is introduced. This can be eliminated by dividing the bob weight into two parts symmetrically placed to the line of stroke. A two-cylinder two-crank engine (fig. 92, B) with cranks opposite is perfectly balanced, and this arrangement has

[1] 'Balancing Marine Engines and Vibrations of Vessels,' 'Trans. Inst. Nav. Arch.,' 1892.

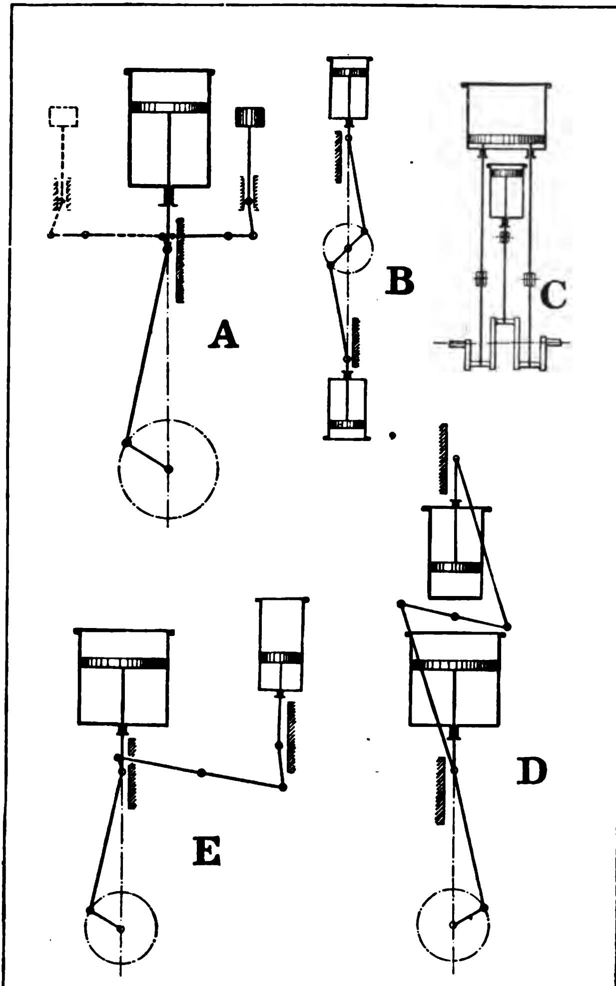


Fig. 92

been adopted for small high-speed engines. Two connecting rods and a pair of cranks must be used for one of the cylinders, and the weights of the moving parts for each cylinder must be the same. If the cylinders are placed side by side with opposite cranks there may be perfect balance as regards primary forces, but the disturbing effect of the secondary forces is about doubled, the secondary forces of both reciprocating masses acting in the same direction. Also an unbalanced couple is introduced, the lever arm of which is the distance between the cylinder centre lines. With cranks at 90°, which is the ordinary arrangement, the engine cannot be balanced so well. The arrangement in fig. 92, C, eliminates the couple, but as regards the secondary forces is no better. The arrangement shown in fig. 92, D, permits an almost perfect balance horizontally and vertically to be obtained, if the revolving unbalanced mass is independently balanced. The reciprocating parts have similar motions in opposite directions. Fig. 92, E, shows an arrangement suggested by Mr. MacAlpine, in which both primary and secondary forces can be balanced. There is an unbalanced couple, but in the case of marine engines this is athwart ship, which is less prejudicial than if it were fore and aft. Applied to triple or quadruple engines this system gives a means of securing very perfect balance. For three-cylinder three-crank engines the best arrangement is to have the cranks at 120°, and the reciprocating parts of all cylinders of equal weight. This arrangement also gives the most uniform turning moment, if the work in the cylinders is equally distributed. With equal crank angles and equal reciprocating weights there is balance of both primary and secondary forces, but not of the couples. These can be balanced if bob weights are introduced fore and aft of the engine, or if two similar three-crank engines are placed in line. If the revolving masses at the three cranks are equal they are balanced as regards forces, but not completely as regards couples. However, as these are

generally small, it is hardly necessary to introduce revolving balance weights. With four-crank engines there are still greater facilities for balancing. Mr. Taylor[1] has pointed out that as regards turning moment when the spacing of the cranks is at 90°, the engine is equivalent to an engine with two opposite cranks. A more uniform turning moment would be obtained if the first crank being at 0°, the second was at 45° or 225°, the third at 90° or 270°, and the fourth at 135° or 315°, that is, provided the work is equally distributed in the cylinders. Also that this arrangement is favourable for balancing.

LOCOMOTIVE BALANCING

104. In the locomotive there are revolving and reciprocating masses not in balance, but the balance weights are necessarily revolving masses; the balancing is therefore imperfect. If even an approximate balance is effected of horizontal forces and couples tending to turn the engine about a vertical axis, then new unbalanced vertical forces and couples tending to turn the engine about a horizontal axis are introduced. If only revolving masses are balanced, there are left horizontal forces and couples due to the reciprocating masses. The actual practice in balancing is intermediate between these limits: the revolving unbalanced masses are, or should be, perfectly balanced, and part of the reciprocating masses is balanced as regards action horizontally. If too much of the horizontal masses is balanced horizontally by the revolving balance weights, the vertical forces introduced may be enough to lift the wheels against the pressure of the springs, which would be dangerous.

The simplest case is the balancing of the driving axle of an engine with single drivers, and the method in this case is easily extended to more complex cases where there are

[1] 'Methods of Balancing Marine Engines,' by D. W. Taylor, 'U.S.N., Am. Soc. of Nav. Arch.,' 1901.

outside cranks and coupling rods. The problem is simplified by these considerations. As the balancing must be imperfect, it is not necessary to consider the secondary action due to the obliquity of the connecting rods in finding the balance weights. When the balance weights are settled the effect of obliquity is easily calculated. The unbalanced masses being symmetrically placed to the middle plane with 90° difference in phase, it is sufficient to consider the action

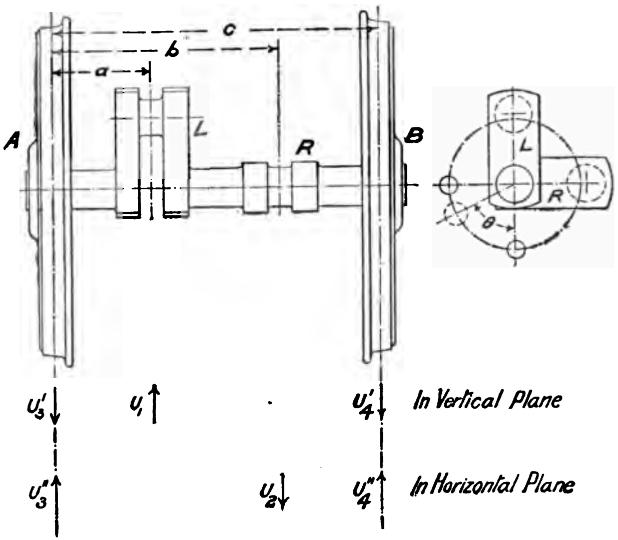


Fig 93

when one crank is horizontal and the other vertical. Fig. 93 shows a driving axle in this position.

Let w_1 = unbalanced revolving mass at each crank, reduced to crank radius, in lbs.

w_2 = reciprocating mass of each engine, in lbs.

$k\,w_2$ = part of reciprocating mass which is to be reckoned as a revolving mass in calculating the balance weights.

r = crank radius, in feet. Commonly two-thirds of the connecting-rod weight is included in w_1 and one-third in w_2. Also most commonly $k = \frac{2}{3}$.

Then treating the masses w_1 and $k\,w_2$ as revolving masses, the equivalent mass at each crank is $U_1 = U_2 = (w_1 + k\,w_2)\,r$ at 1 foot radius. Proceeding as in § 101, if U'_3, U''_3 are the vertical and horizontal equivalents of the left balance weight, and U'_4, U''_4 those of the right balance weight, then for equilibrium of the forces in the vertical plane

$$U'_3 + U_1 + U'_4 = 0,$$

and for the equilibrium in the horizontal plane

$$U''_3 + U_2 + U''_4 = 0.$$

For equilibrium of the couples,

$$U_1\,a + U'_4\,c = 0$$
$$U_2\,b + U''_4\,c = 0.$$

Remembering that in the locomotive the masses are symmetrically placed,

$$U'_3 = -U_1\,\frac{c-a}{c} = -U_1\,\frac{b}{c} = U''_4$$

$$U''_3 = -U_1\,\frac{c-b}{c} = -U_1\,\frac{a}{c} = U'_4.$$

Hence the equivalent single balance weight in each wheel will be

$$U_3 = U_4 = \sqrt{(U'^2_3 + U''^2_3)} = -\frac{U_1}{c}\sqrt{(b^2 + a^2)}.$$

The radii drawn to the balance weights make an angle θ with the direction of the nearest crank, given by the relation

$$\tan\theta = \frac{U'_3}{U''_3} = \frac{b}{a}.$$

105. *Unbalanced forces and couples remaining after approximate balancing.*—Having determined the balance weights by assuming a value for k as above, it is still necessary to examine whether the condition secured is

satisfactory, and it is then no longer necessary to neglect the obliquity of the connecting rod. Consider the position of the cranks shown in fig. 93, the right crank being at the outer dead point. Let $\omega = \pi N/30$ be the angular velocity of the axle, in radians per sec. In the vertical plane the centrifugal forces of the balance weights are $U'_3\, \omega^2/g$ and $U'_4\, \omega^2/g$. The centrifugal force of the revolving mass at the left crank is $w_1\, \omega^2\, r/g$. The inertia forces of the reciprocating masses and the revolving mass at the right crank have no vertical component. Hence the resultant vertical force is

$$\left(U'_3 + U'_4\right) \frac{\omega^2}{g} + w_1 \frac{\omega^2 r}{g} \text{ lbs.}$$
$$= -V = -\frac{\omega^2 r}{g} k\, w_2 \text{ lbs.,}$$

acting downwards. With the right crank at the inner dead point there would be the same unbalanced force acting upwards. As could have been foreseen, since the revolving masses are completely balanced, terms containing w_1 vanish and the unbalanced vertical force is equal to the centrifugal force of a mass equal to the part $k\, w_2$ of the reciprocating mass taken into account in finding the revolving balance weights, and placed at radius r on a crank arm opposite to the vertical crank. If x is the distance at which V acts from the left wheel, taking moments

$$V x = \frac{w_1\, \omega^2\, r\, a}{g} + U'_4\, c,$$

which reduced gives $x = a$. V acts upwards or downwards at the left crank according as that crank points downwards or upwards, and there will be a corresponding force at the right crank when it is pointing vertically. But V at the crank is equivalent to a force V at the centre of the axle and a couple of moment $\frac{1}{2} V\,(b-a)$ tending to rotate the engine about its longitudinal axis, and produce rolling.

Variation of load on the wheels.—It is easy to see that $\pm$ V acting at the left crank produces an increase or decrease $\pm$ V b/c of the pressure of the left wheel on the rail; and an increase or decrease $\pm$ V a/c of the pressure of the right wheel. There are inverse variations of the same amount when the right crank is vertical. This tendency to lift the wheels may be dangerous, and is one reason for not balancing the whole of the reciprocating masses.

The preceding calculation is made for the case of one crank horizontal and the other vertical. The greatest vertical unbalanced force, however, occurs when the engine is in another position. If the calculation were made for the cranks at 45° with the horizontal, the unbalanced force would have been found to be equal to the centrifugal force of two masses each equal to $k w_2$, at radius r, on cranks drawn opposite to the actual cranks. The resultant unbalanced vertical force is then

$$V_m = 1{\cdot}41 \; k w_2 \frac{\omega^2 r}{g},$$

acting at the centre of the axle, and the variation of pressure of each wheel on rail is half this, that is

$$0{\cdot}705 \; k w_2 \frac{\omega^2 r}{g},$$

which generally is rather larger than the variation found above.

106. *Unbalanced forces in the horizontal plane.*—Similarly, in the horizontal plane (fig. 93) the centrifugal forces of the balance weights are $U''_3 \, \omega^2/g$ and $U''_4 \, \omega^2/g$. The centrifugal force of the revolving mass at the right crank is $w_1 \, \omega^2 r/g$ and that at the left crank is zero. The horizontal inertia forces of the reciprocating masses are given by Eq. 1, § 102, putting $\theta = 90°$ or $270°$ for the left engine, and 180° or 360° for the right engine, according as the right crank is on the outer or inner dead centre. Then the

horizontal force at the left engine is $-w_2 \omega^2 r / g n$ in either case and

$$\frac{w_2 \omega^2 r}{g}\left(1 \pm \frac{1}{n}\right)$$

at the right engine, the + sign corresponding to $\theta = 360°$ and the $-$ sign to $\theta = 180°$. Hence the resultant horizontal force is

$$H = \frac{\omega^2}{g}\left\{U''_3 + U''_4 + w_1 r - w_2 r/n + w_2 r\left(1 \pm \frac{1}{n}\right)\right\},$$

which reduces to

$$\text{At } \theta = 180°, \qquad \frac{w_2 \omega^2 r}{g}(1 - k)$$

$$\text{At } \theta = 360°, \qquad \frac{\omega_2 \omega^2 r}{g}\left(1 - k - \frac{2}{n}\right)$$

where the term containing n vanishes if the obliquity of the connecting rod is neglected.

These forces acting at the ends of the stroke of each engine—that is, each of them twice in a revolution—produce the action termed plunging.

Let y be the distance at which this force acts from the left wheel. Taking moments about that wheel,

$$H y = \left\{U''_4 c + w_1 r b - w_2 r \frac{a}{n} + w_2 r b\left(1 \pm \frac{1}{n}\right)\right\}\frac{\omega_2}{g}.$$

Reducing,

$$\text{At } \theta = 180°, \qquad y = \frac{(1 - k)\, b - \frac{a}{n} - \frac{b}{n}}{1 - k}$$

$$\text{At } \theta = 360°, \qquad y = \frac{(1 - k)\, b - \frac{a}{n} + \frac{b}{n}}{1 - k - \frac{2}{n}}$$

If terms depending on the obliquity of the connecting rod are neglected,

$$y = b$$

at each end of the stroke. Hence the unbalanced horizontal force is in that case simply the centrifugal force of a mass $(1 - k) w_2$ equal to the part of the reciprocating mass of one engine neglected in finding the balance weights, and it acts at the centre of and along the crank which is horizontal. It is equivalent to $\{w_2 \omega^2 r (1 - k)\} / g$ lbs. acting at the centre of the engine and producing plunging, and a couple of moment $\{w_2 \omega^2 r (1 - k) (b - a)\} / 2g$ producing sinuous motion. This last action is very prejudicial. It is to keep this couple small enough without increasing too much the variation of load on the wheels that the value of k is determined. Obviously, except that the vertical force must not be enough to lift the wheels, the value of k has to be found by experience.

107. *General result.*—If the reciprocating mass of one engine is w_2, and of this a mass $k w_2$ is treated as a revolving mass and balanced by revolving balance weights, then the unbalanced forces and couples remaining, disregarding the obliquity of the connecting rods, are as follows:—

Vertically, a force $\pm$ V or $\pm$ V_m, according as the cranks are vertical and horizontal or at 45° with the horizontal. V is equal to the centrifugal force of a mass $k w_2$ at the crank pin of a vertical crank. V_m is the resultant of the vertical components of the centrifugal forces of two masses $k w_2$ at the crank pins of the two cranks, when these are in the position making an angle of 45° with the horizontal.

Horizontally, a force H equal to the centrifugal force of a mass $(1 - k) w_2$ at the horizontal crank pin.

If d is the distance between the crank centres, the unbalanced couples are $\frac{1}{2}$ V d producing rolling, and $\frac{1}{2}$ H d tending to cause sinuous motion.

If the lifting force at the wheels is great enough, that is, if $\frac{1}{2}$ V_m is greater than the load on one wheel, the wheel will lift and hammer the rail. The obliquity of the connecting rod chiefly affects the horizontal couple, producing sinuous motion.

In the comparatively simple case considered, these results might have been reached more simply. But the method used is general and can be applied to more complex cases—for instance, to a driving axle with inside cranks and outside cranks and coupling rods. Also for an engine with two or more cranks not at right angles. The distribution of the revolving and reciprocating masses initially into equivalent masses in the plane of the reciprocating pieces and a plane at right angles, is generally sufficient to solve any problem of practical importance.

108. *Coupling rods.*—Outside coupling rods are simply revolving masses, and their weight is to be distributed to the cranks in the proportion of the amount resting on each. A rod resting on two cranks would be distributed one-half to each. A rod resting on three cranks might be distributed in the proportion of $\frac{5}{8}$ths to the centre crank and $\frac{3}{16}$ths to the end cranks.

Slipping.—Let w be the weight on the driving axle. In the worst position this is reduced to $w - v_m$ by the unbalanced vertical force. The resistance to slipping is $\frac{1}{8}$ to $\frac{1}{7}$ $(w - v_m)$. If this is less than the effective effort due to the steam pressure, corrected for the inertia of the reciprocating parts and reduced to the wheel circumference, slipping will occur. This is stated for an engine with single drivers. With coupled wheels, the revolving parts of the coupled axles being balanced, w is to be taken as the total weight on all the coupled wheels.

Reduction of variation of rail pressure by distributing the balance weights.—In the case of coupled engines the variation of rail pressure at the driving axle may be diminished by distributing the mass $k\,w_2$ among all the coupled axles.

CHAPTER VI

CRANKS, LEVERS, AND ECCENTRICS

HAND LEVERS AND WINCH HANDLES

109. Fig. 94 shows an ordinary straight lever for working machinery by hand. The part grasped by the hand may be $1\frac{1}{4}$ in. in greatest and 1 in. in smallest diameter, and 5 ins. long. Let P be the force exerted at the handle, and l the length of the lever. Then P l (nearly) is the greatest bending moment on the arm. Let b be the width and h the thickness of the arm at its largest part. Then (I. § 28),

$$\frac{1}{6} b^2 hf = \text{P}\, l$$

$$b^2 h = \frac{6\,\text{P}\, l}{f}.$$

Let the greatest force, P, exerted by a man be taken at 84 lbs.; and let $f = 9{,}000$ lbs. per sq. in. for wrought iron. Then,

$$b^2 h = \tfrac{1}{18} l \text{ nearly} \quad . \quad . \quad . \quad (1)$$

If $h = \frac{3}{4}$ inch, $b = 0{\cdot}27 \sqrt{l}$. If $h = \frac{1}{2} b$, then $b = 0.5 \sqrt[3]{l}$ and $h = 0{\cdot}25 \sqrt[3]{l}$ nearly. If the flat part of the lever is of uniform thickness, its least width should be half its greatest width, the case corresponding with Case I. Table VII. (Part I.). Let d = diameter of shaft on which the lever is keyed; n = distance from centre of lever to centre of nearest bearing of shaft. Then the shaft is subjected to a twisting moment P l and a bending moment P n, and its strength is determined by the rules in I. § 44 and § 141. The

equivalent bending moment is $P\,(0{\cdot}7\,n + 0{\cdot}41\,l)$ nearly. Hence,

$$d = 0{\cdot}094\sqrt[3]{\{P\,(1{\cdot}4\,n + 0{\cdot}96\,l)\}}$$
$$= 0{\cdot}42\sqrt[3]{(1{\cdot}4\,n + 0{\cdot}96\,l)} \quad . \quad . \quad (2)$$

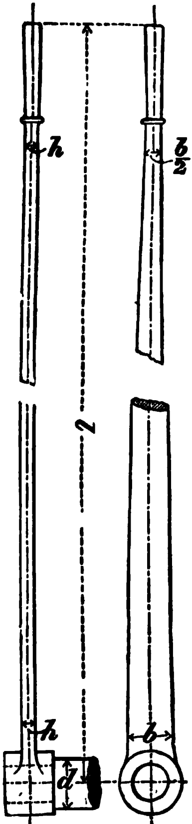


Fig. 94

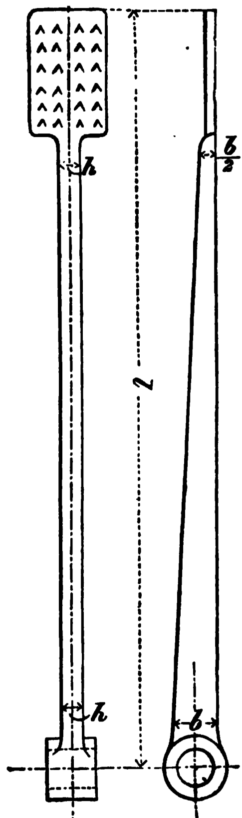


Fig. 95

The part in the eye of the lever may have a diameter $= 0{\cdot}42\sqrt[3]{l}$. The eye of the lever may have a thickness $= 0{\cdot}3\,d$ and a length $= 1$ to $1\frac{1}{4}\,d$.

Fig. 95 shows a foot lever. The foot plate is about 8 ins. by 5 ins., and $\frac{5}{8}$ in. thick. In designing this lever P may be taken at 180 lbs. Then,

$$\left.\begin{aligned} b^2 h &= \tfrac{1}{8} l \\ d &= \cdot 54 \sqrt[3]{(1 \cdot 4 n + 0 \cdot 96 l)} \end{aligned}\right\} \quad . \qquad (3)$$

Fig. 96 shows a winch handle or cranked lever. When this is intended to resist the full force of one man, P may be taken at 84 lbs., and if worked by two men, P = 168 lbs. The mean effort per man in continuous work is only 15 to 30 lbs. The radius r is usually 16 or 17 ins., and the height of the shaft from the ground may be 3 feet to 3 feet 3 ins.

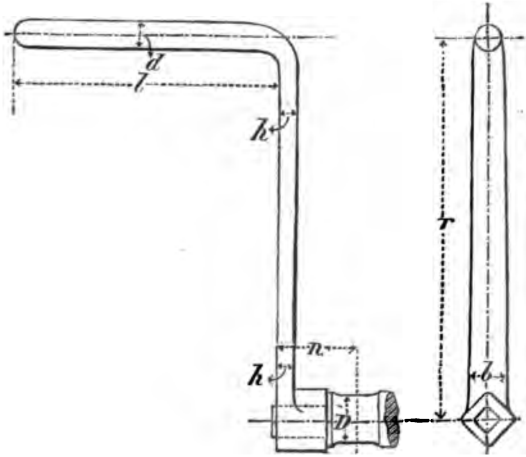


Fig. 96

The length of the handle l may be 10 or 12 ins. for one man and 20 ins. for two men. The pressure on the handle may be taken to act at $\frac{2}{3}$rds of the length. The greatest bending moment at the handle is $\frac{2}{3}$ P l. Then its diameter should not be less than

$$d = 0 \cdot 0947 \sqrt[3]{\tfrac{4}{3} \text{P} l} = 0 \cdot 1042 \sqrt[3]{\text{P} l} \quad . \qquad (4)$$

or say $1\frac{1}{16}$ in. for one man and $1\frac{1}{2}$ in. for two men. The journal of the shaft is subjected to a twisting moment $\text{P}\,r$, and a bending moment $\text{P}\,(\frac{2}{3}\,l + n)$. The equivalent bending moment (I. § 44) is $\text{P}\,(0{\cdot}6\,l + 0{\cdot}9\,n + 0{\cdot}4\,r)$ nearly. Then,

$$\left.\begin{aligned} \text{D} &= 0{\cdot}0947\,\sqrt[3]{\{\text{P}\,(1{\cdot}2\,l + 1{\cdot}8\,n + 0{\cdot}8\,r)\}} \\ &= 0{\cdot}42\,\sqrt[3]{(1{\cdot}2\,l + 1{\cdot}8\,n + 0{\cdot}8\,r)} \text{ for one man} \\ &= 0{\cdot}54\,\sqrt[3]{(1{\cdot}2\,l + 1{\cdot}8\,n + 0{\cdot}8\,r)} \text{ for two men} \end{aligned}\right\} \quad (5)$$

For the part in the eye of the crank the term $1{\cdot}8\,n$ may be omitted. The greatest bending moment on the arm is $\text{P}\,r$, and the twisting moment $\frac{2}{3}\,\text{P}\,l$ nearly. Hence the equivalent bending moment is $\text{P}\,(0{\cdot}9\,r + 0{\cdot}27\,l)$. If b is the breadth and h the width of the arm at the larger end,

$$b^2\,h = \frac{6\,\text{P}}{f}(0{\cdot}9\,r + 0{\cdot}27\,l)$$

$$\left.\begin{aligned} &= {\cdot}056\,(0{\cdot}9\,r + 0{\cdot}27\,l) \text{ for one man} \\ &= {\cdot}112\,(0{\cdot}9\,r + 0{\cdot}27\,l) \text{ for two men} \end{aligned}\right\} \quad (6)$$

Either b or h may be selected and the other obtained from the formula. If the arm is of uniform thickness, its least breadth should not be less than $\frac{1}{2}\,b$, or less than $2\sqrt{\frac{\text{P}\,l}{f\,h}}$ or $0{\cdot}19\sqrt{\frac{l}{h}}$ for one man, and $0{\cdot}27\sqrt{\frac{l}{h}}$ for two men.

In practice $\text{D} = 1\frac{1}{4}$ to $1\frac{1}{2}$ for one man, and $1\frac{1}{2}$ to $1\frac{3}{4}$ for two men. Thickness of eye $\frac{3}{8}$ to $\frac{1}{2}$ D. Length of eye 2 to $2\frac{1}{2}$ ins. Section of arm at eye about $1\frac{3}{8} \times 1\frac{1}{2}$. Diameter of handle about $1\frac{3}{8}$.

Engine Cranks

110. Engine cranks are of cast or wrought iron. A single crank consists of a nave bored to receive the crank shaft, an arm, and a crank pin. If the crank pin is a separate piece, it is fitted into an eye formed at the small end of the crank. Disc cranks have plain circular discs, instead of the ordinary crank arm, and they have the advantage of

being nearly balanced with respect to the crank shaft. A double crank is used when the crank pin cannot be placed at the end of the crank shaft. An eccentric is a crank of peculiar form. It is essentially a crank, with a crank pin, the radius of which is greater than the sum of the crank and crank-shaft radii.

111. *General case. Straining action on crank arm.*—Let fig. 97 represent a crank in any position, and let P be the total pressure on the crank pin. Resolve P into a tangential component T, and a radial component N. Let $a\,b$ be any section of the arm at a distance r from the centre of crank pin, and let m be the distance between centre lines of crank pin and crank arm. Then the straining actions at $a\,b$ which require to be considered are:—

(*a*) A direct pressure (or tension) equal to N.
(*b*) A shear equal to T.
(*c*) A bending moment N m in the plane of the arrow B.
(*d*) A bending moment T r in the plane of the arrow A.
(*e*) A twisting moment T m.

To take into account all these straining actions in several positions of the crank would be laborious. Generally it is sufficient to estimate the strength of the crank in two positions, when the crank is at the dead point and when the crank and connecting rod are at right angles. In the former case, T vanishes and N becomes equal to the greatest piston pressure, or to $N'' = P'' - \frac{W\,V^2}{g\,R}$, which will also for simplicity be denoted by N simply. Here P″ is the initial piston load, W the weight of reciprocating parts, V the velocity of crank pin and R its radius. In the latter case N vanishes and T is equal to 1·02 or 1·03 time the piston load at about mid-stroke.

112. *Strength of the crank.*—Let N be the radial pressure when the crank is at the dead point, and T the tangential

pressure when the crank and connecting rod are at right angles. Let, further,

d, l = diameter and length of crank pin.

D, L = diameter and length of crank-shaft journal.

d', l', t' = internal diameter, length and thickness of small eye of crank.

d'' l'' t'' = internal diameter, length and thickness of large eye of crank.

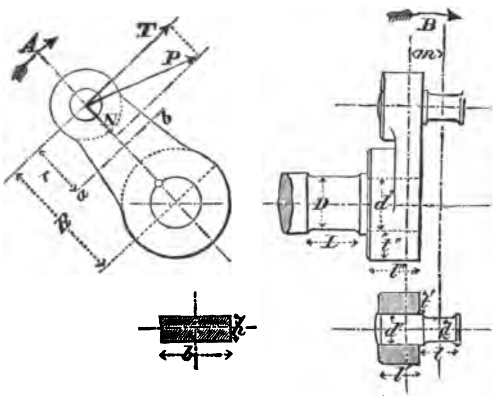


Fig. 97

h, b = thickness and width of arm at any section $a\ b$; the same letters with one accent referring to the section of the arm supposed produced to the centre of small eye, and with two accents the section produced to the centre of large eye.

R = crank radius.

m = distance from centre line of crank pin to centre line ot crank arm.

n = distance from centre line of crank pin to centre line of crank-shaft journal.

113. *Crank pins.*—In Part I., §§ 117 to 124, the conditions to which crank pins are subjected have been dis-

cussed. Most commonly they are designed with reference to a given pressure p in lbs. per sq. in. of projected area of the bearing, and a limiting stress f in lbs. per sq. in. Then if P is the greatest load on the pin in lbs., d the diameter, and l the length of the bearing in inches,

$$l = \mathrm{P}/p\,d$$

$$d = \sqrt[4]{\frac{5\cdot1}{pf}}\,\sqrt{\mathrm{P}}.$$

The pressure p varies from 150 to 200 lbs. per sq. in. in small land engines, to 400 to 800 lbs. per sq. in. in large land and marine engines, and to 1,200 to 1,800 lbs. per sq. in. in locomotives. The following Table may be useful as a general guide. The working stress is to be chosen for stress, changing from tension to compression alternately.

$p =$	200	250	300	400	750	1,000
$\sqrt[4]{\{5\cdot1/pf\}} =$						
For $f =$ 5,000	·0475	·0449	·0429	·0400	·0341	·0318
For $f =$ 7,500	·0429	·0406	·0388	·0361	·0309	·0287
For $f =$ 10,000	·0400	·0378	·0361	·0336	·0287	·0267

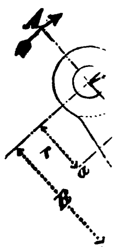
Proportions of wrought-iron end journals or crank pins.

For various intensities of bearing pressure p, and for f = 5,000 lbs. per sq. in.

Load on journal in lbs. P	p = 200		250		300		400		750		1,000	
	d	l	d	l	d	l	d	l	d	l	d	l
1,000	1·29	3·88	1·23	3·26	1·17	2·85	1·09	2·30	0·93	1·44	0·87	1·15
2,000	1·83	5·47	1·74	4·60	1·66	4·01	1·54	3·25	1·32	2·02	1·23	1·63
3,000	2·25	6·67	2·13	5·63	2·03	4·93	1·89	3·97	1·62	2·47	1·50	2·00
4,000	2·59	7·73	2·45	6·53	2·35	5·68	2·18	4·59	1·87	2·86	1·73	2·32
5,000	2·90	8·62	2·74	7·30	2·62	6·36	2·44	5·12	2·09	3·19	1·94	2·58
10,000	4·10	12·2	3·88	10·3	3·71	8·98	3·45	7·24	2·95	4·52	2·74	3·65
20,000	5·80	17·3	5·49	14·6	5·25	12·7	4·88	10·2	4·17	6·40	3·87	5·17
30,000	7·10	21·2	6·72	17·9	6·42	15·6	5·98	12·6	5·11	7·83	4·75	6·32
40,000	8·20	24·4	7·76	20·6	7·42	18·0	6·90	14·5	5·90	9·04	5·48	7·30
50,000	9·17	27·3	8·68	23·0	8·29	20·1	7·71	16·3	6·60	10·1	6·13	8·16

pressure when t
angles. Let, fu

$d, l =$ diam
$D, L =$ diam
$d', l', t' =$ inter
eye
$d'' l'' t'' =$ inter
eye



$h, b =$ thick
the
the
the
the
$R =$ crank
$m =$ distan
line
$n =$ distan
line

113. *Crank*
ditions to which

: modulus of the section is $\frac{1}{6}\ b^2 h$, so that the greatest
ss is (I. § 28)

$$f = \frac{6\ M_e}{b^2\ h} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Me depends on r, the section will increase for a given
king stress f from the crank-pin eye to the crank-shaft

If a value for h or b is selected, then the equations
rmine b or h for any section of the arm. Sections near
eyes may be calculated separately. The greater of the
ies given by (1) or (2) must be taken. The calculations
not quite exact and therefore moderate working stress
uld be taken in the reckoning.

When a crank has a T-form of section, it is but little
ngthened by the feather, but it is more easily cast.

115. *Crank-shaft journal.*—The straining action is
erally greatest when the connecting rod and crank are
ight angles. Then if s is the distance from the centre
ie journal to the plane of rotation of the centre of the
ik pin, and the other notation is as shown in fig. 97,
e is a bending moment $\text{T}\,s$ and a twisting moment T R
the section of the journal. The equivalent twisting
nent is (I. § 44)

$$T_e = \text{T}\,s + \sqrt{\{(\text{T}\,s)^2 + (\text{T R})^2\}},$$

the diameter of the journal is determined by the equa-
(I. § 36)

$$T_e = f\frac{\pi}{16}\,\text{D}^3 = 0{\cdot}2\,f\text{D}^3 \text{ nearly.}$$

The length of the journal is fixed so that the bearing
sure $p = \text{T}/\text{D L}$ or $\text{N}/\text{D L}$, which ever is greatest, does
exceed from 200 lbs. per sq. in. in small fast engines,
50 lbs. per sq. in. in large and slow engines (see I.
1).

116. *Proportions of Cranks.*—The crank is shrunk on to
crank shaft, and the pin is fixed in various ways described
I.

114. *Crank arm.*—In the case of the single crank, fig. 97, when the crank is at the dead centre a section *a b* is subjected to a thrust N and a bending moment N *m*. Then by Eq. 25, I. § 43, p. 109, the greatest stress is

$$f = \text{N}\left(\frac{1}{h\,b} + \frac{6\,m}{b\,h^2}\right) \quad . \quad . \quad . \quad . \qquad (1)$$

So far as this straining action is concerned the arm would be of uniform section throughout.

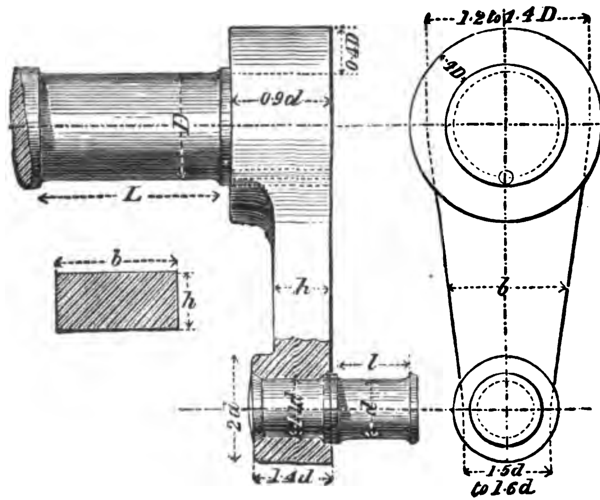


Fig. 98

When the crank and connecting rod are at right angles there is a bending moment T *r* and a twisting moment T *m*. Combining these the equivalent bending moment is (I. § 44, p. 110).

$$\begin{aligned} \text{M}_e &= \tfrac{1}{2}\,\text{T}\,r + \tfrac{1}{2}\sqrt{\{(\text{T}\,r)^2 + (\text{T}\,m)^2\}} \\ &= 0{\cdot}91\ \text{T}\,r + 0{\cdot}41\ \text{T}\,m \text{ nearly.} \end{aligned}$$

The modulus of the section is $\frac{1}{6}\,b^2 h$, so that the greatest stress is (I. § 28)

$$f = \frac{6\,M_e}{b^2 h} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

As M_e depends on r, the section will increase for a given working stress f from the crank-pin eye to the crank-shaft eye. If a value for h or b is selected, then the equations determine b or h for any section of the arm. Sections near the eyes may be calculated separately. The greater of the values given by (1) or (2) must be taken. The calculations are not quite exact and therefore moderate working stress should be taken in the reckoning.

When a crank has a T-form of section, it is but little strengthened by the feather, but it is more easily cast.

115. *Crank-shaft journal.*—The straining action is generally greatest when the connecting rod and crank are at right angles. Then if s is the distance from the centre of the journal to the plane of rotation of the centre of the crank pin, and the other notation is as shown in fig. 97, there is a bending moment $T s$ and a twisting moment $T R$ on the section of the journal. The equivalent twisting moment is (I. § 44)

$$T_e = T s + \sqrt{\{(T s)^2 + (T R)^2\}},$$

and the diameter of the journal is determined by the equation (I. § 36)

$$T_e = f\frac{\pi}{16}\,D^3 = 0{\cdot}2\,f D^3 \text{ nearly.}$$

The length of the journal is fixed so that the bearing pressure $p = T/D\,L$ or $N/D\,L$, which ever is greatest, does not exceed from 200 lbs. per sq. in. in small fast engines, to 450 lbs. per sq. in. in large and slow engines (see I. § 121).

116. *Proportions of Cranks.*—The crank is shrunk on to the crank shaft, and the pin is fixed in various ways described

below. The key in the crank shaft may have a breadth $\frac{1}{3}$ D to $\frac{1}{4}$ D, and a thickness $\frac{1}{6}$ D for small or $\frac{1}{7}$ D for large cranks. With cast-iron cranks it is desirable not to use square cornered keys. The best modern practice is to shrink the crank on the shaft and to have no key. If there is the slightest play of the nave on the shaft the key puts the crank out of truth.

Fig. 98 shows a wrought-iron crank with a section of the

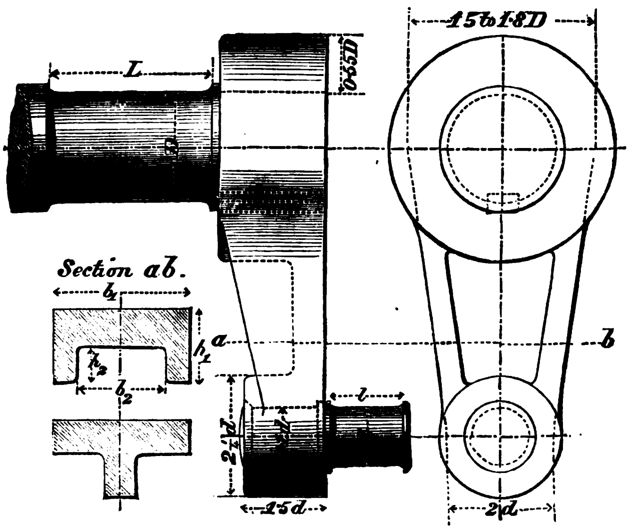


Fig. 99

arm. The arm is sometimes tapered and the back face of the arm is then rounded, so that it forms, in fact, part of a slightly conical surface, turned in the lathe. Fig. 99 shows a cast-iron crank, with sections of arms both trough-shaped and T-shaped.

In quick-running engines it is desirable to balance as directly as possible the weight of the crank and connecting rod. The crank then takes a disc form, as shown in fig. 100.

By hollowing the part of the disc on the crank side and leaving the opposite side full, a surplus weight is obtained which balances the crank pin and connecting-rod end.

The question of the amount of balance weight required involves some difficulty, because parts of the weights attached to the crank pin reciprocate without rotating, and part rotate with the crank pin.

Generally half to two-thirds of the connecting rod is taken as rotating with the crank pin. Then, if W is the weight of the unbalanced rotating mass and r the radius to its mass centre, B the balance weight and ρ the radius to its mass centre,

$$W r = B \rho,$$

which determines the amount of balance weight required. The question of balancing is treated in Chapter V.

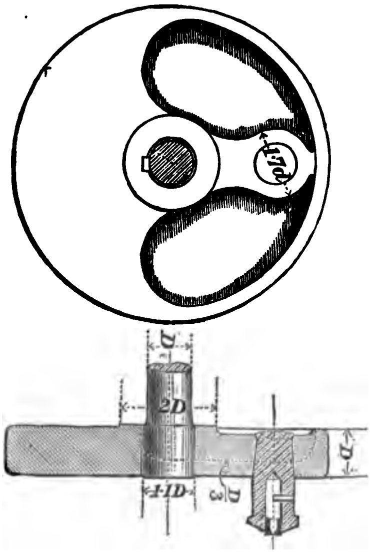


Fig. 100

117. *Crank pin of cranked shaft.*—Let P (fig. 101) be the greatest load on the crank pin. The supporting forces at the shaft journals are $P_1 = P a_2/(a_1 + a_2)$ and $P_2 = P a_1/(a_1 + a_2)$. The greatest bending moment at the centre of the crank pin is $M = P_1 a_1 = P_2 a_2 = (P a_1 a_2)/(a_1 + a_2)$. If $a_1 = a_2$ then $M = P a_1/2$. The bearing pressure $p = P/d l$ should generally not exceed

900 lbs. per sq. in. The diameter is detetermined by the relation,

$$M = f\frac{\pi}{32}d^3 = \frac{fd^3}{10} \text{ nearly.}$$

$$f = 5{,}000 \qquad d = 0{\cdot}126\sqrt[3]{M}$$

$$f = 7{,}500 \qquad d = 0{\cdot}110\sqrt[3]{M}$$

$$f = 10{,}000 \qquad d = 0{\cdot}100\sqrt[3]{M}$$

118. *Marine engine cranked shafts.*—In the case of marine engine cranked shafts, with two or more cranks,

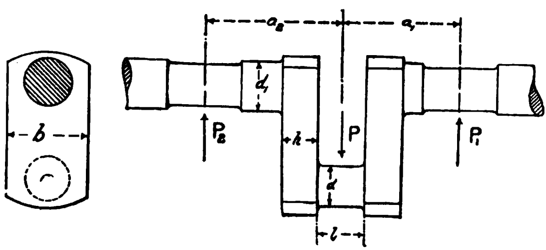


Fig. 101

subjected to both bending and torsion and supported at three or more bearings, the reactions at which are uncertain, the problem of determining the straining action is very complicated and not susceptible of exact treatment. Such shafts must be designed by practical experience, and empirical rules are laid down by the Registry Associations which govern the design. Treating the matter generally, since bending and torsion can be reduced to equivalent torsion, the diameter of the shaft may be

$$d_1 = c\sqrt[3]{\frac{\text{H.P.}}{\text{N}}},$$

where H.P. is the greatest indicated power transmitted and N the revs. per min. Values of c for steel shafts from practice are given in the following table:

	Values of Cranked shaft	Values of Tunnel shaft
Single crank engine	5·1	4·8
Two " "	4·7	4·5
Three " "	4·5	4·3

If the shaft is hollow, let d_1, d_2 be the external and internal diameters, and let $d_2/d_1 = m$. Then

$$d_1 = c \sqrt[3]{\frac{\text{H.P.}}{\text{N}}} \sqrt[3]{\frac{1}{1 - m^4}}.$$

For steel $c = 4·0$ to $4·2$.

If b is the width and h the thickness of the crank arms, then $b^2 t = 0·9$ D to D. The crank-pin diameter and length are equal to D or a little larger.

For high-speed land engines the value of c reaches 7 to $7\frac{1}{2}$.

119. *Built-up steel cranks.*—The difficulty of forging large double cranks has led to the use of built-up cranks

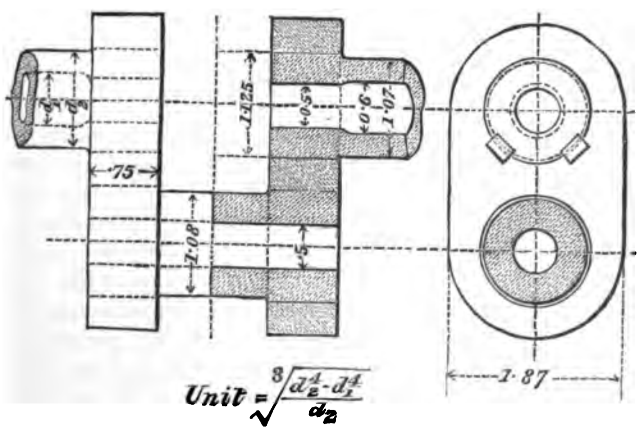


Fig. 102

like that shown in fig. 102, which shows the cranks used in the 'City of Rome' s.s. A double-collared hollow steel

shaft, formed as described in I. Chapter VIII. § 154, is cut in half to form the single-collared pieces. The crank cheeks or webs are first forged solid in the form of slabs, and then a small hole is bored at each end, and enlarged by being forged on a mandril placed on suitable supports ; thus insuring that the metal is thoroughly worked in the most important part. The cheeks are afterwards shrunk and keyed on the cut ends of the half lengths of shaft. The hollow crank pin is drawn to length by forging, and is shrunk in but not keyed into the cheeks.

120. *Failure of crank pins.*—There is some valuable information as to the fractures of crank pins which have occurred in ordinary work in a paper by Mr. M. Longridge ('Proc. Inst. Mech. Eng.,' 1896).

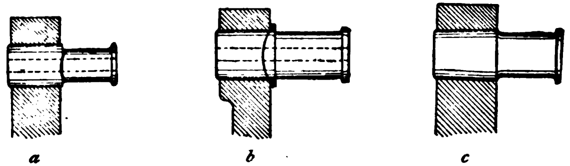


Fig. 103

The commonest fracture is that shown at fig. 103, *a*, starting from the fillet at the root of the pin, where the range of stress due to bending is greatest. This is exactly a case of fatigue, the range of stress being greater than the material would bear. Mr. Longridge gives the case of two Whitworth compressed-steel pins which gave way thus, one after $13\frac{1}{2}$ the other after 6 million revolutions, the calculated maximum stress being 12,000 lbs. per sq. in., and therefore the range of stress 24,000 lbs. per sq. in., or nearly 11 tons per sq. in. A more curious case is shown at fig. 103, *b*, the fracture starting at the angle of the eye and passing through a section greater than that at the root of the pin. It is possible that in this case shrinking-on produced an excessive local stress at the point where the

fracture started. Pins with collars against the eye, held in place by a nut at the back or by riveting over, sometimes break within the eye. This appears to be due to the pin getting loose and the plane of greatest bending being transferred back to the plane where fracture occurs. Fractures have occurred in this way in cases where the greatest calculated stress at the root of the pin was 8,000 to 17,000 lbs. per sq. in.

When the root of the crank pin is conical it must generally be fixed by a key, which is objectionable. The best method is to make the root cylindrical as in fig. 103, *c*, a form which can be more accurately made than the conical form, and to fix it in the eye by forcing it in by an hydraulic press or by shrinking. The removal of the pin for repair is then easy.

Eccentrics

121. An eccentric is a modified crank, chiefly employed to drive the slide valve of steam engines. It is really a crank and connecting rod, with a crank pin enlarged, so as to include the crank shaft within its section. The eccentric consists of a sheave, which is virtually a crank pin, and a strap and rod which is virtually equivalent to a connecting rod. The sheave is most commonly of cast iron, and is often cast in two parts connected by bolts. In very hard-worked eccentrics the sheave may be of wrought iron, case-hardened. When the sheave is in two parts, the smaller may be of wrought and the larger of cast iron. The strap is in two parts, and is prevented from slipping sideways by a flange or flanges, or it has internally a spherical surface fitting on the sheave. The strap is of brass, of cast iron, or of wrought iron lined with brass or with white metal. It is doubtful if any eccentric strap wears as well as one of cast iron. The friction of the eccentric is much greater than that of a crank, and it is therefore not used where ordinary cranks can be applied.

The distance between the centres of the crank shaft and eccentric sheave is termed the 'eccentricity,' the 'radius,' or the 'throw' of the eccentric. Let this be denoted by r, and let d be the diameter of the shaft on which the eccentric is fixed. Then the least diameter suitable for the eccentric sheave is about

$$= D = 1{\cdot}2\,d + 2\,r + \tfrac{3}{4}.$$

122. *Width of bearing surface of eccentric sheave.*—The sheave is properly a journal of much enlarged diameter. In such a case, as the velocity of rubbing is much greater than for ordinary journals, the pressure on the bearing surface must be less. The rule which is most trustworthy is that in I. § 115, Eq. 4, and § 122.

Let Q be the greatest thrust exerted by the eccentric, usually to overcome the friction of a slide valve; let b be the width of eccentric sheave and N the number of revolutions per minute. Then,

$$b = \frac{Q\,N}{\beta}.$$

As will be seen in I. § 122, the constant β has a wide range of values, depending on the accuracy of workmanship, the perfectness of lubrication, &c. For ordinary crank pins β ranges from 60,000 to 200,000, the smaller value for small crank pins of quick-running engines, and the larger for large crank pins of slow engines. For eccentrics $\beta = 60{,}000$ to 100,000. There is, however, a difficulty in determining the thrust Q exerted, and to this rather than to fault in the rule the apparent discrepancy in the values of β in some practical cases is probably due.

123. *Effort required to move a slide valve.*—Suppose a very simple case, fig. 104, a flat valve covers a port to a passage which can be closed by a stop-cock, A. The pressure on the back of the valve of area Ω is p_1, and that in the port of area ω is p_2 when the cock is open. It is found that when the cock is closed no effort is required to

remove the valve. Leakage under the valve establishes equilibrium of pressure above and below. If the cock is opened an effort is required, but there is still a fluid film between the faces of valve and seat which exerts some average pressure p. Then the load on the valve is

$$p_1 \Omega - p_2 \omega - p (\Omega - \omega).$$

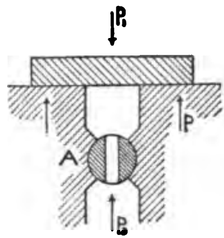


Fig. 104

The pressure p is not known, but it must lie between p_1 and p_2 and therefore the load on valve must lie between

$$(p_1 - p_2) \omega \text{ when } p = p_1$$

and

$$(p_1 - p_2) \Omega \text{ when } p = p_2,$$

and the effort to move the valve must lie between $\mu (p_1 - p_2) \omega$ and $\mu (p_1 - p_2) \Omega$, where μ is the coefficient of friction.

124. *Friction of a slide valve.*—The friction of a slide valve must depend on the total pressure between the valve and seating, which varies with the position of the valve relatively to the ports; and on the coefficient of friction, which depends on the state of lubrication of the surfaces. The pull or thrust in the valve rod is further increased at the beginning of a stroke and decreased towards the end by the inertia of the valve and other reciprocating parts, which in quick-running engines may amount to a considerable force.

To simplify matters consider first the valve in mid-stroke, fig. 105. The total pressure between valve and seating consists :—(*a*) Of the pressure on the back of the valve. If p_1 is the steam-chest pressure (absolute) this amounts to p_1 B L lbs. ; (*b*) Of the relieving pressure on the exhaust space. If p_2 is the absolute pressure in the exhaust space this amounts to $p_2\ b\ l$ lbs.; (*c*) Of the relieving pressure on the steam port which has just closed to

admission. The pressure will not sensibly differ[1] from the pressure p_1 in the steam chest. The amount of this is then $p_1\,a\,l$ lbs.; (*d*) The pressure in the port which has just closed to exhaust. This pressure will not much differ from the pressure p_2 in the exhaust space. This will amount therefore to $p_2\,a\,l$ lbs.; (*e*) There is the pressure between the valve faces and the seating faces separated by a fluid layer of steam or lubricant.[2] What the intensity of this pressure is, is unknown, but its mean value must lie between p_1 and p_2. If, as a rough approximation, this pressure is taken at $(p_1 + p_2)/3$, then the total pressure is $\frac{1}{3}(p_1 + p_2)(\mathrm{BL} - b\,l - 2\,a\,l)$. Hence the total pressure of the valve on its seat is

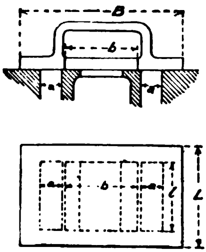


Fig. 105

$$\mathrm{P} = p_1 \mathrm{BL} - p_2 b l - p_1 a l - p_2 a l - \tfrac{1}{3}(p_1 + p_2)(\mathrm{BL} - b l - 2 a l)$$
$$= p_1 \{ \tfrac{2}{3}\mathrm{BL} - \tfrac{1}{3} a l + \tfrac{1}{3} b l \} - p_2 \{ \tfrac{1}{3}\mathrm{BL} + \tfrac{2}{3} b l + \tfrac{1}{3} a l \},$$

and the frictional resistance to motion is $\mu\,\mathrm{P}$ where μ is the coefficient of friction.

Mr. J. A. F. Aspinall has made some extremely valuable experiments on the friction of locomotive slide valves ('Proc. Inst. of Civil Eng.,' xcv. 167, and cxxxiii. 13). A small hydraulic cylinder and piston formed part of the valve rod, and indicator diagrams were taken from the ends of this during the motion of the valve. These corrected for friction and inertia gave the total slide-valve friction. For the mid position of the valve the inertia correction is unnecessary. Simultaneously diagrams were taken from the

[1] Strictly the pressure will be rather less than p_1, as the valve has closed by the amount of outside lap and expansion has begun.

[2] Professors Callendar and Nicholson have shown that probably in all cases there is a leakage of steam under the slide valve. Consequently there must be pressure between the valve and seating faces.

cylinder and steam chest, so that the pressures p_1 and p_2 could be ascertained. Three valves were tried, an Allen valve of gunmetal, a gunmetal ordinary valve, and a cast-iron ordinary valve. The following results are given by Mr. Aspinall for the valves working at full stroke, or are inferred from data given in the paper:

	Valve	B	L	b	l	a	p_1	p_2	Friction of motion at mid-stroke lbs.
I.	Gunmetal Allen	10	$16\frac{1}{2}$	6*	$13\frac{1}{2}$	$2\frac{1}{8}$	148	15	1,321
II.	Gunmetal plain	10	$16\frac{1}{2}$	6	$13\frac{1}{2}$	$1\frac{5}{8}$	148	15	1,096
III.	Cast iron plain	10	$16\frac{1}{2}$	6	$13\frac{1}{2}$	$1\frac{5}{8}$	138	15	982

* The area of passage through the Allen valve is added to the port area.

Calculating by the formula above, the total pressure of the valve on its seat is in I., 17,011 lbs.; in II., 17,663 lbs.; in III., 16,353 lbs. Hence the coefficients of friction are in I., 0·078; in II., 0·062; in III., 0·060.

Mr. Aspinall, reckoning with gauge pressures instead of absolute pressures, and neglecting the pressure between valve faces and seating, gets the values $\mu = 0{\cdot}068$, 0·054, and 0·051. But though the calculation is in any case dependent on an assumption, the formula above is likely to be the more exact.

The experiments showed a distinct decrease of friction with increased lubrication. Mr. Aspinall estimates the work expended in friction of two valves and eccentrics to amount to 2,723 foot lbs. per revolution. In a second series of experiments on valves working on a horizontal face, the friction was sensibly greater than that of the valves in the first series with vertical face. This appears to be due to the lubrication being less perfect. The total force to

move the valve varied from 964 lbs. to 2,452 lbs. in different cases. Some experiments with balanced valves of the same size gave resistances varying from 760 to 941 lbs. only, showing the great advantage as far as friction is concerned of a pressure-relief ring on the back of the valve. There are not data in the paper for calculating the friction by the method given above. But Mr. Aspinall's own estimate, neglecting the pressure between valve and seating faces, is that the coefficient of friction varied from 0·041 to 0·112.

These results give only the friction of the valve at mid-stroke with valves in excellent working order. The friction, however, varies a good deal in different positions of the valve; the friction at starting is greater than the friction of a valve which has been in motion, and times must occur with any valve when the lubrication is imperfect. But all these circumstances must be taken into account in considering the strength of the valve gear.

Hence, in designing a valve gear so as to provide sufficient strength in all contingencies, it will be assumed that the load P on the valve is calculated as above and that the coefficient of friction μ is taken at 0·2. Then the effort which the parts of the valve gear may be called on to resist is

$$Q = \mu P = 0{\cdot}2\, P \text{ lbs.},$$

acting along the valve rod.

125. *Inertia forces.*—The inertia forces due to acceleration are greatest at the ends of the valve stroke and may be neglected at mid-stroke. If W is the weight of the valve and parts between the valve, and the section considered which reciprocate with it, then the inertia forces at the ends of the stroke are approximately

$$\pm \frac{W}{g} \frac{v^2}{r};$$

where r is the eccentric radius and v the velocity of the

centre of eccentric sheave. If the eccentric makes n revs. per sec. then $v = 2\pi r n$. In many cases where the slide valve moves slowly the inertia forces are not large. But in quick-revolution engines and locomotives the inertia forces are too large to neglect in designing the valve gear. The total thrust or pull on the valve gear at the ends of the stroke is

$$Q' = \mu P' \pm \frac{W}{g} \cdot \frac{v^2}{r},$$

where P' is the valve load when the valve is at the end of its stroke. In the case of piston valves, the friction is small, but as the valves are heavy the inertia forces may be very large, and cases have occurred where trouble has arisen from not fully considering them.

126. *Proportional unit.*—In the following figures the proportional unit taken is

$$k = c\sqrt{Q}$$

where $c = 1/100$ for cast iron, $1/250$ for wrought iron, and $1/350$ for steel.

127. *Radius of eccentric.*—Let w be the greatest width of port opened to steam; l, the lap of the valve; r, the radius of the eccentric,

$$r = w + l,$$

w is in some cases the whole width of the steam port, but in quick-running engines the opening to steam is less than the opening to exhaust. This is secured by making w about $\frac{2}{3}$rds of the width of the port. The external lap l may vary from $\frac{1}{3}$th of the width of the port to the whole width of the port, according to the amount of expansion required.

The proportions of slide valves will be given in a later chapter.

128. *Proportions of sheave.*—The width b of the bearing surface of the sheave may be taken equal to $2k$, or $2\frac{1}{4}k$,

unless independently determined by the rule above. The diameter of the bolts connecting the two parts of the sheave may be 0·85 k to k, and the cotter in these bolts may have a width equal to their diameter and a thickness equal to $\frac{1}{4}$ of their diameter. The set screws may be 0·7 k diameter, or if there is only one it may be k in diameter. If the sheave has flanges to retain the strap, their projection may

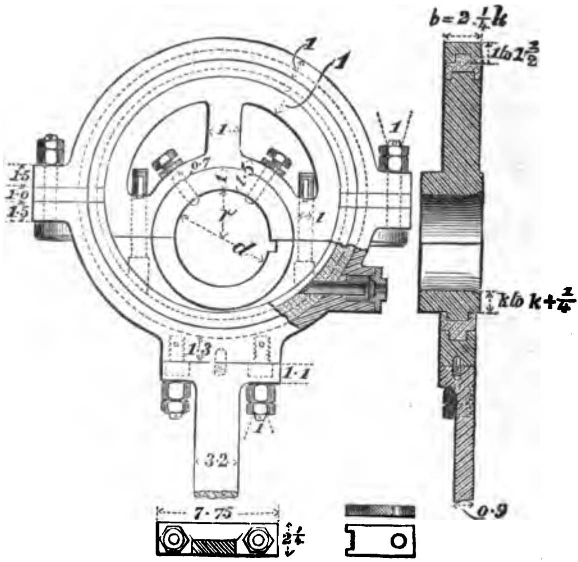


Fig. 106

be 0·4 k, and their thickness 0·3 k. The smaller part of the sheave, when it is in two parts, is generally of wrought iron in small eccentrics. The least thickness of this part, where the shaft comes near the edge of the sheave, should be at least k for wrought iron and $1\frac{1}{4}$ k for cast iron.

129. *Proportions of strap.*—The strap thickness varies very much. For wrought iron it may be from 0·5 k in large

to k in small eccentrics. For gunmetal it should be from 0·625 k to $1\frac{1}{4}$ k. For cast iron from 0·75 k to 1·5 k. The brass lining may be about $\frac{1}{8}$th the strap thickness in

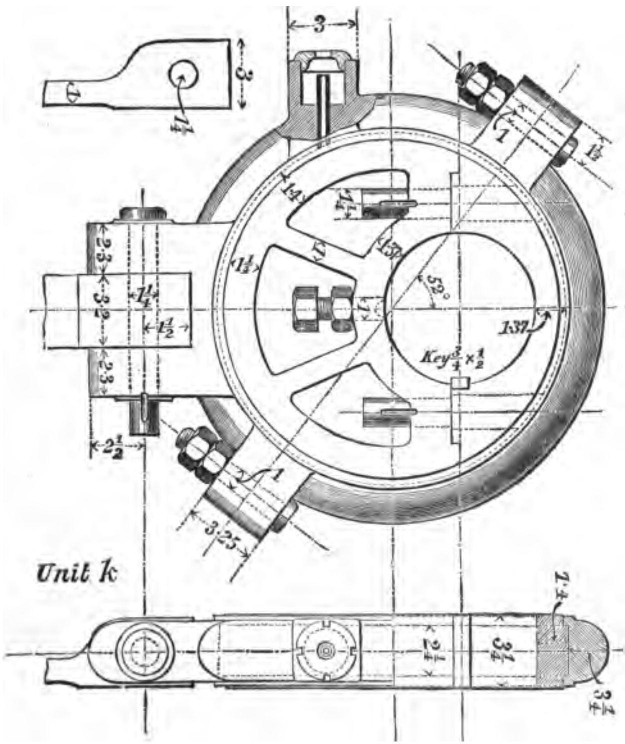


Fig. 107

large eccentrics, and in other cases its thickness may be $\frac{D}{40} + \frac{1}{8}$.

When the strap is recessed to fit a projection on the sheave, the depth of the recess may be $\frac{5}{7}$ths of the thickness

of the brass, and its width $0{\cdot}3\,b$. The corresponding recess in the brass may be of the same depth, and its width $b - \frac{5}{8}$ to $b - \frac{7}{8}$.

With eccentric straps made in the ordinary manner there is a tendency to open at the joints slightly, causing cutting and heating. This may be avoided by making the

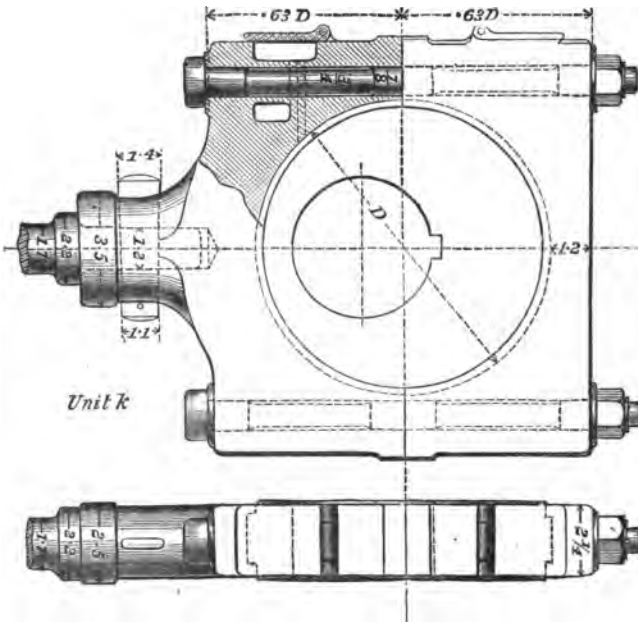


Fig. 108

strap of the form shown in fig. 108.[1] The bolts are long and brought as close as possible to the sheave. The bolts bear in the strap only at the middle and ends, and are turned down to the diameter at bottom of screw thread in intermediate parts. This gives a little elasticity to the

[1] The eccentric shown in fig. 108 was designed by Mr. Druitt Halpin.

bolts and prevents the fatigue of the metal at the last screw thread. An oil box can be formed in each half of the strap.

130. *Proportions of eccentric rod.*—The eccentric rod is very commonly attached to the eccentric strap by a T-end, and has at the other an eye to receive the pin of the valve rod. At its smaller or eye end it may have a width of $1{\cdot}8\,k$ and a thickness $0{\cdot}55\,k$. It tapers in width about $\frac{1}{2}$ in. per foot of length to the T-end, the thickness being constant. The bolts in the T-end may be of the same size as the strap bolts.

Fig. 107 shows a link-motion eccentric for a locomotive having both sheave and strap of cast iron, and so arranged as to be easily taken apart.

In both fig. 106 and fig. 107 the eccentric sheave is shown divided into two parts for convenience of fixing. When the eccentric can be put on from the end of the shaft this is not necessary.

Although the proportional figures may be taken as a guide in designing, it is very desirable afterwards to examine if the strength of bolts, pins, &c., is sufficient, by calculating the section required from the effort Q, found as stated above, by the ordinary rules of strength of materials.

CHAPTER VII

CONNECTING RODS

131. Connecting rods are the pieces which connect a rotating crank with a reciprocating piece, such as a piston or pump plunger. A link connecting two cranks is generally termed a coupling rod. In the older steam engines the connecting rod was often of cast iron, and frequently it had a cross-shaped section in order to get sufficient stiffness without too much increasing the section. Now most connecting rods are of wrought iron or steel. For engines of moderate speed the connecting rod is generally of circular section, tapered from the centre towards the ends or from the small end towards the big end, with a view of lightening the rod without much diminishing its resistance to bending. In high-speed engines, as, for instance, in locomotives, greater resistance to bending in the plane of oscillation is obtained by making the rod of rectangular or approximately rectangular section. Sometimes even an **I**-shaped section is adopted in locomotive coupling and connecting rods, part of the sides of a rectangular rod being cut away by milling to leave an **I**-shaped section.

The ends of a connecting rod are fitted with brass steps and adjusting arrangements for neutralising wear, and these are fitted to the crank pin at one end and the crosshead pin at the other. If, in consequence of adjusting the brasses, the connecting rod is altered in length, the clearance at the two ends of the cylinder is altered. To prevent this, it is desirable to make the adjustments for wear at the two

ends of the rod in such a way that tightening up the brasses at one end lengthens the rod and tightening at the other shortens it.

In consequence of the varying obliquity of the connecting rod, the position of the piston is not the same for corresponding crank angles in the forward and return strokes. With a very short connecting rod this has a prejudicial effect on the distribution of steam, and at the same time the pressure on the guides becomes excessive. Hence, usually in steam engines the connecting rod is $4\frac{1}{2}$ to $6\frac{1}{2}$ cranks in length. For engines at high speed a long connecting rod is specially desirable. In the Westinghouse single-acting engine the line of stroke passes to one side of the centre of the crank shaft. The effect of this is to diminish the obliquity of the connecting rod (or virtually to lengthen it) during the working stroke and to increase the obliquity in the non working or return stroke.

132. *Notation.*—The following notation will be used in the discussion which follows:

P = the axial thrust or tension along the rod in lbs.
I = the smallest moment of inertia of the middle section of the rod about an axis through its centre of figure.
A = area of cross-section in sq. ins.
l = length of rod between crank-pin and crosshead-pin centres in ins.
r = radius of crank in feet.
$\nu = l/r$.
w = average weight of rod per in. length in lbs.
m = factor of safety (including allowance for straining actions neglected).
ω = angular velocity of crank in radians per sec., considered uniform.
N = revolutions of crank per minute.

v = velocity of crank pin in feet per sec.

$$v = \omega r$$

$$\omega = 2\pi N/60 = \pi N/30.$$

G = weight of a cubic in. of the material of the rod in lbs. (G = 0·282 for iron or steel.)

133. *Axial straining forces acting on the connecting rod due to the effort at the crosshead pin.*—There are two positions of the connecting rod for which it is desirable to consider the straining action :—(*a*) When the crank is at a dead centre the effort transmitted from the piston is greatest ; (*b*) When the connecting rod and crank are at right angles the bending forces are greatest.

(*a*) *Crank at dead centre.*—Let p be the initial effective steam pressure on the piston. For a condensing engine this may be taken as the absolute boiler pressure less 2 to 4 lbs. per sq. in., and for a non-condensing engine the absolute boiler pressure less 16 lbs. per sq. in. For a compound engine p is the greatest difference of pressure on the two sides of the piston, estimated or found from an indicator diagram. Let D be the diameter of the cylinder in ins. Then

$$P = \frac{\pi}{4} D^2 p \text{ lbs.} \qquad \qquad (1)$$

is the piston load transmitted to the crosshead. If the engine is starting slowly this is the axial thrust or tension on the rod, and this must in any case be provided for. When the engine is in motion this thrust is modified by the inertia of the crosshead, piston rod, and piston. Let W be the weight of these parts in lbs.

Then at the beginning of a stroke, the axial force is

$$P' = P - \frac{W}{g} \cdot \frac{v^2}{r}\left(1 \pm \frac{1}{\nu}\right) \text{ lbs.} \qquad \qquad (2)$$

where the + sign refers to the inner and the − sign to the outer dead point. At the end of a stroke the axial force is

increased by the inertia force, but P is diminished in consequence of the expansion of the steam. Since at times the inertia force must be small, P and not P′ should be used in calculating the strength of the rod.

(*b*) *Crank and connecting rod at right angles.*—In this case the effective steam pressure p' is less than the initial steam pressure in consequence of expansion, and must be estimated or taken from a diagram. Also the inertia force of the reciprocating parts is practically zero. In consequence of the obliquity of the rod the axial thrust or tension is greater than the piston load by about 3 to 5 per cent. Hence the axial force is now

$$P_1 = 1{\cdot}03 \text{ to } 1{\cdot}05 \frac{\pi}{4} D^2 p' \text{ lbs.} \quad . \quad . \quad . \qquad (3)$$

SIMPLE TREATMENT OF THE STRENGTH OF CONNECTING AND COUPLING RODS.

134. Connecting rods are subjected to alternate thrust and tension and to bending forces, due to the inertia of the rod, alternately acting in opposite directions in the plane in which the rod oscillates. The load transmitted to the rod from the piston varies with the varying steam pressure, and the inertia forces vary with the position of the mechanism and the speed of the engine. Consequently, to determine the straining action in the condition most unfavourable to the strength of the rod is a very complicated problem. When the engine runs at moderate speed the inertia forces may be allowed for in the factor of safety, and a simple treatment of the problem is possible.

In general the greatest axial thrust or tension occurs with the crank at the dead centre, and its value P has been found in Eq. 1. In the case of a coupling rod the axial force must be determined with reference to the proportion of the whole piston load which is transmitted through it.

135. *Resistance of connecting rod to tension.*—The minimum section of the rod must be sufficient to resist the

greatest tension. Let $d_{\min}$ be the least diameter of a rod of circular section ; f the safe working stress. Then

$$\frac{\pi}{4} d^2_{\min} f = \text{P} = \frac{\pi}{4} \text{D}^2 p$$

$$d_{\min} = \frac{1}{\sqrt{f}} \cdot \text{D} \sqrt{p} \quad . \quad . \quad . \quad (4)$$

From Table II., p. 5, Case C, the safe stress f is 1,400 for cast iron, 5,000 for wrought iron, and about 6,000 for steel. The stresses at the smallest sections of actual rods are usually rather less than these and will be fairly represented by taking

$$\left. \begin{aligned} d_{\min} &= 0{\cdot}0299 \text{ to } 0{\cdot}0327 \text{ D} \sqrt{p} \text{ for cast iron} \\ &= 0{\cdot}0158 \text{ to } 0{\cdot}0173 \text{ D} \sqrt{p} \text{ for wrought iron} \\ &= 0{\cdot}0138 \text{ to } 0{\cdot}0151 \text{ D} \sqrt{p} \text{ for steel} \end{aligned} \right\} (5)$$

If the section is rectangular, of breadth b and height h, it is only necessary to make

$$(b\, h)_{\min} = \frac{\pi}{4} d^2_{\min}.$$

136. *Stability of connecting rods treated as columns resisting a thrust.*—When, as is most commonly the case, the rod is long in proportion to its diameter, its resistance to lateral buckling must be considered.

Let d be the diameter of the rod at the centre. Then for bending in the plane of oscillation it may be considered a strut hinged at the ends, and it corresponds to Case II., Table VIII., Part I., p. 95. Let P be the initial load, taken as $(\pi \text{D}^2 p)/4$ lbs., and m a factor of safety determined by calculation from actual rods which have worked satisfactorily. Let E = coefficient of elasticity of the material = 28,000,000 for iron or steel; l = length of rod between centres of pins; I = least moment of inertia of cross-section at middle of rod about an axis through its centre of figure. Then the general relation for the greatest load consistent with stability is

$$m\,\text{P} = \frac{m\,\pi\,\text{D}^2 p}{4} = \pi^2 \frac{\text{E I}}{l^2} \quad . \quad . \quad . \quad (6)$$

Case I.—Rod of circular section, diameter at middle of length $= d$; $I = 0{\cdot}05\, d^4$

$$\left.\begin{aligned} P &= 13{,}810{,}000\, \frac{d^4}{m\, l^2} \\ d &= 0{\cdot}0164\, \sqrt[4]{(P\, l^2)}\, \sqrt[4]{m} \end{aligned}\right\} \quad . \quad . \quad (7)$$

The value of the apparent factor of safety m varies very greatly in different practical cases. It includes the real factor of safety and an allowance for the inertia straining actions neglected in calculating P. In ordinary land engines $m = 20$ to 25, but in some high-speed engines $m = 10$. In marine engines $m = 30$ to 40. In locomotives m is as low as 2 to 4.

$m =$	2	3	4	6	10	15	20	30	40
$\sqrt[4]{m} =$	1·19	1·32	1·41	1·56	1·78	1·97	2·11	2·34	2·51
$·0164\sqrt[4]{m} =$	·0195	·0216	·0231	·0255	·0291	·0323	·0346	·0383	·0411

As d varies as the fourth root of m, a great variation in m does not make a very great difference in d.

In rods of circular section the diameter near the crosshead is sometimes reduced to $0{\cdot}7\, d$, and that at the crank-pin end to $0{\cdot}8\, d$. In other cases, especially in the case of quick revolutions, the rod is simply tapered from the crosshead end to the crank end—that is, the diameter near the crosshead is $0{\cdot}7\, d$, and that at the crank-pin end about $1{\cdot}3\, d$.

Case II.—Similar rod of rectangular section, breadth b and height in the plane of rotation h. Let $h = x\, b$, where x varies in practice from $1\frac{1}{2}$ to 2. For bending in the direction in which the rod is thinnest $I = \frac{1}{12} b^3 h = \frac{1}{12} x\, b^4$.

$$\left.\begin{aligned} P &= 23{,}000{,}000\, \frac{x\, b^4}{m\, l^2} \\ b &= 0{\cdot}0144 \sqrt[4]{P\, l^2}\, \sqrt[4]{(m/x)} \end{aligned}\right\} \quad . \quad . \quad (8)$$

$m =$	2	3	4	6	10	15	20	30	40
	Values of $0{\cdot}0144\ \sqrt[4]{(m/x)}$								
$x = 1\frac{1}{2}$	·0154	·0171	·0184	·0204	·0231	·0256	·0275	·0303	·0326
$1\frac{3}{4}$	·0148	·0164	·0177	·0195	·0223	·0246	·0264	·0292	·0315
2	·0144	·0159	·0171	·0190	·0210	·0239	·0256	·0283	·0303
$2\frac{1}{2}$	·0136	·0151	·0162	·0180	·0204	·0226	·0241	·0267	·0288

In some locomotive coupling rods the rectangular section is reduced to an **I**-section by milling out a recess on each side of the rod. This reduces the weight and therefore the inertia forces, causing bending. The rod is strengthened rather than weakened by the removal of material.

It will no doubt appear strange that the apparent factor of safety m varies so much, and that it has the lowest values in locomotive rods where the speeds and inertia forces are greatest. But a partial explanation at least can be given. In land engines there is not much object in reducing the weight of the rod, and, on the other hand, if the apparent factor of safety is chosen greater, the weight of the rod and the bending forces are increased and the increase of the real factor of safety is not so great as that of the apparent factor.

In marine engines there is no great reason for saving weight; the speeds are fairly high, and the rods are usually of a circular section, which is not the best to resist the bending action in the plane of oscillation. In high-speed engines, and especially in locomotives, it is very desirable to keep the weight of the rod as small as possible, in order to reduce the inertia forces; and since in these cases the rods are generally rectangular, their strength in the plane of oscillation is very much greater than that normal to the plane of oscillation. But the equations above give the strength of the rod as regards lateral bending in the direction in which the rod is weakest, while the inertia forces act in the plane of oscillation. Hence in the locomotive the extra straining action due to the high speed is allowed for by giving the section a form such that it has extra strength to resist the bending in the plane of oscillation which the high speed entails.

137. *Inertia bending forces acting on a connecting rod.*—The big and little ends of the rod do not much affect the bending action. Let w be the average weight of the rod

per in. length, so that if A_m is an average section (excluding the ends), $w = G\,A_m$.

When the crank and connecting rod are at right angles (fig. 109) the acceleration of the crank pin C in the direction C O is v^2/r feet per sec. per sec. At a point distant x from D, the acceleration normal to the rod is approximately $x\,v^2/l\,r$, so that the distribution of acceleration is represented by a triangle D E C, the acceleration being the breadth of the triangle normal to the rod. The inertia force of an

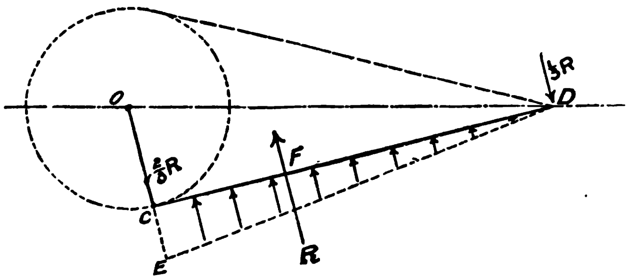


Fig. 109

in. length of rod at x from D, is $\frac{w}{g}\,\frac{v^2}{r}\,\frac{x}{l}$ lbs. Hence the resultant force R normal to the rod is

$$R = \frac{w}{2\,g}\,\frac{v^2}{r}\,l.$$

It acts at a point F at a distance $\frac{2}{3}\,l$ from D. The reaction at the crank pin C is $\frac{2}{3}\,r$, and at the crosshead pin D is $\frac{1}{3}$ R. To the right of a point x from D, the forces acting are $\frac{1}{3}$ R and the centrifugal load $\frac{w}{g}\,\frac{v^2}{r}\,\frac{x}{l}\,\frac{x}{2}$ at $\frac{2}{3}\,x$ from D. Then the bending moment is

$$M = \frac{1}{3}\,R\,x - \frac{w}{g}\,\frac{v^2}{r}\,\frac{x}{l}\,\frac{x}{2}\,\frac{x}{3} \qquad . \quad . \quad . \qquad (9)$$

$$= \frac{w\,v^2}{g\,r} \cdot \frac{l}{6}\,(l^2\,x - x^3)$$

The bending moment is greatest when $d\,\text{M}/d\,x = 0$

$$\frac{d\,\text{M}}{d\,x} = l^2 - 3\,x^2 = 0$$

$$x = l/\sqrt{3} = 0{\cdot}577\,l.$$

The bending moment at the section where it is greatest is, putting this value of x in (9),

$$\text{M}_{\max} = \frac{1}{16}\,\frac{w\,v^2}{g\,r}\,l^3 \quad . \quad . \quad . \quad . \qquad (10)$$

If the section of the rod is circular and d is its diameter at $0{\cdot}577\,l$ from D, the stress at the edges of the section, in the plane of oscillation, is

$$f_b = \text{M}_{\max}/(0{\cdot}098\,d^3) \quad . \quad . \quad . \qquad (11)$$

If the section is rectangular, of breadth b and depth h in the plane of oscillation

$$f_b = \text{M}_{\max}/(\tfrac{1}{6}\,b\,h^2) \quad . \quad . \quad . \quad . \qquad (12)$$

138. *Inertia bending forces on a coupling rod.*—In the case of a coupling rod connecting two equal cranks, each

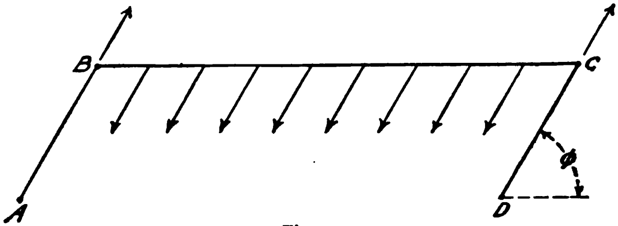


Fig. 110

element of mass describes an equal circle relatively to the crank shafts.

Let w be the weight of the rod per in. length. In the position C B of the coupling rod (fig. 110) the radial acceleration of any point is v^2/r and its component normal to the rod is $v^2/r \times \sin\phi$.

The bending force normal to the rod reckoned per in. of length is

$$\frac{w}{g}\frac{v^2}{r}\sin\phi,$$

and the greatest bending moment at the centre of the rod is

$$M = \frac{w\,v^2}{g\,r}\frac{l^2}{8}\sin\phi.$$

This is greatest when $\phi = 90°$ and then

$$M_{max} = \frac{w\,v^2}{g\,r}\frac{l^2}{8} \text{ in. lbs.} \qquad (13)$$

From this the section of the rod can be calculated by Eqs. 11 and 12 above.

139. *Greatest stress in the connecting rod when the rod and crank are at right angles.*—Let d be the diameter of the rod if circular, b and h its breadth and height if rectangular, at the section at which the bending moment is greatest. For all practical purposes this section may be taken at four-tenths of the length of the rod from the crank pin. The modulus of the section in the plane of oscillation is then either $0{\cdot}098\,d^3$ or $\frac{1}{6}\,b\,h^2$ (I. Table V., p. 66).

The thrust in the rod has been found to be (Eq. 3, § 133),

$$P_1 = 1{\cdot}03 \text{ to } 1{\cdot}05\,\frac{\pi}{4}\,D^2 p'.$$

where D is the diameter of piston and p' the effective pressure at mid-stroke. The stress in the rod due to this thrust is

$$f_t = P_1/\left(\frac{\pi}{4}\,d^2\right) \text{ or } P_1/(b\,h) \qquad (14)$$

The greatest bending moment is given by Eq. 10, § 137. The stress due to this is

$$f_b = M_{max}/(0{\cdot}098\,d^3) \text{ or } M_{max}/(\tfrac{1}{6}\,b\,h^2) \qquad (15)$$

Hence, the greatest stress in the rod is (I. § 43)

$$f = f + f \qquad (16)$$

The greatest stress found in actual rods varies a good deal. In stationary engines the stress is often 2,000 to 3,000 lbs. per sq. in. with wrought iron, and 4,000 to 6,000 lbs. per sq. in. with steel. In locomotives with steel rods the stress is often 6,000 to 7,000 lbs. per sq. in. The variation of these values is greater than can be explained rationally. When a connecting rod has been designed by the empirical rules (2) or (3), it is desirable to determine the stress f as a check on the result, at least in the case of high-speed engines.

With approximation enough for practical purposes, $w = \pi \mathrm{G} d^2/4 = 0{\cdot}221\, d^2$ lbs. for a circular section rod; and $w = \mathrm{G}\, b\, h = 0{\cdot}282\, b\, h$ for a rectangular section rod. Using these values,

For a circular section rod,

$$f = 1{\cdot}05 \frac{\mathrm{D}^2 p'}{d^2} + 0{\cdot}00429 \frac{v^2 l^2}{r d} \quad . \qquad (18)$$

For a rectangular section rod,

$$f = 0{\cdot}82 \frac{\mathrm{D}^2 p'}{b h} + 0{\cdot}00329 \frac{v^2 l^2}{r h} \quad . \qquad (19)$$

Connecting-rod Ends

140. Connecting-rod ends for steam engines are commonly termed 'big ends' and 'little ends,' the former being the bearing for the crank pin and the latter for the crosshead pin. The 'big ends' are distinguished as open ends in which the journal is confined by a strap and cotter or cap and bolts, and box or solid ends in which the metal of the rod takes the strain, and there is no loose strap or cap. Many breakdowns are due to the giving way of open-ended connecting-rod straps, or of the gibs and cotters securing them.[1] Such straps, if used, may be secured by bolts in

[1] Longridge, 'Proc. Inst. Mech. Eng.,' 1896, p. 560.

addition to the gibs or by bolts alone; but solid ends are better.

The small end is made in two ways. Sometimes it is forked and the crosshead pin is fixed in it, the bearing for the pin being formed in the crosshead, and the working surface of the pin being between the jaws of the fork. In other cases the crosshead pin is fixed in the crosshead, and the small end of the connecting rod is single ended and has a bearing for the pin.

141. *Crosshead pin.*—In most cases the crosshead pin is a neck journal, and therefore of less diameter than an overhung crank pin. The small oscillating motion of the rod on the pin, or of the pin on its bearing in the crosshead, is unfavourable to good lubrication; on the other hand the rubbing speed is low. The limit of pressure on the bearing is about 800 lbs. per sq. in. in ordinary engines, and as much as 1,500 to 2,000 lbs. per sq. in. in locomotives. Let l_1 be the length and d_1 the diameter of the crosshead pin; p_1 the pressure on the bearing surface; P the greatest load transmitted from the piston (Eq. 1). Then (I. § 123)

$$\left.\begin{aligned} l_1 &= \mathrm{P}/p_1 d_1 \\ \text{and } d_1 &= \sqrt[4]{\frac{1\cdot 28}{p_1 f}}\,\sqrt{\mathrm{P}} \end{aligned}\right\} \quad . \quad . \quad (20)$$

The working stress f is generally about 5,000 for wrought iron and 7,500 for steel.

$f =$	5,000	7,500	10,000
	Values of $\sqrt[4]{(1\cdot 28/p_1 f)}$		
$p_1 =$ 800	·0238	·0215	·0200
= 1,200	·0215	·0194	·0181
= 1,500	·0203	·0184	·0171

If the length of bearing l_1 of the crosshead pin is equal to its diameter, then two conditions must be satisfied—namely,

$$\left.\begin{aligned} d_1 &\geqq \sqrt[4]{\frac{1\cdot 28}{p_1 f}}\,\sqrt{\mathrm{P}} \\ l_1 &\geqq \sqrt{(\mathrm{P}/p_1)} \end{aligned}\right\} \quad . \quad . \quad . \quad (20a)$$

142. *Proportions of steps.*—The ends of connecting rods are designed to receive crank pins or neck journals, and are fitted with gunmetal steps similar to those used for pedestals (Part I., Chap. IX.). The unit for the proportional numbers relating to the steps in connecting rods is

$$t = 0{\cdot}08\, d + \tfrac{1}{8} \quad . \quad . \quad . \quad . \qquad (1)$$

where d is the diameter of an ordinary crank pin supporting the thrust transmitted by the connecting rod. When the connecting rod is attached to a journal of greater size than is sufficient for the thrust P of that connecting rod only,

$$t = {\cdot}007\sqrt{\mathrm{P}} + \tfrac{1}{4} \text{ to } {\cdot}012\sqrt{\mathrm{P}} + \tfrac{1}{4} \quad . \quad . \qquad (2)$$

The flanges of the steps are of very variable thickness, but very often the space between the flanges of the steps in which the connecting-rod end is placed is $\frac{7}{10}$ths of the length of the journal.

143. *Strength of connecting-rod ends.*—It is difficult to give rational rules for the strength of the parts which form a connecting-rod end, because various contingencies, such as inaccuracy of fitting, may alter the distribution of stress unfavourably to the resistance of the rod. The following, without being intended to give fixed rules, may be taken as an example of the kind of analysis a draughtsman would make in order to satisfy himself as to the sufficiency of the dimensions he proposed to adopt.

The greatest piston load acting along the rod is

$$\mathrm{P} = \pi\, \mathrm{D}^2 p/4 \text{ (Eq. 1, § 133).}$$

But to allow for contingencies, let the parts be designed for a load $m\,\mathrm{P}$, where $m = 1\tfrac{1}{4}$ to $1\tfrac{1}{2}$.

The strap is subject to tension only. It may therefore carry a working stress f of about 10,000 lbs. per sq. in. for

wrought iron or 13,200 lbs. for steel. Its breadth being β and thickness δ, fig. 111,

$$\beta\delta = \frac{m\,\mathrm{P}}{2f} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

$$\beta\delta = 0{\cdot}0000625 \text{ to } 0{\cdot}0000750\ \mathrm{P} \text{ for wrought iron}$$
$$= 0{\cdot}0000473 \text{ to } 0{\cdot}0000568\ \mathrm{P} \text{ for steel.}$$

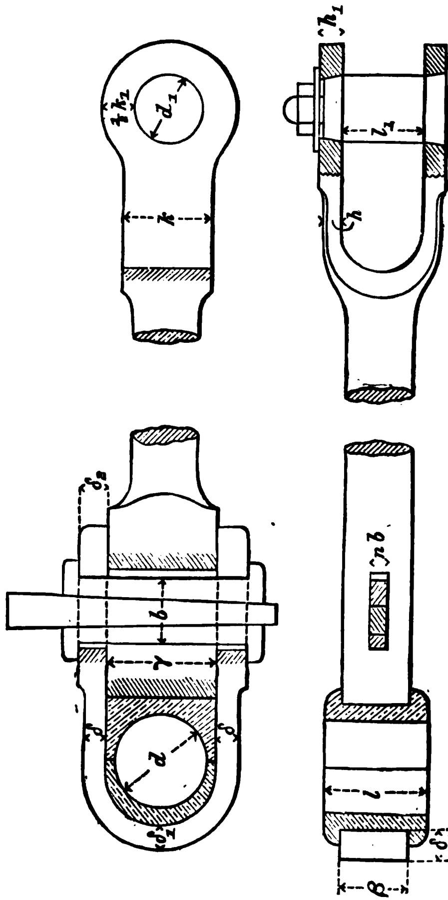


Fig. 111

The thickness δ_1 is taken 1·2 to 1·5 δ to allow for the undeterminable bending action which arises out of the strap tightening down on the brass when the load comes on. The thickness δ_2 is taken about 1·3 δ partly to make up for the metal cut out to form the cotter hole and partly to give bearing surface enough for the crushing pressure of the cotter.

The gibs and cotter should be proportioned by the rules in I. § 105, p. 225. If b is the width of gibs and cotter, and $n\,b$ their thickness, then usually $n = \frac{1}{4}$. Here the stress on the cotter acts in one direction only. For the shearing section

$$2\,n\,b^2 f_s = m\,\text{P}$$

$$b = \sqrt{\frac{m}{2\,n f_s}}\sqrt{\text{P}} \qquad . \qquad . \qquad (4)$$

Putting $n = 1\frac{1}{4}$, $m = 1\frac{1}{4}$ to $1\frac{1}{2}$, $f_s = 8{,}000$ for wrought iron and 13,200 for steel.

$$\sqrt{\frac{m}{2\,n f_s}} = 0{\cdot}00442 \text{ to } 0{\cdot}00484 \text{ for wrought iron}$$
$$= 0{\cdot}00344 \text{ to } 0{\cdot}00377 \text{ for steel.}$$

For the other end of the rod the total section through the fork is $2\,k\,h$, and as this is subjected alternately to tension and pressure we should take $f = 5{,}000$ for wrought iron and 6,600 for steel. Then

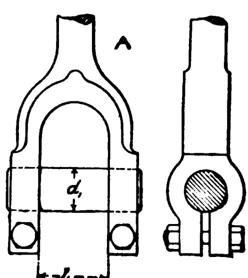


Fig. 112

$$k\,h = \frac{m\,\text{P}}{2 f} \qquad . \qquad . \qquad (5)$$
$$= {\cdot}000125 \text{ to } {\cdot}000150\ \text{P for wrought iron}$$
$$= {\cdot}000095 \text{ to } {\cdot}000114\ \text{P for steel.}$$

The section $k_1\,h_1$ through the pin is increased to about $1\frac{1}{4}$ $k\,h$ to allow for bending stresses due to the distribution of the load over the inner surface of the eye.

Figs. 111 and 112 show two ways of fastening the cross-head pin in the forked end of a connecting rod. The latter, which is the method adopted by Bollinckx, is both the simplest and the safest.

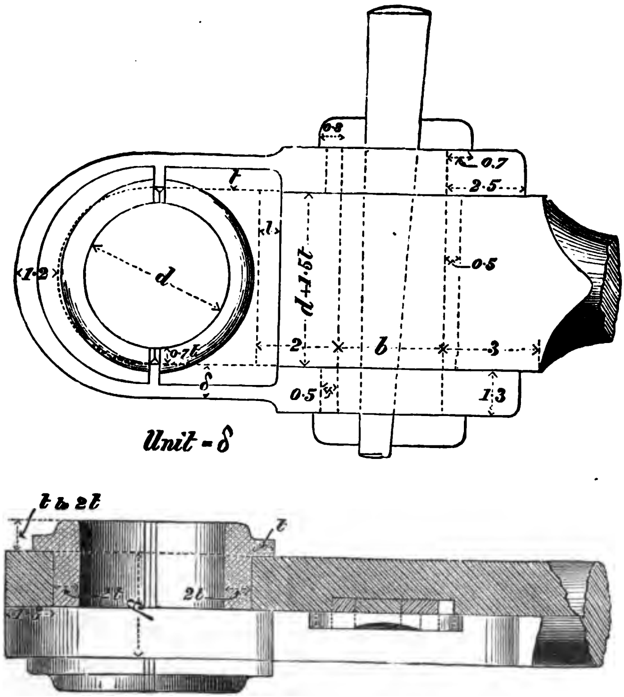


Fig. 113

144. *Forms of connecting-rod ends.*—Fig. 113 shows a very common form of connecting-rod big end, having a loose strap confining the brasses, kept in place by gibs and cotter. It will be seen that tightening the cotter shortens the connecting rod. The strap-end form is not well

adapted for high-speed engines, because the transverse forces due to the inertia of the rod tend to open the strap. The strap is of wrought iron or steel. Its section may be determined by the rule above, but cases will be found in practice where 50 or 100 per cent. more section has been given.

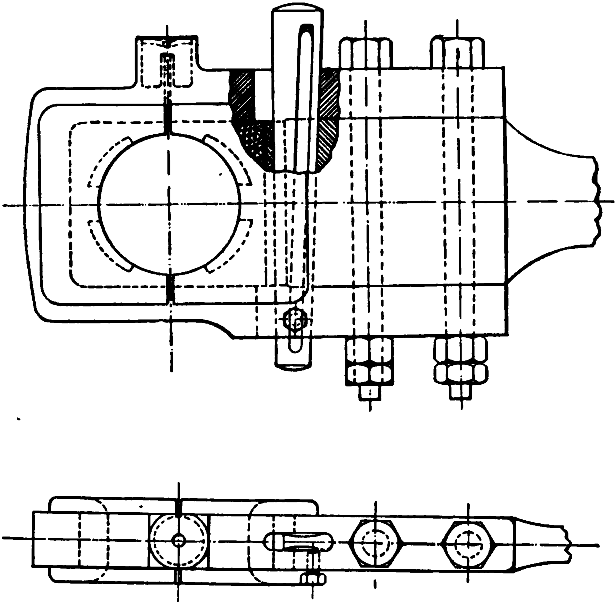


Fig. 114

In the figure a proportional unit $\delta = 0{\cdot}15\, d + 0{\cdot}2$ has been taken. The inclination of the sides of the cotter may be $\frac{1}{20}$ to $\frac{1}{18}$ on each side, if there is no locking arrangement, and $\frac{1}{6}$ if there is a set screw.

Although it is very convenient to design machine parts such as these by proportional figures, it is always desirable

to check the dimensions of the most important sections by calculating the stress on them. The stress should not exceed the values in Table II., p. 5, after ample margin has been left for straining actions neglected. Saving of weight

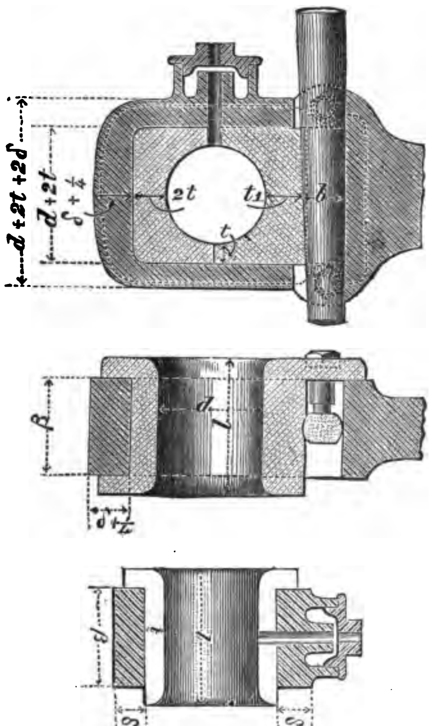


Fig. 115

is not very important in such machine parts as these, and often in practice a very large factor of safety will be found, especially in small engines.

Fig. 114 shows a modification of the ordinary strap end

connecting rod, which is nearly as safe as a box end. The strap is fastened to the rod by bolts, which prevent the displacement of the strap by resistance to shear and also prevent the opening of the strap. The cotter in this case has only the function of adjusting the gunmetal step.

145. *Box end.*—Fig. 115 shows a connecting-rod end having no loose strap. The brass steps have a thickness $2\,t$ opposite the key, and $t_1 = 6\,t - \frac{1}{2}$ next the key. At the sides the thickness is reduced to t. The thickness and overlap of the flanges of the steps may be $\frac{1}{5}\,l - \frac{1}{8}$, so that the width of the box may be $\beta = \frac{3}{8}\,l + \frac{1}{4}$. The flanges of the steps are partially removed on one side to allow their insertion in place. The thickness δ of the sides of the box may have the same value as the thickness of strap in the last case. The mean breadth of the cotter is $0{\cdot}6\,\beta$ and its thickness $0{\cdot}3\,\beta$, and it tapers 1 in 12 on each side. It is secured by two set screws; diameter of set screws = cotter thickness. Unlike the last form, this connecting rod is lengthened when the cotter is tightened. But it may be arranged with the cotter on the other side of the brass steps, and then it is shortened by tightening the cotter. A coupling rod should have one end arranged in the former and one in the latter method. Then the length of the rod is not much altered by tightening the cotters. The proportional unit for this figure is $\delta = 0{\cdot}15\,d + 0{\cdot}2$.

Fig. 116 shows a locomotive connecting-rod end which, whilst it is adjusted like a box end, can be separated from the shaft like a strap-ended rod. The block forming the end is held in place by a stout pin having a slight taper, with nuts at each end. The wedge is tapered 1 in 16, and is fixed by two peg pins driven through the adjusting bolt. The ends of the adjusting bolt are left long, and have double nuts at each end to facilitate the adjustment of the position of the wedge. The flanges of the steps are large, to cover the spaces in the wrought-iron frame of the end. In this case $\beta = 2{\cdot}3\,\delta$,

for the proportions given on this rod, t is obtained by Eq. 1, § 142, and the unit for the other proportional numbers is δ. The value of m is about 3·12.

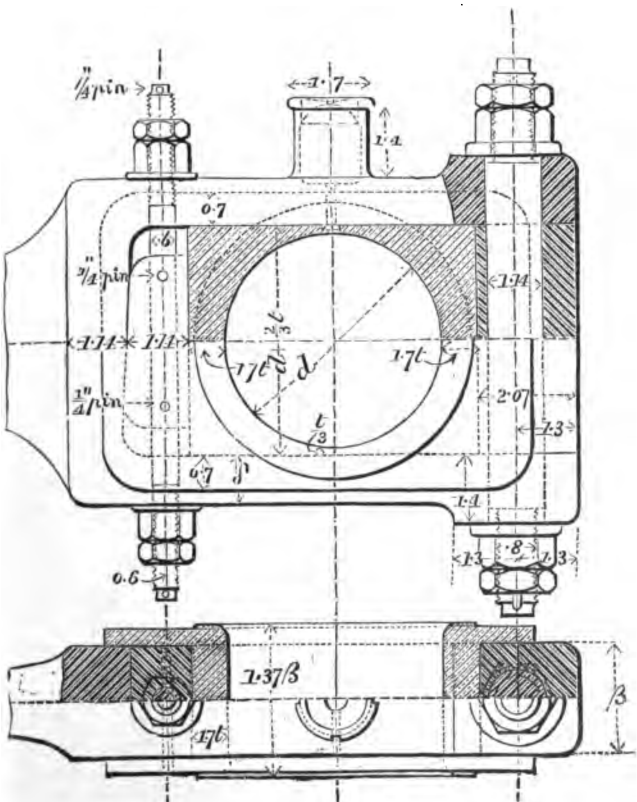


Fig. 116

146. *Marine engine connecting-rod end.*—Fig. 117 shows another form of connecting-rod end. This is of simple and massive form, and is often used in marine engines. The

brasses are lined with white metal or Babbitt's metal cast in shallow recesses.

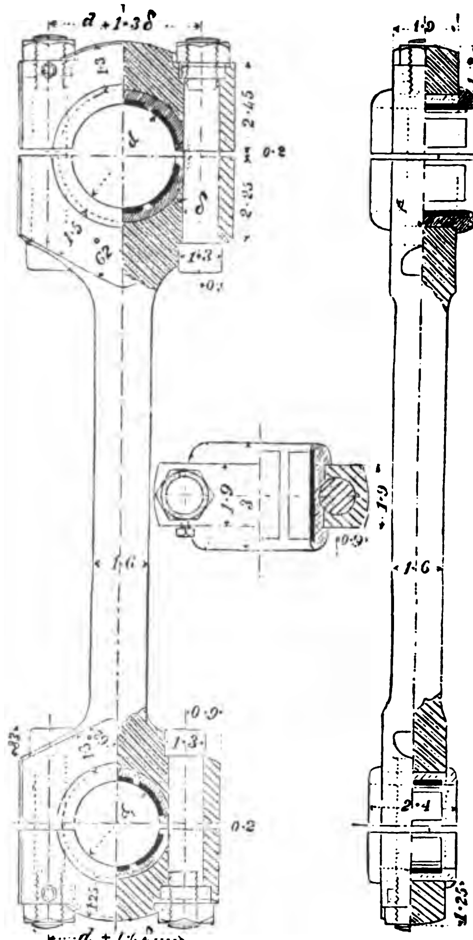


Fig. 117

If δ_1 is the diameter of the bolt at the bottom of the screw thread and f the working stress,

$$\frac{\pi}{4} \delta_1^2 f = m \text{ P}.$$

The diameter δ at top of threads will be nearly enough for the present purpose $1{\cdot}12\ \delta_1$. Hence

$$\delta = \sqrt{\frac{1{\cdot}25\ m}{{\cdot}785 f}} \sqrt{\text{P}} \quad . \quad . \quad . \qquad (6)$$

Putting $f = 5{,}000$ for wrought iron and 6,600 for steel

$$\delta = 0{\cdot}2000 \text{ to } 0{\cdot}0218 \sqrt{\text{P}} \text{ for wrought iron}$$
$$= 0{\cdot}0174 \text{ to } 0{\cdot}0190 \sqrt{\text{P}} \text{ for steel.}$$

It is a good thing to turn part of the shank of the bolt to a diameter equal to that of the bottom of the screw thread. This gives the bolt a little elasticity without weakening it. The proportional numbers in the figure are reckoned to the unit $7/8\ \delta$ or δ_1.

Fig. 118 shows a somewhat similar big end for a connecting rod. Here the brasses are designed so that they can be machined all over and are cored internally to diminish weight. The bolts are turned down so as to bear only at those points where they most effectively support the brasses and caps and prevent lateral movement.

Figs. 119, 120, show connecting rods designed by Mr. W. F. Mattes.[1]

In fig. 119 the end has jaws long enough to hold both brasses. The cap hooks over projections on the jaws, turned concentrically with the rod, and the cap is bored to fit the projections. Thus the bolts are relieved of any straining action except the longitudinal tension, which they are best capable of resisting. The cap is lighter than in the ordinary marine-engine type. As compared with a cap secured by a transverse bolt as in fig. 116, the forging of the

[1] 'Trans. Am. Soc. Mechanical Eng.,' ix. p. 467.

head is simpler and the bolts in tension are more secure than the eyes in the other form which hold the bolt in shear. As compared with a solid-head rod, the open end is frequently more convenient. In fig. 119 the wear is intended to be taken up by liners. In fig. 120 a wedge is introduced, and tapped bolts are used instead of through bolts to secure the

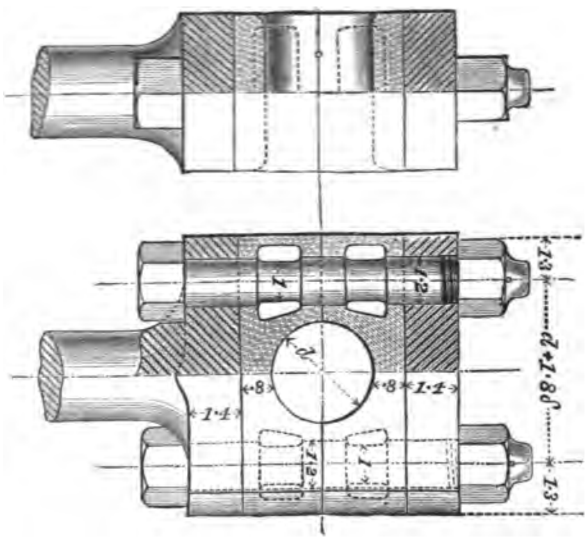


Fig. 118

cap. Bolts and rod are of forged steel. The brasses are of cast steel lined with Babbitt metal.

147. Fig. 121 shows a connecting-rod small end, from a design by Mr. Halpin. Part of the pin on each side is planed away and the brasses undercut. As the pin only vibrates through a small angle, it overruns the edges of the brasses, which prevents shoulders being formed by wear. The spaces allow lubricant to reach the wearing surfaces easily and the difficulty of an oval pin is avoided. Solid bushes instead

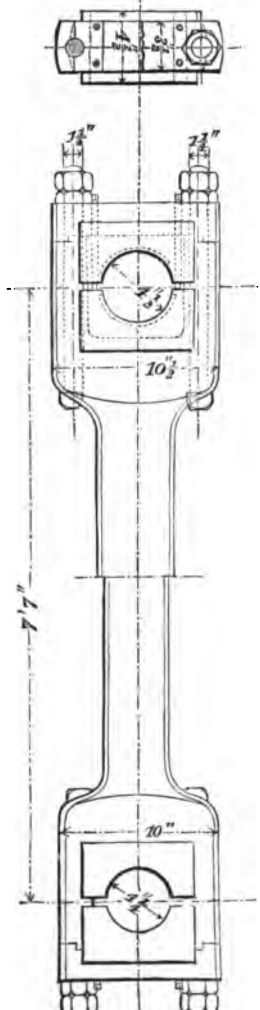


Fig. 119

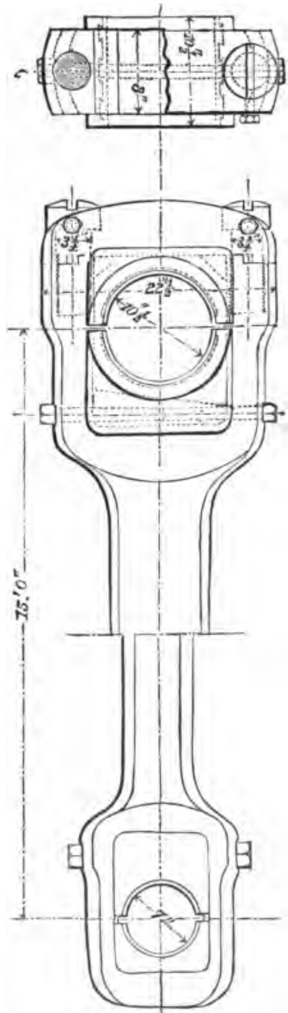


Fig. 120

of two steps are sometimes used for connecting-rod small ends. They are cheaper, and the wear is not very great if there is good lubrication.

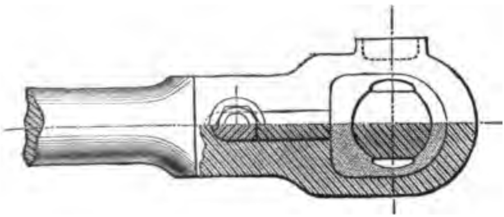


Fig. 121

Fig. 121A shows an ordinary locomotive coupling rod, with solid eyes and cylindrical gunmetal bushes. The rod is I-shaped to secure bending strength. The bushes are pressed into place and kept from revolving by a tangent pin.

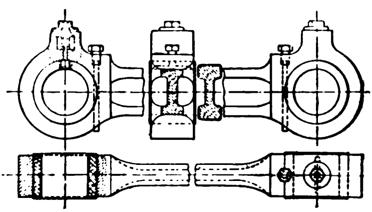


Fig. 121A

Fig. 122 shows a coupling - rod joint which may serve as an example of a journal bearing where there is not a great amount of motion and wear. This joint is intermediate in construction between a common knuckle-joint and a connecting-rod end. It has bushes to diminish friction and wear, but these are not divided, so that there is no adjustment after wear has taken place. The crank pin turns in a brass bush, which is protected by an outer steel bush. Both brass and steel bush are fixed in the forked-rod end by small snugs, and the solid-rod end turns on the steel bush. The pin in a joint of this kind is often larger than is necessary for strength, because, by using a large pin with a small intensity of pressure between the rubbing surfaces,

there is less danger of squeezing out the lubricant. The pin is of steel. The proportions may be

$$t_1 = t_2 = 0{\cdot}1\,d + \tfrac{1}{8}$$
$$a_1 = 0{\cdot}3\,d$$
$$a_2 = 0{\cdot}8\,d$$
$$t_3 = b\,h/4\,a_1.$$

For the brasses of parallel motion bars, which should be capable of being tightened without altering the position of

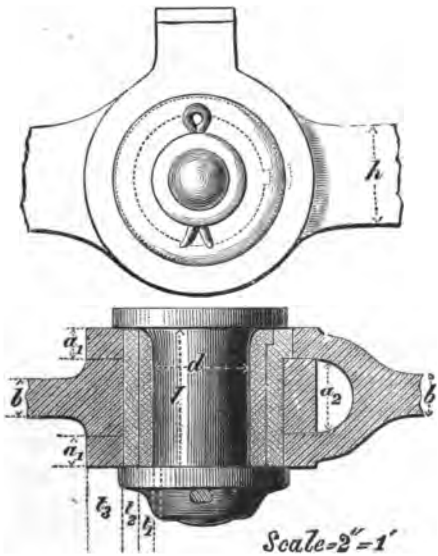


Fig. 122

the centres of rotation, the following ingenious plan has been suggested by Mr. Candlish (' Engineering,' xxxii. 461). The motion bars are fitted with bushes tapered internally to fit conical journals, and parallel externally, with a feather to prevent the bushes from turning in the rod eyes. By regulating the position of the bushes by double nuts they can be adjusted when worn without interfering with the centres of motion.

CHAPTER VIII

CROSSHEADS AND SLIDES

CROSSHEADS

148. 'Crosshead' is the name given to the part which connects together the piston rod and connecting rod of a steam engine, and with which is also connected the guiding arrangement either of slide blocks or parallel motion bars. It consists essentially of a socket to which the piston rod is keyed, and a crosshead pin, forming one or more journals on which the connecting rod works. The crosshead either forms a slide block or has the slide blocks attached to it.

The design of the crosshead depends primarily on the arrangement of the slides which guide the piston-rod end. In the older engines and some modern engines there are two slides (formed by four slide bars), one on each side of the crosshead. Now frequently there is only one slide formed by slide bars above and below the crosshead, or the crosshead is formed into a slipper slide block guided in a channel on one side of the crosshead.

The dimensions of crosshead pins have been discussed in Chapter VII. § 141. The crosshead pin is sometimes fixed in the crosshead, and the connecting-rod end rotates on it; sometimes the pin is fixed in the connecting-rod end and rotates in a bearing in the crosshead. When the connecting-rod end is forked the latter plan is best, as it secures more uniform stress in the jaws of the connecting rod. Figs. 127, 129, show crossheads forked, with pins on

which the connecting rod works; fig. 128, a forked connecting rod with bearings for journals on the crosshead pin; fig. 132, a forked connecting rod with fixed pin and the bearing formed in the crosshead.

The sliding surfaces which receive the lateral thrust of the connecting rod and prevent bending of the piston rod are sometimes formed on the crosshead itself, fig. 130; sometimes on slide blocks attached to the crosshead by journals, fig. 127. If the wearing surfaces of the slide blocks are large there is sometimes no adjustment for wear. Sometimes a simple adjustment is obtained by putting thin washers or lining pieces in the supports of the slide bars; by removing these the slide bars are made to come closer together. More generally, in modern engines the slide block has one or more adjusting wedges fixed by screws or cotters.

149. *Forces acting at the crosshead.*—Let D be the diameter of the cylinder, p the effective steam pressure,

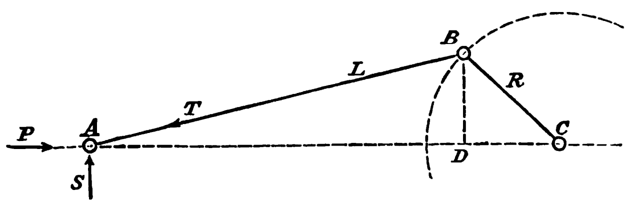


Fig. 123

then the pressure on the crosshead due to the piston load is, with accuracy enough for the present calculation,

$$P = \frac{\pi}{4} D^2 p \qquad \text{. . . . .} \qquad (1)$$

This is balanced at the crosshead by a thrust T along the connecting rod and a lateral reaction S due to the pressure on the slide blocks. Let R be the crank radius; $L = n R$

the connecting-rod length ; θ the crank angle with the line of stroke. Then

$$\text{P} : \text{T} : \text{S} :: \text{AD} : \text{AB} : \text{BD}$$
$$:: \sqrt{n^2 - \sin^2\theta} : n : \sin\theta$$

If we treat P as constant through the stroke

$$\text{S} = \text{P}\frac{\sin\theta}{\sqrt{(n^2 - \sin^2\theta)}} \quad . \quad . \quad . \quad (2)$$

This is a maximum when ABC is a right angle, and then the greatest pressure on the slide block is

$$\text{S}_{max} = \text{P}/n \text{ nearly} \quad . \quad . \quad . \quad . \quad (3)$$

Lengthening the connecting rod diminishes this pressure, and that is one of the reasons for adopting a long connecting rod. If values of S are set up along the stroke they form an approximate ellipse. Hence with a constant piston load, the mean pressure on the slide bars is

$$\text{S}_{mean} = \frac{\pi \text{P}}{4n} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Usually, however, the steam pressure diminishes through the stroke and S_{max} and S_{mean} have values less than those given above. There are also the inertia forces which have been neglected. These do not much affect the greatest slide-block pressure, which occurs near mid-stroke.

150. *Cottering the piston rod into the crosshead.*—Mr. Longridge ('Proc. Inst. Mec. Eng.,' 1896, p. 548) has pointed out that many accidents occur from the bad fitting of cotters. If the taper of the cotter differs slightly from that of the cotter-hole, the pressure is concentrated on one edge of the rod, instead of being distributed over its section, as at *a*, fig. 124. If the cotter is flexible enough to bend, however slightly, the pressure is concentrated at points *b b*, *c c*, fig. 125. The cotter should therefore be deep enough to be rigid, and should be accurately fitted. The cotter-hole

should have its edges rounded as at *d*, fig. 126, and the cotter is best made of the section shown at *e*. Mr. Longridge recommends that the piston-rod end should be

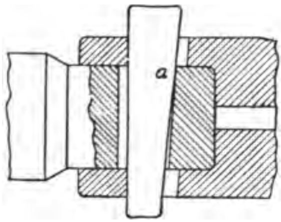


Fig. 124

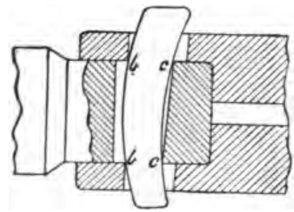


Fig. 125

cylindrical and not tapered, and butted against the bottom of the socket. If the rod is reduced in section at the

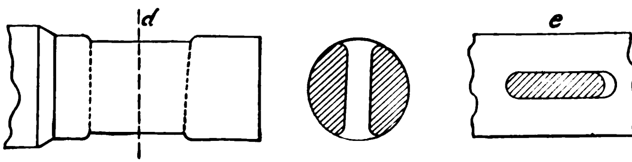


Fig. 126

crosshead, the change of section should be made by a gentle taper.

151. *Forms of crosshead.*—Fig. 127 shows a simple crosshead for an arrangement of four slide bars. The crosshead is of wrought iron, cottered to the piston rod, and having a forked end embracing the connecting rod. A pin passing through the crosshead forms a neck journal for the connecting rod, and at the same time two end journals on which the slide blocks are fixed. The slide blocks are simple cast-iron blocks. In large engines these blocks have brass faces on the rubbing surfaces. The pin must be fixed in the jaws of the crosshead by a small key, shown in the end view, which prevents the rotation of the

pin. For the connecting-rod journal $d = l$. The unit for the proportions of the other parts of the crosshead is d.

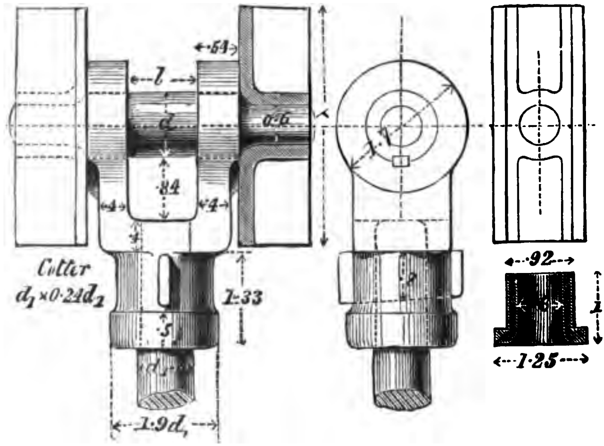


Fig. 127

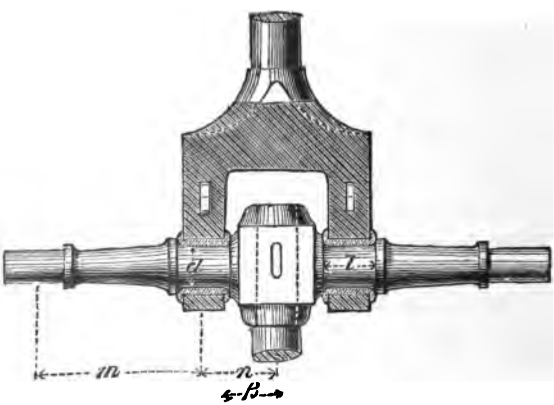


Fig. 128

Fig. 128 shows the form of the crosshead pin when the crosshead has a single end and the connecting rod is

forked. Each connecting-rod end is designed as above described, but for half the total thrust in the rod.

Fig. 129 shows a simple crosshead equivalent to that in fig. 127, but arranged with two slide bars only, above and below the crosshead. The slide blocks are of cast iron, and the slide bars of steel. The design has, however, a fault not uncommon in crossheads. In order that the

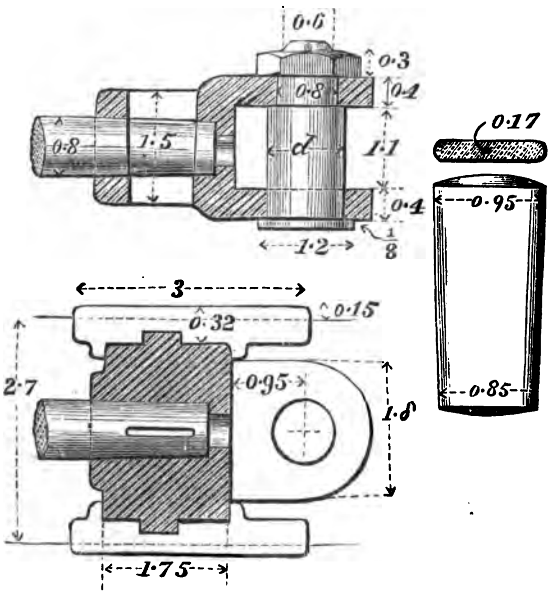
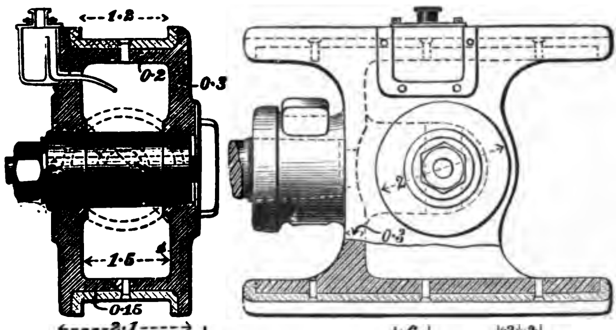


Fig. 129

vertical pressure may be uniformly distributed over the slide block, the centre of the crosshead pin should be over the centre of the slide block. If it is not so, either the pressure is very unequal, or it is only prevented from being so by the stiffness of the piston rod. Besides the bad distribution of the pressure causing increased wear, it tends to force out the lubricant.

Figs. 130, 131, show two forms of crosshead applicable when there are two slide bars in the plane of oscillation of the connecting rod. The piston-rod socket is propor-

Fig. 130



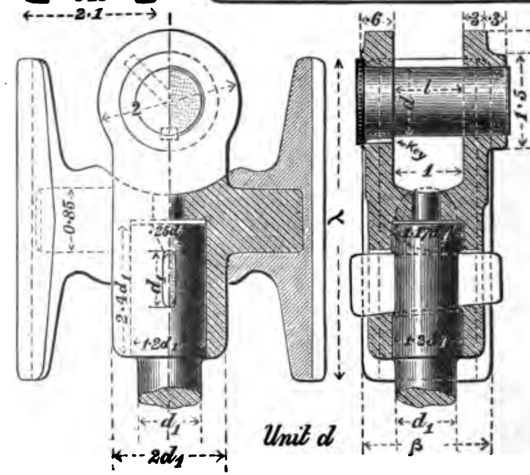


Fig. 131

tioned to the piston-rod diameter, d_1. In both these examples the piston rod is enlarged at the crosshead end. This involves a split stuffing-box. The unit for the

remaining parts is the crosshead-pin diameter, *d*. In fig. 130 the crosshead is entirely of wrought iron, except the brass faces attached by set screws to the rubbing surfaces. The crosshead pin is kept in place by a T-headed bolt, which passes completely through it. The ends of the pin are tapered, and rotation of the pin is prevented by friction of the tapered parts.

In fig. 131 the crosshead of wrought iron and the slide blocks are separate, and of cast iron. The crosshead pin is kept in place by a split pin, and rotation is prevented by a small key inserted on one side.

Fig. 132 shows a crosshead designed by Mr. Stroudley for a slipper slide. The crosshead is of wrought iron

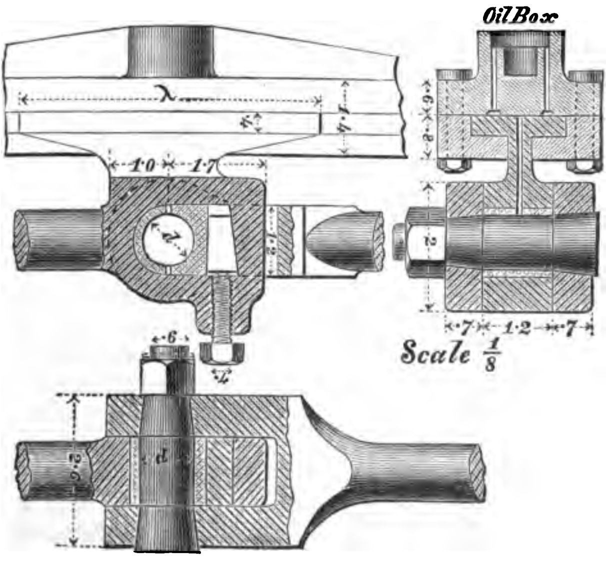


Fig. 132

forged in one piece with the piston rod and slide block, and the connecting rod is forked at the end, and embraces the

crosshead. The steps of the crosshead pin are of gun-metal, or of case-hardened wrought iron, and are tightened by a wedge and set screw. The crosshead pin, of case-hardened wrought iron, is fixed to the jaws of the connecting rod. In this case the slipper slide block is over the crosshead. The reason of this is that in locomotives the principal pressure on the slide, when the engine is running forwards, is upwards.

Fig. 133 shows a crosshead for a single slide bar, designed by Mr. Adams for the engines of the Great Eastern Railway. The slide block is in two parts, of cast iron. A projection below receives the end of the piston rod cottered in a conical hole. Forked jaws carry the crosshead pin. The connecting-rod small end is single with a solid bush. Six $\frac{7}{8}$ in. bolts connect the two parts of the slide block. The slide bar is 8 in. $\times$ 3 in. of steel, with holes through it to permit the oil to reach the under side. The pressure on the slide block is about 40 lbs. per sq. in.

Slide Bars

152. In most linkwork arrangements it is necessary to guide the ends of some of the bars, so as to constrain them to move in straight lines. This can be done by an arrangement of links forming what is termed a 'parallel motion.' Into the construction of parallel motions no elements enter which have not already been discussed. A parallel motion may be made to guide a given point with great accuracy and with very little friction. On the other hand, it is from the point of view of mechanical construction a somewhat complicated arrangement, and if the links alter in length by wear it no longer properly answers its purpose. Hence, parallel motions have been to a great extent superseded by a simpler arrangement of straight-guiding surfaces termed 'slides.' Slide bars and slide blocks waste more work in friction than parallel motions, unless, indeed, the latter are out of adjustment.

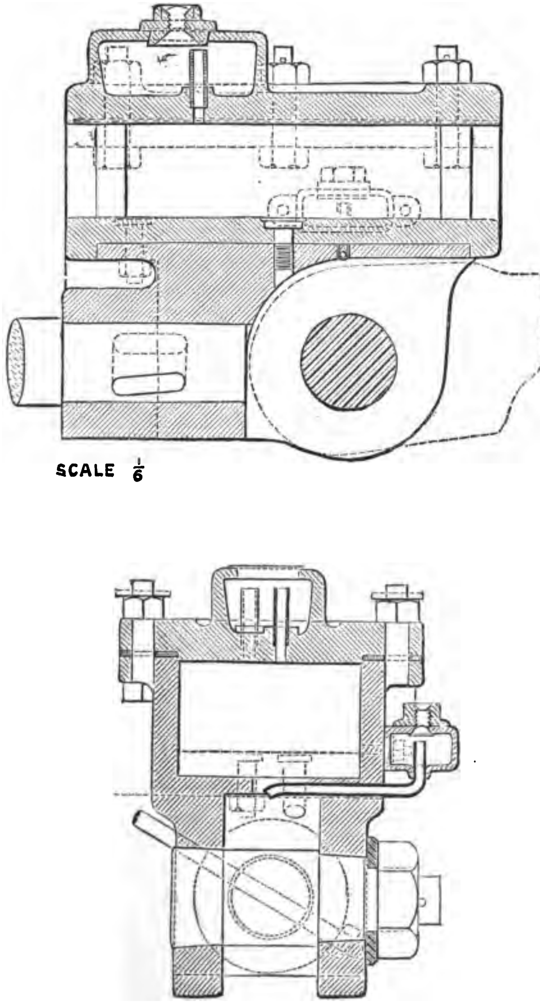


Fig. 133

But they are more cheaply and easily constructed, and the waste of work is not serious. Hence, slides are now almost always used to guide the end of piston rods.

In ordinary stationary horizontal engines the crank throws over in the forward stroke from the cylinder towards the crank shaft and passes under the crank shaft in the return stroke. Then the principal pressure on the slides is downwards, both in the forward and return stroke. There is some advantage in this, because the lower slide bar is most easily lubricated. In locomotives, since the cylinders are forward of the driving axle, the reverse condition holds. The principal pressure is upwards in forward running, and is only downwards when the engine is reversed. In marine engines the slide-bar surfaces, which take the principal pressure when going ahead, are often made larger than those which have to take the pressure when going astern.

Fig. 134 shows a pair of ordinary slide bars with the slide block between them. The bars are of T-section and are

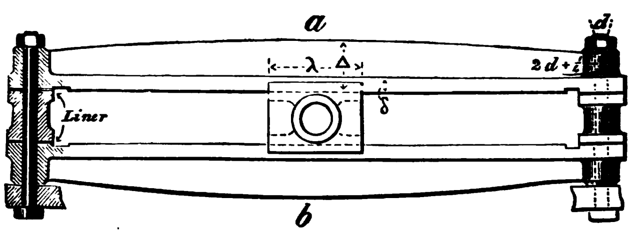


Fig. 134

spaced apart at the ends by distance pieces. Thin washers of liners are introduced between the distance pieces and the bars, so that, when the bars and slide block are worn, the bars can be brought closer together.

The bars are notched at the ends, and the slide block passes the edge of the notch at each stroke. This prevents the formation of a ridge at the end of the stroke, in consequence of the wear of the bar. Ample provision must be

made for lubricating the bars. The slide blocks may be of cast iron or of gunmetal. They fit on journals at the end of the crosshead pin. The arrangement will be understood, if the crosshead and slide blocks in fig. 127 are compared with the slide block and slide bar in fig. 134.

Fig. 135 shows slide bars of wrought iron (sometimes case-hardened) or steel slide bars. In the arrangement here shown the bars are above and below the crosshead.

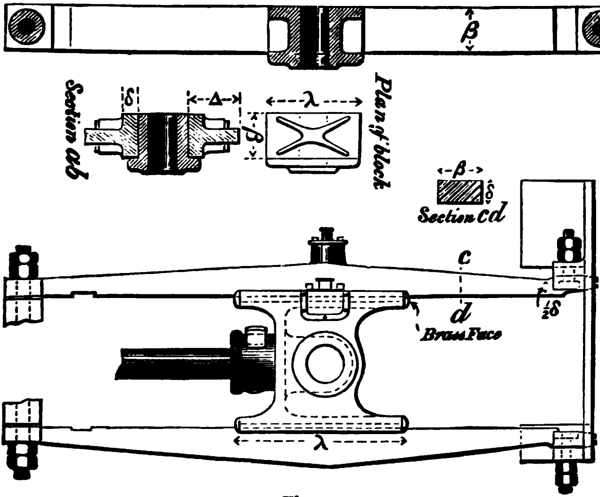


Fig. 135

The bars are rectangular in section, and thickest at the centre where the thrust is greatest. The crosshead here shown has brass faces. With this kind of arrangement, in large engines, provision is made to neutralise the wear of the bars by separating the surfaces of the slide blocks.

Fig. 136 shows an American slide bar of cast iron [1] partly chilled on the surface. The chilling has been effected in diagonal strips, leaving equally wide spaces of unchilled cast

[1] Rigg, 'Steam Engine,' p. 285.

iron between. The arrangement seems well adapted to secure good lubrication, and at the same time a sufficient area of very hard surface.

A slipper slide is sufficiently shown in the arrangement already described (fig. 132). The slide block is a T-shaped

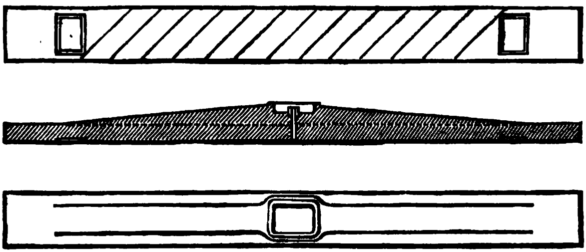


Fig. 136

piece, forged in one with the crosshead, and this is guided in a groove formed by a flat slide bar and two L shaped bars. Other forms of slide bar are used, the sliding surfaces being sometimes wedge-shaped and sometimes cylindrical.

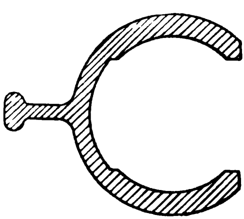


Fig. 137

In very many stationary engines the working surfaces of the slide bars are cylindrical, fig. 137. This form has great advantages. The slides can be bored out concentric with the cylinder, and re-bored with little trouble when worn. Further, a rotation about the axis of the piston rod is possible if the crank shaft gets out of level from wear or accident.

153. *Wearing surfaces of slides and slide blocks.*—Slides and slide blocks should be so designed that the wear is very small, because adjustment is always troublesome. It has been shown above (§ 149) that the mean pressure on the slide

block, normal to the sliding surface, is $\pi \text{P}/4n$, under the assumption that the variation of steam pressure and the inertia forces are disregarded. Now let μ be the coefficient of friction and v the mean velocity of the piston and crosshead. Then the work wasted in friction is $\text{U} = \mu \frac{\pi}{4} \frac{\text{P}}{n} v$ foot lbs. per second. Both the wear and the heating must be supposed proportional to U. Now let a be the area of the slide block (or two slide blocks) supporting the lateral thrust of the connecting rod, and let t be the thickness worn off in any given length of time. Then

$$a t \text{ varies as } \mu \frac{\text{P}}{n} v, \qquad . \quad . \quad (5)$$

and for any given depth of wear a should be proportional to the piston load P and piston speed v, and inversely as the ratio of connecting rod to crank. We should arrive at the same result if we consider that h thermal units were conducted or radiated from each unit of slide-block surface per second and that the rise of temperature should be limited.

It has been usual to design the slide-block surface with reference to the maximum pressure P/n calculated from the greatest piston load due to the initial steam pressure. But even so, great discrepancies occur in different cases in practice. Mr. Rigg has given a series of cases ('Treatise on the Steam Engine,' p. 124) of slide blocks of stationary, marine, and locomotive engines, in which the slide-block pressure ranges from 22 to 126 lbs. per sq. in. In some marine engines the surface of the slide for going astern, which is seldom used or only for short periods, is so small that the pressure is 400 lbs. per sq. in. Perhaps in good practice in ordinary cases, the pressure per sq. in. of slide-block surface may be taken at 30 to 60 lbs. per sq. in. in slow or moderate speed engines, and at 12 to 36 lbs. per sq. in. in quick-rotation engines.

Let β be the width and λ the length of the slide block, then the area supporting the pressure is $a = \beta\lambda$, if there is one slide block and $a = 2\beta\lambda$ if there are two slide blocks. Let y be the intended pressure per sq. in. of slide-block surface. The area a should be so arranged that

$$a = \frac{P}{n y} = \frac{\pi}{4} \frac{D^2 p}{n y}, \quad . \quad . \quad (6)$$

where in good practice y ranges from 30 to 50 in stationary engines, from 40 to 60 in good locomotives, and from 40 to 100 in large marine engines. The lower values should be chosen when the speeds are high.

The slide-bar surfaces are usually of cast iron, except in locomotives, where steel is often used. The rubbing surfaces of the slide blocks are sometimes of cast iron, sometimes of gunmetal. Oil-grooves are formed on the slide-block surface, and sometimes shallow grooves are planed across it to hold lubricant. These depressions are not reckoned as diminishing the slide-block surface.

154. *Strength of the slide bar.*—Let b and c be the distances from the centre of the connecting-rod eye to the points of support of the slide bar, when the crank and connecting rod are at right angles. Then the greatest bending moment on the slide bar, immediately under the connecting rod, is

$$M = \frac{P}{c} \cdot \frac{b c}{b + c} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Hence, if the section of the bar is rectangular of breadth β and thickness δ,

$$M = \tfrac{1}{6} \beta \delta^2 f = \frac{P}{c} \cdot \frac{b c}{b + c},$$

$$\delta = \sqrt{\left\{ \frac{6}{f} \frac{P}{n \beta} \frac{b c}{b + c} \right\}}$$

$$= k \sqrt{\left\{ \frac{P}{\beta} \frac{b c}{b + c} \right\}} \quad . \quad . \quad (8)$$

The limiting stress should be taken at 6,000 lbs. for wrought iron or steel, to allow for the straining actions due to reaction and to secure stiffness ; and at about 3,000 lbs. for cast iron. Hence,

$n =$	$3\frac{1}{2}$	4	5	6	
$k =$	·0169	·0158	·0141	·0129	for wrought iron,
$=$	·0239	·0224	·0200	·0183	for cast iron.

The T-shaped section for cast iron is more rigid, but not much stronger than if the feather were omitted.

When the slide bars are horizontal, the weight of the connecting rod, crosshead, &c., rests on the lower bar. If, then, the engine runs only in one direction, it may be arranged so that the thrust due to the pressure transmitted acts on the upper bar, provided at least that the crank is driven by the piston, and that the crank does not for part of the stroke drag the piston. Then the weight and thrust partially neutralise each other, and friction and wear is diminished. If the engine runs in both directions, but more constantly forwards than backwards, the surface of the slide block, which receives the thrust when running forwards, is often greater than that which receives the thrust when running backwards. This is the case in fig. 132. When the engine runs forward, the thrust is upward ; when running backward, the thrust is downward.

In slide bars having large surfaces and well lubricated the wear is small. Hence, sometimes adjustments to neutralise wear are omitted.

CHAPTER IX

PISTONS AND PISTON RODS

155. A piston, or plunger, is a sliding piece which is either driven by fluid pressure or acts against fluid pressure as a resistance. Pistons and plungers are commonly circular in section, and are guided by cylindrical bearing surfaces, so as to reciprocate in a straight path. But other forms of piston are occasionally used.

A plunger is a single-acting piston—that is, a piston receiving the action of the fluid on one face only—and it is guided, not by the cylinder itself, but by a stuffing-box in the cylinder cover. The bearing surface of the plunger therefore requires to be longer than the stroke. The stuffing box forms the only joint requiring attention to keep it staunch, and it is accessible without removing the plunger. A piston is equivalent to a short plunger entirely contained within the cylinder and guided by it. The force is transmitted through a piston rod of relatively small area. Hence the piston has two faces on which the fluid pressure can act, and it is usually double-acting. With a piston there are two joints requiring to be kept staunch, one within the cylinder and one where the rod passes through the cylinder cover. A large hollow piston rod is termed a 'trunk.' The pistons of pumps are often termed 'buckets.'

156. *The volume swept through* by an ordinary piston is the product of the transverse section of the piston normal to the direction of motion and the length of its path. With an incompressible fluid, such as water, the volume swept through

is the volume of water lifted, in the case of a pump; or acting on the machine, in the case of a pressure engine or ram.

Work done on a piston.—The work done on a piston by fluid pressure is the product of the volume swept through by the piston and the intensity of the fluid pressure. If the fluid pressure is variable, the mean intensity of the fluid pressure is to be taken. If the work is to be in foot lbs., the volume swept through may be in cubic feet and the pressure in lbs. per sq. foot, or the volume swept through in units of 12 cubic ins. and the pressure in lbs. per sq. in.

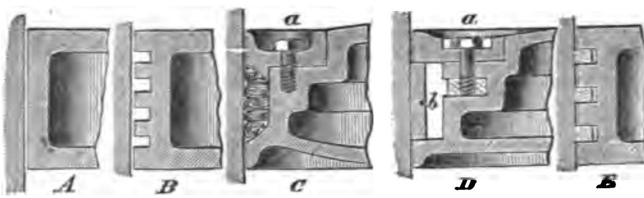
Velocity of piston.—Ordinarily a piston drives or is driven by a crank, rotating with nearly uniform velocity. Then the motion of the piston is approximate harmonic motion varying from rest at each end of the stroke to a maximum near mid-stroke. The acceleration is greatest at the beginning of the stroke, vanishes near mid-stroke, and changes sign and increases to another maximum at the end of the stroke.

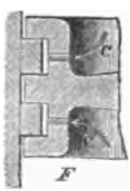
157. *Influence of the weight of the piston on the crank-pin pressure.*—When a piston is driven by a constant pressure, it is generally desirable to make the piston as light as possible, because the inertia of the piston causes the piston effort to be more irregular than it otherwise would be. When, however, the pressure on the piston varies, the inertia of the piston may be used to diminish the variation of the piston effort and to make the total pressure on the crank pin nearly uniform.

Usually, when the inertia of the reciprocating parts is intended to equalise the effort on the crank pin in expansive engines, the weight, w, of the piston, piston rod, and cross-head, reckoned per sq. in. of piston area, is so adjusted that at the intended speed of the engine $w v^2/g \text{R} =$ about $p/2$, where v is the velocity of the crank pin, R the radius of the crank, and p the initial steam pressure. (Units, feet and lbs.)

158. *Construction of piston.*—Various arrangements have been adopted to diminish the leakage between the piston and the sides of the cylinder in which it slides. The piston may be simply turned to fit the cylinder accurately (A, fig. 138); but, however good the fit at first, the wear of the cylinder and piston will gradually enlarge the clearance between them, and the leakage will steadily increase. If a series of recesses are cut round the piston circumference (B, fig. 138), the leakage for any given width of clearance

Fig. 138





space is less, because the fluid loses its energy of motion, at each sudden enlargement of the section of the annular space between the piston and cylinder, through which it is escaping. Pistons of this kind are used for quick-running pumps, where a small leakage is not very prejudicial. Leakage may be prevented by placing, in a recess in the piston, a packing of gasket or tallowed rope (C, fig. 138). This soft and elastic packing is compressed against the cylinder by a junk ring, shown at *a*, which is fixed by studs or set screws. As the gasket wears away it can be replaced, and thus the permanent staunchness of the piston is secured. Pistons of this kind are not now used for steam cylinders, though they are still employed for air pumps and cold-water pumps. The objection to them is that the repacking of the piston is troublesome, and the friction of the piston is considerable; also with high-pressure steam

hemp packing is charred. To diminish the wearing away of the gasket, a face ring or spring ring, shown at *b*, was introduced (D, fig. 138), made of cast iron and divided on one side, to allow it to expand to the cylinder diameter as it wore away. The space behind the spring ring was at first filled by gasket packing, but it was found better to substitute steel springs for gasket, which retain their elasticity much longer, and press the spring ring outwards quite as effectively. In small pistons the elasticity of the spring ring itself is sufficient to maintain contact with the cylinder. The spring ring is then free from the piston. Various arrangements of this kind have been used. Sometimes the spring ring is a cast-iron ring, of uniform or varying thickness. Ramsbottom's rings are shown at E, fig. 138. These consist of a continuous spiral-steel ring of three coils, or much more commonly of three separate steel rings, each split on one side. The rings are initially of one-tenth larger diameter than the cylinder, and, when compressed within it, press outwards with sufficient force to prevent leakage. The width of the rings (parallel to the axis of the cylinder) is about $0{\cdot}014\,D + 0{\cdot}08$, and the thickness in the plane of the piston $0{\cdot}025\,D$, where D is the diameter of the cylinder. Ramsbottom's rings are usually of steel, but sometimes of gunmetal and in large sizes of cast iron. It is stated that Mr. Ramsbottom found such rings tight against 100 lbs. steam pressure, when the radial pressure of the rings was $3\frac{1}{2}$ lbs. per sq. in. of surface. The rings were bent before being placed in the cylinder to a curve determined experimentally. A turned circular split ring of the required section was strained by weights acting on strings passing over pulleys, the total amount of the weights being the total amount of the required radial pressure of the ring. The curve was marked and the ring bent to the curve. Then, when compressed to its original circular form in the cylinder, it exerted a uniform radial pressure. Cast-iron rings answer very well for small spring rings. They retain

their elasticity till half worn through. Cast-iron rings are sometimes of uniform thickness, but very often they are one-half thicker at the middle of the ring than at the ends where the ring is split. At F, fig. 138, is shown an arrangement for admitting the steam pressure in the cylinder to the back of the rings. In principle this is a good arrangement, but in this form it does not succeed very well, and is not very often adopted. In fig. 139 is shown a modern form of piston, in which two spring rings with inclined inner surfaces are pressed radially and axially by a coiled spring of circular section. In Stroudley's piston the steam is admitted to the back of the spring ring on the opposite side to that on which the steam acts. This ring, passing a little beyond the bored part of the cylinder, prevents the formation of a shoulder at the end of the cylinder. Bramah's cup leather is a perfectly successful application of the same principle. For pumps and blowing cylinders, wood blocks have been used to replace the spring ring. The packing of a piston may or may not share with the piston rim the pressure of the piston against the cylinder sides, whether due to the weight of the piston, as in horizontal engines, or to other causes.

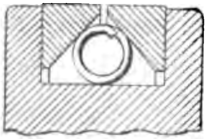


Fig. 139

The spring rings or metallic packing of pistons may be of cast iron, of wrought iron, of steel, or of gunmetal. For steam cylinders cast iron wears better than wrought iron, and about as well as gunmetal. Gunmetal is chiefly employed in pumps and in pistons of complicated types, the action of which would be impaired by corrosion. Steel is a good material for packing, especially where considerable elasticity is necessary. Cast-iron and wrought-iron rings may be made more elastic by hammering.

159. *Strength of pistons.*—Pistons are of a complicated form, and it is not easy to determine their strength theo-

retically. If the piston were a simple metal disc, supported at the centre and uniformly loaded, the greatest stress would be

$$f = k \frac{D^2}{t^2} p \quad . \quad . \quad . \quad (1)$$

where D is the diameter of the piston, t its thickness, p the greatest difference of the pressures on the two sides,[1] estimated per unit of area, and k is a constant. Putting $f =$ 8,000 for wrought iron and 3,000 for cast iron, we get

$$\left.\begin{aligned} t &= \cdot 0051 \text{ D} \sqrt{p} \text{ for wrought iron} \\ &= \cdot 0083 \text{ D} \sqrt{p} \text{ for cast iron} \end{aligned}\right\} . \quad (2)$$

These values of t will be taken as empirical units for the proportions of pistons. Since, however, the form of pistons varies greatly, and also the conditions under which they work, the draughtsman should not depend solely on the following proportional figures, but should deduce the proportional figures for himself, from good examples of pistons of a similar kind to the one he is designing.

160. *Locomotive pistons.*—Fig. 140 shows two forms of piston used in locomotives. One is constructed chiefly of cast, the other chiefly of wrought iron. Wrought iron is preferred by some engineers on account of its toughness and strength. But cast iron is much cheaper and answers well. Steel castings are now often used and are much lighter than cast iron. The spring rings in both cases are of cast iron and require no springs or packing. These rings are of uniform section, about 1½ in. wide by ½ in. thick, in pistons of average size. The split is made with a half lap, to prevent leakage at that point. The rings are sprung into the recesses in the piston, and should be so placed that the splits in the two rings are on opposite sides of the piston.

[1] That is, p is very nearly the initial absolute steam pressure in the case of a condensing engine, and the initial steam-gauge pressure in a non-condensing engine.

This equalises the wear of the cylinder. A small screw is sometimes used to prevent the rings turning round in the grooves. The piston rod is screwed into the wrought-iron piston and fixed by a split pin. In the case of the cast-iron piston the rod is slightly coned at the end, and, when in

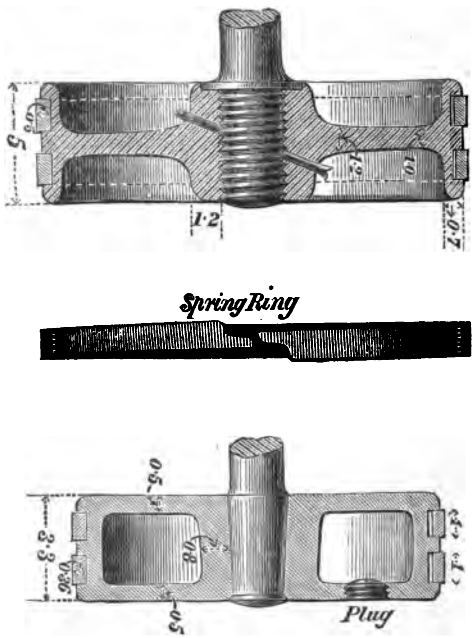


Fig. 140

place, is riveted over. The holes filled by screw plugs are intended for the removal of the sand core after casting.

Fig. 141 shows another locomotive piston. In this three spiral springs are placed behind the spring ring, and assist the elasticity of the latter in keeping the piston tight. A brass tongue-piece prevents leakage at the joint in the spring ring. The piston rod has a strong taper to enable it

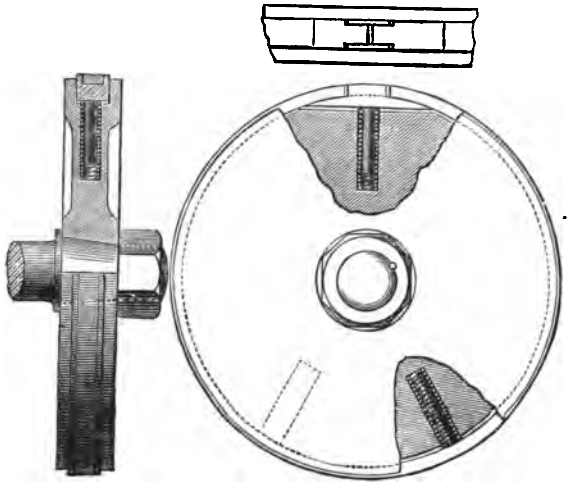


Fig. 141

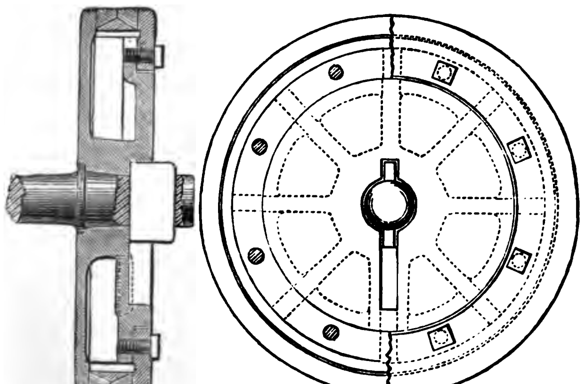


Fig. 142

to be easily removed, and it is secured by a screwed end and nut. The spiral springs are so placed as to prevent the body of the piston bearing on the bottom side of a horizontal cylinder.

161. *Stationary engine pistons.*—Fig. 142 shows one form of stationary engine piston. It is made of cast iron, with a junk ring to confine the metallic packing. The packing consists of three cast-iron rings of the sectional form shown. The outer rings are turned $\frac{1}{16}$ in. larger than the cylinder diameter, and are split. The inner ring may or may not be split. By screwing down the junk ring the two outer rings are forced outwards, as they slide down the conical surfaces of the inner ring, and thus any desired amount of pressure can be obtained between the piston and cylinder. The inner ring has sometimes been made in the form of a spiral spring. It then presses the outer rings both apart and outwards. In this piston the rod is tapered at the end and fixed by a cotter.

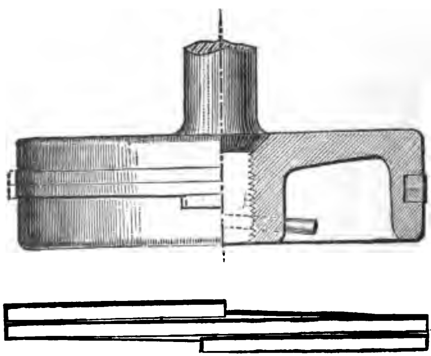


Fig. 143

162. *Joy's piston* (fig. 143).—The piston is a simple block, into which the piston rod is screwed and pinned. The diameter of this block is $\frac{1}{16}$ in. less than the bore of the

cylinder. A recess is cut with a tool set to $\frac{1}{2}$-in. pitch, and making 3 ins. more than two revolutions. The cast-iron ring from which the packing is made is turned and bored, $\frac{5}{8}$ in. thick, and $\frac{3}{4}$ in. larger in diameter than the cylinder. It is then placed on a mandrel, and a spiral groove cut with a $\frac{1}{4}$-in. tool set at $\frac{5}{8}$-in. pitch, so as to form a spiral spring of $\frac{5}{8}$ in. by $\frac{1}{2}$ in. in section. The spiral spring gives probably a more uniform pressure on the cylinder than a simple ring, and its axial elasticity prevents its knocking in the groove.

163. *Marine-engine pistons.*—Marine-engine pistons are often of very large size, and are usually of cast iron, of a box shape and stiffened by numerous ribs. Fig. 144 shows a piston of this kind. The spring ring of cast iron is of uniform thickness. Leakage at the split is prevented by a brass tongue-piece, fixed to one end of the spring ring by screws. The split in the spring ring is shown square across the spring ring. But it is better to cut it obliquely so that the edges may not score the cylinder. The spring ring is pressed outwards by numerous plate-springs, placed in recesses cast in the rim of the piston. The strength of these springs is such that they exert a radial pressure of about 2 lbs. per sq. in. of the bearing surface of the spring ring. The spring ring and springs are kept in place by a junk ring. This last is attached to the piston by bolts, which have brass nuts placed in recesses behind the plate springs. To prevent these bolts slacking back, in consequence of the vibration of the piston, various locking arrangements are used. In the piston shown (a type used by Messrs. Humphreys and Tennant), a securing ring bears against the heads of all the junk-ring bolts. This ring is attached to the piston by studs, the nuts being fixed by split pins. Holes are cast in the piston for clearing out the loam core, and these are afterwards fitted with screw plugs or plugs secured by screwed pins.

In this piston the rod is tapered and the piston is secured to it by a large nut on the upper side of the piston. When

such a piston works in a horizontal cylinder, a block placed between the spring ring and piston body on the bottom side of the piston keeps the latter from bearing on the cylinder.

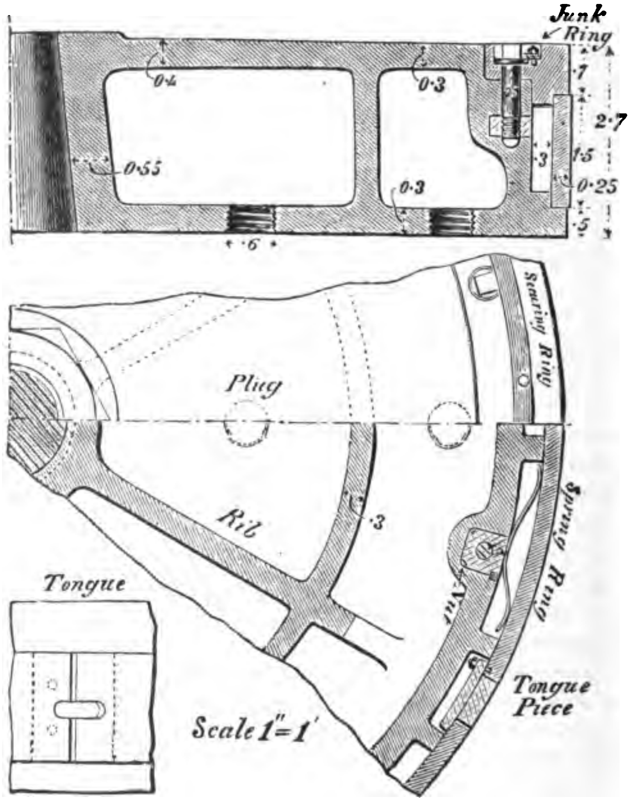


Fig. 144

This block may extend round about one-fourth of the circumference.

164. *Cast-steel pistons.*—Large pistons are now often made of steel castings, which are stronger and one-third

lighter than cast-iron pistons. Fig. 145 shows a cast-steel piston such as is now adopted in marine engines. The unit for the proportional figures is $t = 0{\cdot}0015\, d \sqrt{p}$ in large to $0{\cdot}003\, d \sqrt{p}$ in small engines, but the thickness is in no

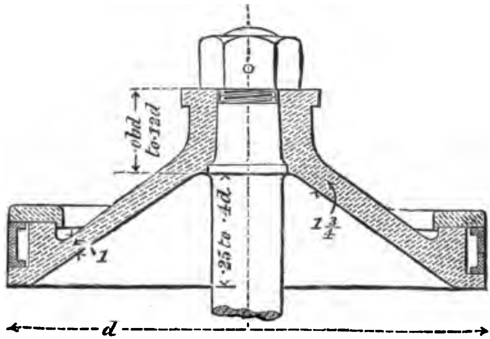


Fig. 145

case less than $\frac{5}{8}$ in. The conical form gives strength and rigidity, and is at the same time easy to cast. The projection on the boss facilitates lifting the piston. The angle of cone is sometimes altered so that all the piston rods may be of the same size and length. The taper of the piston rod in the boss is 1 in 3 to 1 in 4, and the packing rings are pressed out by springs so that the pressure is about 3 to 5 lbs. per sq. in. of packing-ring surface.[1]

165. *Hydraulic pistons.*—Fig. 146 shows a combined piston and plunger with a hat-leather arrangement for preventing leakage. The fluid pressure acting on the inside of the flexible leather cups, aided by their own elasticity, makes

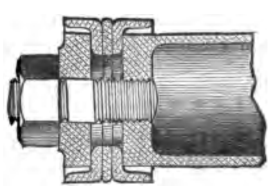


Fig. 146

[1] There is an important paper on conical pistons, with an investigation of their strength, by M. Kraft, 'Proc. Inst. Civil Eng.,' cxxvii.

an exceedingly staunch joint, whatever the pressure may be. The hat-leathers are so arranged that one acts when the piston moves in one direction, the other when the piston moves in the reverse direction.

The forms of leather packing for hydraulic cylinders will be described more fully in the next chapter.

THEORY OF SPRING RINGS FOR PISTONS

The object of the spring ring is to produce a uniform radial pressure sufficient to prevent leakage. It is known that under certain conditions the spring ring collapses, and then steam passes from the front to the back of the cylinder. The amount of radial pressure necessary to prevent this collapse must be determined experimentally. It is not at present very accurately known. It appears, however, that a radial pressure reckoned per sq. in. of surface of piston ring which is very considerably less than the steam pressure is sufficient to prevent this collapse of the piston ring. In any case it is desirable to determine what forms of spring ring secure a uniform radial pressure and what amount of radial pressure is to be expected from any given piston spring ring.

166. *Flexure by bending of a bar initially curved.*—In the case of a straight bar subjected to a bending moment, it is known (see I. § 27) that the bar takes a curvature given by the equation

$$M = \frac{E I}{\rho},$$

where M is the moment of the bending forces on one side of a section taken normal to the axis of the bar, I is the moment of inertia of the section, and ρ is the radius of curvature of the bent bar at the section.

Let fig. 147 represent a bar initially curved, and let G G′ be a line passing through the centres of gravity of sections normal to G G′. Then if P Q, P′ Q′ are two normal sections

at a small distance apart, and if P Q, P′ Q′ meet at C, C G $= \rho$ is the radius of curvature at P Q of the unstrained bar. Now let a bending moment M be applied to the right of P Q. The particles initially in P′ Q′ will be found after bending in a new plane P″ Q″ still normal to the axis of the bar. The radius of curvature after bending will be C′ G $= \rho'$. Then on the same assumptions made in treating straight bars, it will be found that

$$M = E\,I\left(\frac{1}{\rho'} - \frac{1}{\rho}\right) \quad . \qquad (1)$$

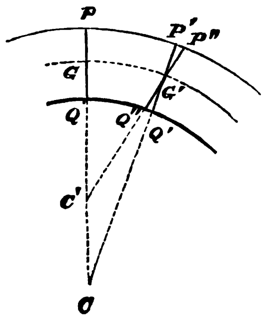


Fig. 147

that is, to pass from straight bars to curved bars, we have only to substitute in the equation for the curvature due to bending $\frac{1}{\rho}$, the augmentation of curvature $\frac{1}{\rho'} - \frac{1}{\rho}$.

167. *Theory of a cast-iron spring ring of unequal thickness.*—Let A B be a portion of a spring ring, which, initially circular on the outside, has been sprung into a cylinder of smaller diameter than itself. Let I be the moment of inertia of a cross-section at B, which without sensible error may be taken normal to the outside curve of the ring. Let ρ be the radius of the outside of the ring initially, and r its radius when sprung into the cylinder. Let b be the breadth, t the thickness of the ring at B. From the conditions in which the ring is placed, it is necessary to make the breadth b of the ring constant, and it is required to determine the variable thickness t of a ring which will produce a uniform pressure

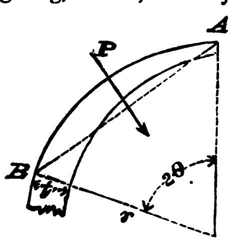


Fig. 148

of p lbs. per sq. in. on the surface of the cylinder into which it is sprung.

The pressure of the ring on the cylinder per unit of length is $p\,b$, and the resultant of the uniform pressure from A to B is $p\,b \times$ chord A B, or

$$P = 2\,p\,b\,r \sin\theta.$$

The moment of this about B is

$$2\,p\,b\,r^2 \sin^2\theta.$$

Putting this value of the bending moment in the equation above,

$$2\,p\,b\,r^2 \sin^2\theta = E\,I\left(\frac{1}{r} - \frac{1}{\rho}\right) \quad . \quad . \quad (2)$$

But for a rectangular section (I. Table V., p. 66),

$$I = \frac{b\,t^3}{12}.$$

$$\frac{1}{r} - \frac{1}{\rho} = \frac{24\,p\,r^2 \sin^2\theta}{E\,t^3} \quad . \quad . \quad (3)$$

For $2\,\theta = 180$, let t_1 be the thickness of the ring,

$$\frac{1}{r} - \frac{1}{\rho} = \frac{24\,p\,r^2}{E\,t_1^{\,3}} \quad . \quad . \quad . \quad (4)$$

Hence, for uniform pressure of ring on cylinder,

$$\frac{t}{t_1} = \sqrt[3]{(\sin^2\theta)} \quad . \quad . \quad . \quad (5)$$

$2\,\theta =$	$t =$
10°	0·197 t_1
20°	·311
40°	·489
60°	·630
80°	·745
100°	·837
120°	·908
140°	·960
160°	·990
180°	1·000

From which table the thickness at different points in the ring can be calculated when t_1 is known.

Fig. 149 shows a ring drawn from these values. It is found that about $\frac{2}{3}$ of the inside curve of the ring agrees very closely with a circle struck from a point C at a distance O C $= a$ from the centre of the outside of the ring. It is easy to show that $a = 0{\cdot}206\ t_1$ nearly. The ring can there-

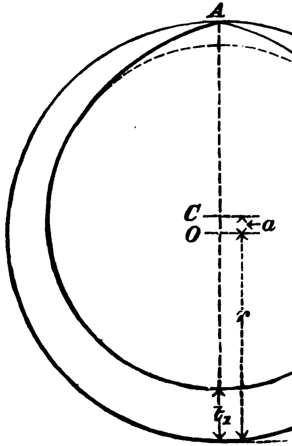


Fig. 149

fore be bored out to a radius $r - 0{\cdot}794\ t_1$, and the points near A then thinned to agree with the proportions above.

From Eq. 3,

$$p = \frac{\mathrm{E}\, t_1^{\,3}}{24\, r^2}\left(\frac{1}{r} - \frac{1}{\rho}\right) \quad . \quad . \quad . \qquad (6)$$

The stress in the spring ring at its thickest part is (I. § 28),

$$f = \frac{\mathrm{M}}{\mathrm{Z}} = \frac{6\,\mathrm{M}}{b\, t_1^{\,2}}.$$

But the bending moment is

$$\mathrm{M} = \mathrm{E\,I}\left(\frac{1}{r} - \frac{1}{\rho}\right) = \frac{\mathrm{E}\, b\, t_1^{\,3}}{12}\left(\frac{1}{r} - \frac{1}{\rho}\right).$$

Inserting this value

$$f = \frac{E\,t_1}{2}\left(\frac{1}{r} - \frac{1}{\rho}\right) \quad . \quad . \quad . \quad . \quad . \qquad (7)$$

or, using the value in (6),

$$f = 12\,p\,\frac{r^2}{t_1^{\,2}} \quad . \quad . \quad . \quad . \quad . \qquad (7a)$$

$$t_1 = r\sqrt{\frac{12\,p}{f}} \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

$$\frac{\rho}{r} = \frac{E\,t_1}{E\,t_1 - 2\,f\,r} \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

For instance, for cast iron let $E = 17{,}000{,}000$; $f = 5{,}000$ lbs. per sq. in., as the stress is constant and of one kind; $p = 3\frac{1}{2}$ lbs. per sq. in. Then $t_1 = 0{\cdot}092\,r$ and $\rho/r = 1{\cdot}007$.

With the same data, except that $p = 2$ lbs. per sq. in., $t_1 = 0{\cdot}069\,r$, and $\rho/r = 1{\cdot}009$.

Assuming $\rho/r = 1{\cdot}05$, $E = 17{,}000{,}000$ and $f = 5{,}000$ lbs. per sq. in., then t_1 cannot be greater than $0{\cdot}0123\,r$ and the pressure per sq. in. of ring on cylinder would be only $0{\cdot}064$ lbs. per sq. in.

Lastly, if t_1 is assumed equal to $0{\cdot}05\,r$, $f = 5{,}000$ and $E = 17{,}000{,}000$, then $\rho/r = 1{\cdot}012$ and $p = 1{\cdot}04$ lbs. per sq. in.

Sometimes two eccentric rings are used placed one inside the other, as shown in fig. 150. The proportions of such rings given by Von Reiche are:

Total thickness of two rings	$\delta = 0{\cdot}8 + 0{\cdot}12\,r$
Greatest thickness of outside ring	$\frac{2}{3}\,\delta$
Least " "	$\frac{1}{3}\,\delta + 1{\cdot}2$
Greatest thickness of inside ring	$\frac{2}{3}\,\delta - 1{\cdot}2$
Least " "	$\frac{1}{3}\,\delta$
Breadth of rings . . .	$0{\cdot}12\,r + 0{\cdot}9$

It is easy to see that such rings will produce approxi-

mately the same pressure on the cylinder as a single ring designed as described above, and of a maximum thickness equal to the total thickness of the two rings.

The breadth of the ring does not affect the intensity of pressure against the cylinder, nor for a given pressure per unit of area will a narrow ring wear longer than a wide one. On the other hand, a wide ring will wear the cylinder more than a narrow one.

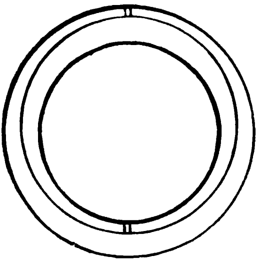


Fig. 150

The theory above is simplified from an investigation by Prof. Robinson,[1] which contains also an attempt to determine theoretically the proper pressure of the ring on the cylinder to prevent leakage. This latter part of the investigation does not appear to the author to be well based.

168. *Theory of a spring ring of uniform thickness.*—A spring ring of uniform thickness may be made to give a uniform pressure on the cylinder, if it is initially bent to a form of varying curvature. Let A B C, fig. 151, be the spring ring when sprung into the cylinder, A′ B C the ring when unstrained. Let b be the breadth and t the thickness of the ring; I the moment of inertia of a cross-section; ρ the radius of curvature of a point B when the ring is unstrained; r the radius of the cylinder into which the ring is sprung. The resultant pressure between A and B is as before,

$$2\, p\, b\, r \sin\theta.$$

Its moment about B is

$$M = 2\, p\, b\, r^2 \sin^2\theta.$$

Inserting this in the equation above,

$$2\, p\, b\, r^2 \sin^2\theta = E\, I \left(\frac{1}{r} - \frac{1}{\rho}\right)$$

$$\frac{1}{r} - \frac{1}{\rho} = \frac{24\, p\, r^2 \sin^2\theta}{E\, t^3}.$$

[1] Van Nostrand's Magazine, June 1881.

Hence

$$\rho = \frac{E\,t^3\,r}{E \qquad\qquad r^3 \sin^2\theta} \quad . \quad . \quad (10)$$

Let E = 30,000,000 for steel, and let $t = 0{\cdot}04\,r$, which is about the usual proportion in practice. Then

$$\rho = \frac{80\,r}{80 - p \sin^2\theta} \quad . \quad . \quad . \quad (11)$$

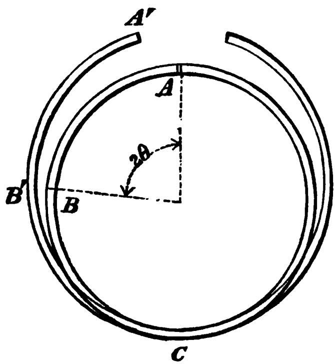


Fig. 151

The following table gives values of ρ for $\theta = 0°$ to $90°$, or $2\,\theta = 0°$ to $180°$, and for $p = 3\frac{1}{2}$ and $p = 14$,

$2\,\theta =$	$\rho =$ for $p = 3\frac{1}{2}$	$= \rho$ for $p = 14$
0°	1·000 r	1·000 r
30	1·003	1·011
60	1·012	1·047
90	1·021	1·096
120	1·033	1·152
150	1·042	1·195
180	1·045	1·214

The form to which the spring ring ought to be bent in

order that when sprung into the cylinder the pressure may be uniform, may be obtained thus :

Let o be the centre of the cylinder (fig. 150). Divide its circumference at $a\, a'\, a''$ . . . . into 12 equal parts. Take $a\,c =$ the value of ρ for $2\,\theta = 180$, and with centre c draw the arc $a\,b'$. Make $a\,b' = a\,a'$ and join $b'\,c$. Take $b'\,c' = \rho$

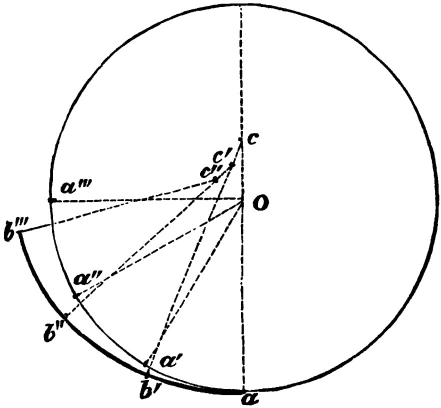


Fig. 152

for 150°, and with centre c' draw the arc $b'\,b''$. Proceeding thus, the whole curve can be drawn.

Fig. 153 shows the curves drawn for the values of ρ for $p = 3\frac{1}{2}$ and $p = 14$ given above. The former curve agrees roughly with a circle of radius $\frac{1}{26}$th greater than the cylinder radius; the latter curve with a circle of radius $\frac{1}{7}$th greater than the cylinder radius. But it will be seen that both curves deviate considerably from circular curves, and hence rings made by turning them to a circular form and cutting out a portion cannot give a nearly uniform distribution of pressure.

In the Willan's engine piston there is an internal spring ring of varying thickness and two thin uniform outer rings, each of half the axial width of the inner ring. The rings

are so placed and pinned together that the three splits in the rings break joint. The junk ring is of sheet metal and

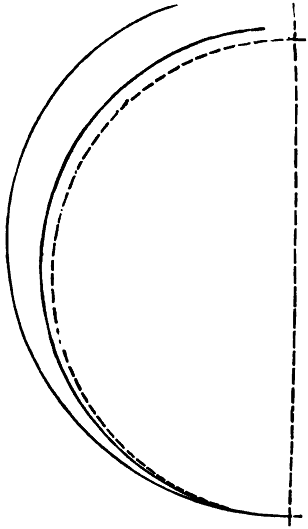


Fig. 153

thin, so that under the steam pressure it springs and grips the spring rings.

Piston Rods

169. Piston rods are subjected alternately to compression and tension. In horizontal engines of large size, the weight of the piston and rod produces bending, and extra strength is then required. In oscillating engines, the resistance of the cylinder to angular acceleration also causes bending of the piston rod. In most engines the piston has a single piston rod, but there are some types of engine (return connecting-rod engines and some forms of tandem compound engine, for instance) in which there are two piston rods. It is more difficult to fit two rods accurately, so that they shall be

equally strained, and hence double-piston rods should have some extra margin of strength.

For the minimum section of the piston rod through the crosshead cotter-hole, or at the bottom of the screw thread at the piston nut, seeing that the stress acts only in one direction, a stress might be allowed of 10,000 lbs. per sq. in. for wrought iron, or 11,400 for soft steel (Table II. p. 4). But it is probably impossible to adjust a cotter so that the stress is uniformly distributed, and hence a lower value of the mean working stress must be taken. Let D be the piston diameter in ins., p the greatest difference of pressure on the two sides in lbs. per sq. in., a the section of the rod after deducting cotter-hole, m a factor of safety to allow for straining actions due to inertia, water in the cylinder, and unequal distribution of stress, f the safe working stress on the material given above. Then the piston load due to steam pressure is $(\pi/4)\,\mathrm{D}^2 p$, and the section of the rod is determined by the relation

$$f a = m \frac{\pi}{4} \mathrm{D}^2 p.$$

It appears that m should not be taken less than 2, although cases will be found of rods working with a smaller margin of security. However, taking $m = 2$ and $f = 10{,}000$ for wrought iron, and 11,400 for steel—

$$\begin{aligned} a &= 0{\cdot}000157\ \mathrm{D}^2 p \text{ for wrought iron,} \\ &= 0{\cdot}000138\ \mathrm{D}^2 p \text{ for steel.} \end{aligned}$$

If $d_{\text{min.}}$ is the minimum diameter of the piston rod, and the net section after deducting the cotter-hole is taken to be $0{\cdot}535\, d^2_{\text{min.}}$, then

$$\begin{aligned} d_{\text{min.}} &= 0{\cdot}0171\ \mathrm{D}\sqrt{p} \text{ for wrought iron,} \\ &= 0{\cdot}0160\ \mathrm{D}\sqrt{p} \text{ for steel.} \end{aligned}$$

170. *Size of cotter for piston rod.*—For the piston load given above and using the rules in Part I. p. 217, for

cotters strained alternately in opposite directions, we get

$$m \frac{\pi}{4} D^2 p = 2\, b\, t f_s$$

where b is the width, t the thickness of the cotter, and $f_s = 4{,}000$ for wrought iron and 5,300 for steel. It appears that for the cotter the stress is pretty uniformly distributed, so that $m = 1\frac{1}{4}$. Hence

$$b\,t = 0{\cdot}000122\ D^2 p \text{ for wrought iron,}$$
$$= 0{\cdot}000092\ D^2 p \text{ for steel.}$$

If $b = 4\,t$, a common proportion,

$$t = 0{\cdot}00552\ D\sqrt{p} \text{ for wrought iron,}$$
$$= 0{\cdot}00479\ D\sqrt{p} \text{ for steel.}$$

171. *Strength of piston rod considered as a strut.*—The rules above fix the minimum section of the piston rod, but the body of the rod has to be of greater section to secure stiffness and resistance to bending. The rod is sometimes treated as a long column, but this is unsatisfactory, partly because the precise condition of loading at the ends is unknown, and partly because there are transverse bending actions, due for instance to the slide bars being slightly out of line with the cylinder, which it is impossible to calculate. It is enough therefore to design the rod for a moderate value of working stress, shown by experience to be safe. Table II. p. 5, shows that for stress alternating from tension to thrust 4,300 to 5,700 lbs. per sq.. in. should be safe for wrought iron or steel. But if the load is taken to be $\frac{1}{4}\,\pi\, D^2 p$, where p is the greatest excess of forward pressure over back pressure, all the straining action is not included and a lower working stress is desirable. In Mr. Longridge's table of piston-rod breakdowns,[1] there are rods on which the calculated stress ranged from 3,000 to 8,000 lbs. per sq. in., and in one case to 11,000 lbs. per sq. in. Taking the

[1] 'Proc. Inst. Mech Eng.,' 1896, p. 564.

load on the rod to be $\frac{1}{4}\pi D^2 p$, the following values were found for the working stress in some actual engines. In short-stroke direct-acting engines, $f = 3{,}000$ to $3{,}600$; in return connecting-rod engines, $f = 2{,}400$ to $3{,}000$; in moderately long-stroke horizontal engines, $f = 2{,}000$ to $2{,}500$; in oscillating engines, $f = 2{,}000$. Calculations based on treating the rod by the formula for columns, appear to the author to be misleading.

If d is the diameter of the rod, either of iron or steel, D the piston diameter, p the greatest excess of forward over back pressure, then

$$d = k\,D\sqrt{p}$$

$f =$	$k =$
2,000	0·0224
2,500	0·0200
3,000	0·0182
3,500	0·0169
4,000	0·0158

Very often the minimum diameter of the rod, allowing for the cotter-hole, is also sufficient for the diameter of the body of the rod. In compound engines for reasons of convenience the piston rods are often all of the same diameter. Then the rod must be determined for the piston which has the greatest load.

172. *Modes of fixing piston to piston rod.*—For pump buckets and other cases where a very accurate fit is not required the rod may have a shoulder, a parallel part in the piston and a nut at the back of the piston (fig. 154, A). With steam pistons, which must be steam-tight, in some cases the rod is turned slightly larger than the hole in the piston. Then the piston is heated and shrunk on, and the rod riveted over (fig. 154, B). The diameter of rod may be made 1·0025 times the diameter of the hole bored in the piston. To facilitate the removal of the rod from the piston, the part of the rod in the piston is in part or wholly tapered.

If the taper extends through the piston the taper is 1 in 32 to 1 in 16 (that is, the total taper for the two sides). Sometimes the rod has a short tapered part with a taper of 1 in 4, till the section of the rod is reduced to three-fourths its full section and the remainder is parallel and not tightly

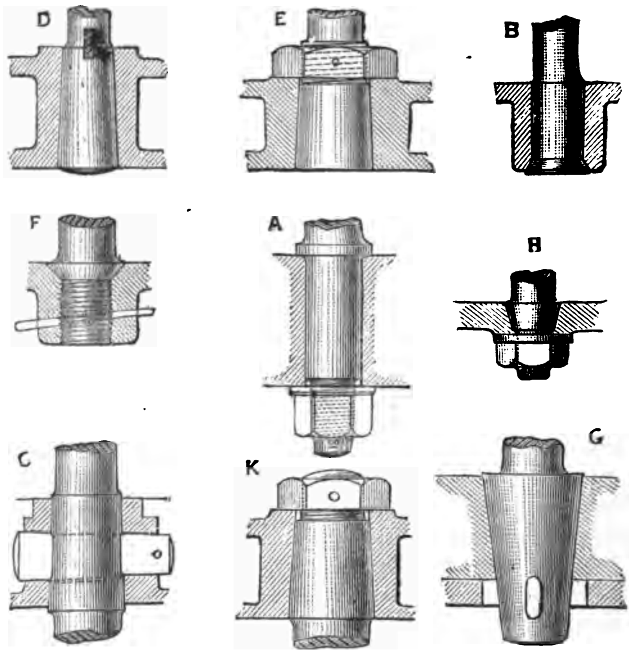


Fig. 154

fitted. In Mr. Stroudley's piston (fig. 154, F) there is a still sharper taper, the rod being enlarged to permit it. In all these cases a cotter or nut is required to fix the rod in the piston, and in either case stresses are induced of unknown amount in driving the cotter or screwing up the nut. M. Bollinckx prefers to make the piston rod cylindrical and

to force the piston by hydraulic pressure, noting by a pressure gauge exactly the force applied. It may be assumed that the piston cannot be forced off by a less effort than that applied in driving it on. No cotter or nut is used.

CHAPTER X

STUFFING-BOXES

173. Stuffing-boxes are used to prevent leakage of steam or water at the points where moving parts pass through the sides of vessels containing fluids. Thus a stuffing-box is used where the piston rod of an engine passes through the cylinder cover, or where a rotating shaft passes through a centrifugal pump case. In ordinary stuffing-boxes, soft packing is used to prevent leakage, but various forms of metallic packing have also been employed.

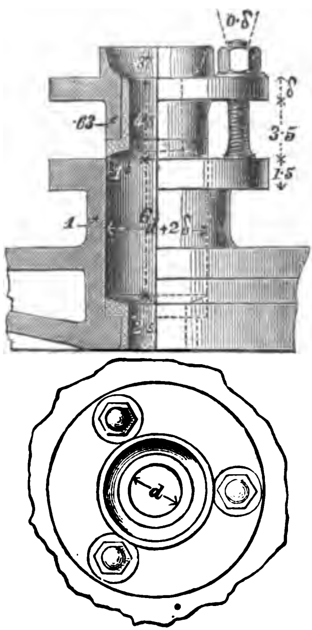


Fig. 155

In fig. 155 is shown a stuffing-box for a vertical rod, and in fig. 156 a stuffing-box for a horizontal rod. In both cases the stuffing-box is cast on the cylinder cover. The stuffing-box is larger than the rod which traverses it, by the space necessary for the soft packing. At the bottom of the box there is a brass bush, which, being softer than the rod, preserves the

latter from injury. When the bush has worn oval, it is easily replaced by a new one. To keep the packing in place and to compress it sufficiently to prevent leakage, a loose piece termed a 'gland' is used. This is entirely of brass (fig. 156) or bushed with brass (fig. 155), and often has an oil-box formed in it (fig. 156). The gland is forced down on the soft packing by two or more bolts or studs. The larger the amount of packing, the longer it will work before requiring renewal. Hence for quick-running rods more packing space should be allowed than for slower rods.

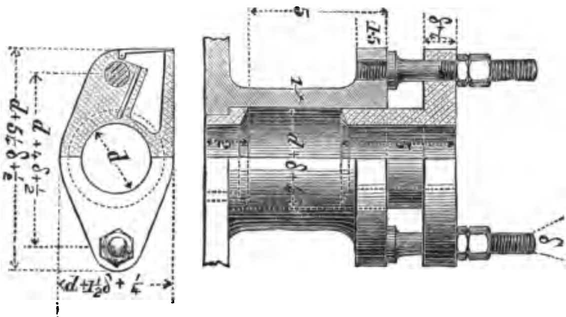
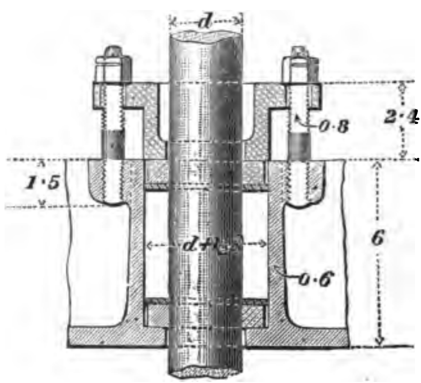


Fig. 156

174. *Proportions of stuffing-box and gland.*—Let d be the diameter of the rod or shaft traversing the stuffing-box. The diameter δ of the gland bolts may be $= \frac{1}{4} d + \frac{1}{4}$, if there are two; and $= \frac{1}{8} d + \frac{1}{4}$, if there are three. Then δ will be taken as the unit for the proportions of the box. The thickness of packing in the box is very variable. In ordinary stuffing-boxes, not of very great diameter, the packing thickness varies from $\frac{1}{2}\delta$ to δ; for larger rods the thickness of packing may be 0·4 to 0·5 $\sqrt{d}$. But the width of packing space should be such that commercially obtainable packings fit it. In large stuffing-boxes for trunks or hollow piston rods a less thickness is employed. The length of the box is also variable. The greater the

length of box, the less frequently will it be necessary to renew the packing. On the other hand, the space available for the stuffing-box is sometimes restricted. From 5 δ to 8 δ is an average length. The thickness of the stuffing-box flange may be $1\frac{1}{4}\,\delta$ to $1\frac{1}{2}\,\delta$, and the thickness of the gland flange may be δ for cast iron, or $1\frac{1}{4}\,\delta$ for brass. If an oil-box is cast in the flange, the thickness is somewhat greater. The length of gland may be $\frac{3}{4}$ to $\frac{4}{5}$ the stuffing-box length. The thickness of the stuffing-box should not be less than $\frac{1}{5}\sqrt{d}$ or less than $\frac{3}{8}\,\delta$. The length of the



Unit $\frac{1}{4}d + \frac{1}{4}$

Fig. 157

brass bush may be about $2\frac{1}{2}\,\delta$, but when the stuffing-box serves to guide the rod, as in oscillating engines, a much greater length of bush is used.

175. *Yarrow's stuffing-box.*—As ordinarily constructed the gland of a stuffing-box fits the piston rod, and at the same time it is fixed in position by the stuffing-box. Hence it must act as a guide to the piston rod. The gland ought, therefore, to be exactly co-axial with the cylinder and parallel to the line of stroke of the slide block. If initially or in

consequence of wear these conditions are not satisfied, the piston rod must bear heavily on one side of the gland, and will wear it oval.[1] In quick-running engines the piston rod will be heated on the side bearing on the gland, and this will cause it to bend in a direction which increases the injurious pressure. Messrs. Yarrow have found it advantageous to guide the parts connected by the piston rod by the piston

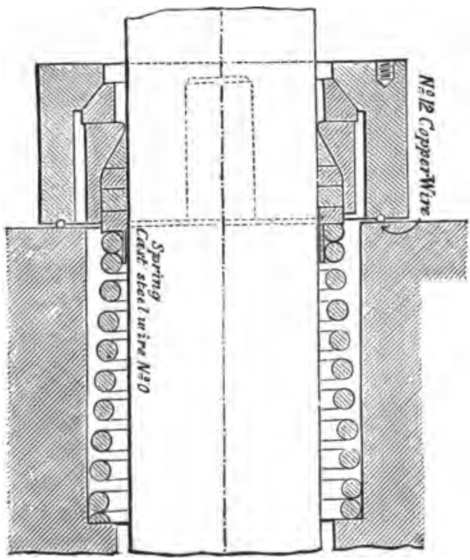


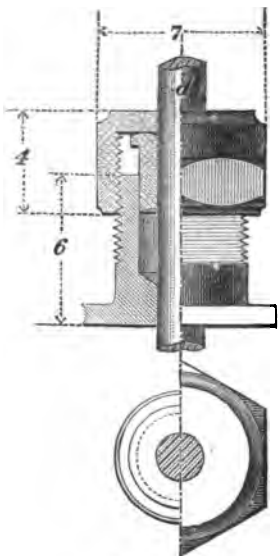
Fig. 158

and slide block only, and to construct the stuffing-box so as to permit a small lateral adjustment of position of the piston rod without causing pressure and heating. Fig. 157 shows this stuffing-box. The gland is bored out a little larger than the piston rod (about $\frac{1}{8}$ in. larger) and the ordinary fixed bush

[1] It is usual to make the gland a little smaller than the stuffing-box, so as to permit play to the rod.

at the bottom of the stuffing-box is dispensed with. Two rings fit closely to the piston rod, but are turned a little smaller than the stuffing-box, and between these and the soft packing are two thin washers fitting the stuffing-box, but $\frac{1}{8}$ in. smaller than the rod.

176. *Stuffing-boxes with metallic packing.*—Packing consisting of metallic rings in place of the ordinary soft packing has often been tried with more or less success. It should require much less attention to keep in order than soft packing. Fig. 158 shows one good form for piston-rod stuffing-boxes. Three Babbitt metal split rings grasp the piston rod. These fit inside a cup which presses on a ring having a spherical surface fitting the fixed part of the stuffing-box. Hence the piston rod can move a little sideways by displacing the cup and angularly by displacing the ring in its spherical seating, without causing leakage. The flat and spherical joints between the cup and ring, and between the ring and seating, are carefully ground, so as to be steam tight in any position. The Babbitt metal rings are kept in place in the cup by a spiral spring acting on a distance piece which is inserted a little way into the cup. The pressure of the steam acts with the spring in keeping the rings in place. The essential feature is that steam tightness is obtained without hindering the unavoidable lateral movement of the piston rod. The wear of the piston rod with this packing is very small. No doubt this form of packing



Unit $\frac{1}{4}d + \frac{1}{4}$

Fig. 159

is greatly superior to soft packing. It requires, however, the best workmanship, and is necessarily expensive in first cost.

177. *Small stuffing-box.*—For small rods, such as those round the spindles of valves and cocks, the form of stuffing-box shown in fig. 159 is used. The stuffing-box has a screw thread on the outside, and a six-sided cap fits over the gland and is screwed to fit the stuffing-box. Taking the unit as $\delta = \frac{1}{4} d + \frac{1}{4}$, the internal diameter may be $2\frac{1}{2} \delta$, the external diameter $5\frac{1}{2} \delta$, and the other proportions as given in figure.

178. *Packing for stuffing-boxes.*—A loose kind of hemp rope termed 'spun yarn,' steeped in melted tallow, was commonly used for stuffing-box packings. Occasionally brass turnings were sprinkled over the spun yarn, to cause the packing to wear longer. Packings of indiarubber wrapped in canvas, termed 'elastic core packing,' and metallic packings of woven wire, are also used. These are formed into ropes of such a form that when cut of a length equal to the circumference of the rod, they form a ring exactly fitting the stuffing-box. Lately asbestos prepared in discs or ropes has been used. Elastic core or asbestos packing is much better than spun yarn for engines working with high-pressure steam.

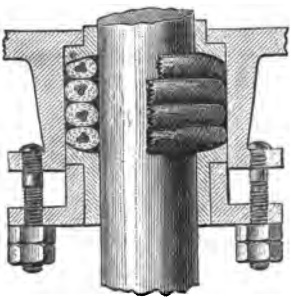


Fig. 160

Fig. 160 shows the most common mode of using a coil of indiarubber or asbestos packing in stuffing-boxes. The advantage of asbestos is that it stands the high temperature of steam at the high pressures now used without injury.

Glands when worn, if not bushed at first, are bored out, fitted with a bush having a collar at the inner end, and the bush is then bored.

For hydraulic purposes, Mr. Tweddell has found hemp packing in ordinary stuffing-boxes efficient under pressures of 1,500 to 2,000 lbs. per sq. in.; and where the rod passing through the stuffing-box is continuously at work, the hemp packing gives less trouble than a cup-leather, and is much more easily replaced when worn.

179. *Straight line engine bushing for piston rods.*—In the very interesting engine designed by Mr. J. E. Sweet there is no stuffing-box for the piston rod, but only a Babbitt metal bushing (fig. 161) bored slightly larger than the rod, so as to be a free sliding fit. The bushing rests in a spherical seat so as to be free to accommodate itself to the piston rod. The bushing is long and light, and held in place by a gland and screw cap. Leakage is prevented, or

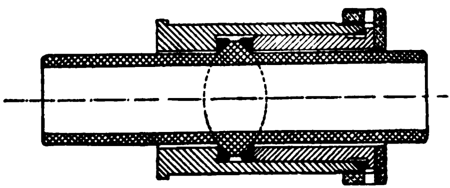


Fig. 161

at any rate reduced to a negligible amount, as the time of a stroke is too short for steam to leak through. The bush can be compressed when worn so as not to leak, and is not costly to replace.

180. *Friction of stuffing-boxes.*—Information as to the friction of stuffing-boxes is very scanty. Prof. Benjamin ('Proc. Am. Soc. Mech. Eng.,' xxi. 292) has made some direct experiments on the friction of stuffing-boxes with various forms of soft packing. He measured directly the power required to reciprocate a 2-in. rod in a stuffing-box at 200 revolutions per minute with a stroke of 4¼ ins. or a mean speed of 140 feet per minute. The steam pressure at the back of the stuffing-box was 50 lbs. per sq. in. and the gland bolts were lightly tightened so as merely to pre-

vent leakage, or in some cases merely to prevent more than slight leakage. The results reduced show that the frictional resistance of the rod varied with different packings from 12 to 94 lbs.; mean for the softer packings $16\frac{1}{2}$ lbs. When the gland bolts were tightened by a force of 14 to 16 lbs. at the end of a 7-in. wrench, the frictional resistance varied from 77 to 125 lbs. With the softer packings the resistance was nearly independent of the steam pressure at the back of the box, and with the harder it increased nearly directly as the steam pressure. Driving the 2-in. rod at 600 feet per minute the average power expended in the friction of one stuffing-box would be about 0·3 horse-power.

181. *Cup-leather packing.*—When great hydraulic pressure is to be resisted, a peculiar packing is used, invented by Bramah, and already alluded to (§ 165). The leakage is prevented by a flexible leather ring, kept in contact with the piston rod or ram on one side and the cylinder on the other by the fluid pressure. In its original form, leather was moulded into an annular shape in plan and to a U-shape in section (fig. 162). This ring is placed in a recess in the cylinder or in a stuffing-box, in such a way that the fluid has free access to its interior; the fluid pressure acting within the ring presses it against the plunger and the sides of the recess, and this, aided by the elasticity of the ring, makes a perfectly tight joint. When the cup-leather is large, it is provided with an internal brass ring. At one time a thin guard ring of brass was used on the edge most liable to wear. Mr. Tweddell introduced this guard ring, but it is no longer used. A packing of hemp or cotton is sometimes used as a bed for the leather.

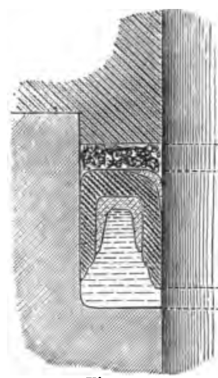


Fig. 162

Fig. 163 shows all the ordinary forms of hydraulic leather packing now used : *a* is a cup-shaped leather used for pump buckets ; *b* is a form sometimes termed a 'hat' leather ; *c* is the ordinary Bramah cup-leather or double cup-leather.

Mr. Welch gives the following rules for the proportions of cup-leathers for great pressures. Let D be the diameter

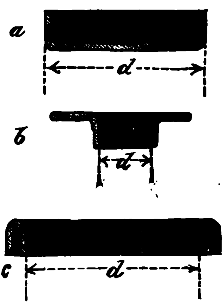


Fig. 163

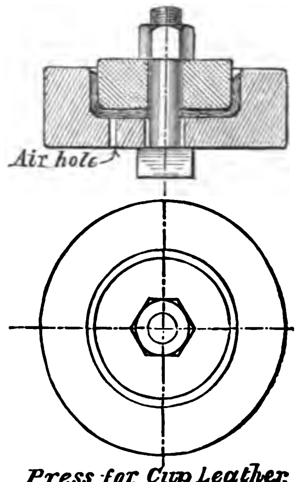


Fig. 164

of the ram or plunger. Then the thickness of the leather should be

$$t = 0{\cdot}156\ D^{0{\cdot}28} \quad . \quad . \quad . \qquad (1)$$

$$\log.\ t = \frac{7}{25} \log.\ D - 0{\cdot}8069,$$

and the width and depth of the ring measured outside should be each $2\frac{1}{2}\ t$. ('Proc. Inst. Mech. Eng.', 1876.)

D =	3	6	9	12	15	18	21	24
t =	0·212	0·258	0·288	0·313	0·333	0·351	0·366	0·380

Fig. 164 shows a press for moulding cup-leathers.[1] The best oil-dressed leather is steeped in warm water, and then forced gradually into the mould and left till it is again hard.

182. *Friction of cup-leathers.*—According to experiments by Mr. John Hick, the friction of a press cup-leather on a ram D ins. in diameter with a pressure of p lbs. per sq. in. is $F = c D p$, where c is a constant. But the whole pressure on the ram is $\frac{1}{4} \pi D^2 p$. Hence the fraction of the ram pressure expended in friction is $4 c / \pi D$. According to the experiments of Mr. Hick, c has values from 0·03 to 0·05 when p is in lbs. per sq. in. and D is in ins. These results are only applicable for large pressures in the ram cylinder. Some experiments on cup-leather friction were given in 'Engineering,' June 15, 1888, and in these it appeared that the friction was 4 to 9 per cent. of the ram pressure for leathers in good condition. In one case the friction amounted to 19 per cent. of the ram pressure. With a ram 36 ins. in diameter and a pressure of 130 lbs. per sq. in. in the ram cylinder, the author found a cup-leather friction amounting to from 9 to 15 per cent. of the load on the ram.

[1] Anderson, 'Chatham Lectures on Hydraulic Machinery.'

CHAPTER XI

FLYWHEELS

183. For most of the purposes for which engines are used, considerable regularity of speed is important. Now there is not in general an equilibrium between the effort exerted by the engine and the resistance overcome. Both the effort and resistance vary, and though for any sufficiently long period the mean effort and the mean resistance must be equal, there is during the period an alternate excess of effort or of resistance, producing a fluctuation of speed. It is to moderate this fluctuation of speed that heavy flywheels are used, which alternately store and restore a portion of the energy of the engine.

The causes of these temporary fluctuations of speed are chiefly: (*a*) the variation of the effective steam pressure driving the piston; (*b*) the variation in rotative engines of the leverage at which the piston pressure acts in rotating the crank; (*c*) periodical variations in the resistance overcome, such as the variation, during a stroke, of the pressure on a pump driven by the engine. We have not here to do with permanent causes of alteration of speed such as variation of boiler pressure, or permanent alteration of work being done by throwing machines into or out of gear.

184. *Flywheel radius and speed.*—The energy of motion of a steam engine at any given moment is partly the energy of reciprocating pieces, partly the energy of revolving pieces. The reciprocating pieces have variations of speed definitely connected with the piston positions, and since the whole

energy stored in, or restored by, these is not a large fraction of the whole energy of motion, we may neglect the small differences of speed of these in different strokes. Treating the reciprocating pieces as always having the same speed at the same points of the stroke, the velocity being calculated as if the crank pin revolved uniformly, the forces producing changes of energy of motion in the reciprocating pieces can be dealt with, without appreciable error, as additions to or deductions from the steam pressure on the piston. If the indicator diagram is corrected for the inertia of the reciprocating pieces (§ 86, p. 133), then the influence of these on the fluctuations of speed of the engine need not further be considered.

Of the rotating pieces there is one, the flywheel, of such very large weight and moment of inertia, that the effect of all the others on the fluctuations of speed may be disregarded. Let W_r be the weight of the rim, W_a the weight of the arms and nave, ρ_r the radius of gyration of the rim, and ρ_a the radius of gyration of the arms. Let ΔE be a quantity of energy stored or restored during a change of angular velocity from ω_1 to ω_2. Then (units, lbs., and feet)

$$\Delta E = \left(W_r \rho_r^2 + W_a \rho_a^2\right) \frac{\omega_2^2 - \omega_1^2}{2g}.$$

If the rim is of rectangular section and internal and external radii R_1 and R_2, then

$$\rho_r^2 = \frac{R_1^2 + R_2^2}{2}.$$

In any case, if R is the radius to the centre of figure of the rim, $R = \frac{1}{2}(R_1 + R_2)$ nearly, and ρ_r will not in ordinary cases differ much from R. Also as the weight of the arms is a good deal less than that of the rim, and ρ_a does not greatly differ from 0·577 R, we may take, for practical purposes,

$$\Delta E = (W_r + \tfrac{1}{3} W_a) R^2 \frac{\omega_2^2 - \omega_1^2}{2g} \quad . \qquad (1)$$

If v_1, v_2 are the velocities at the radius R, this becomes

$$\Delta E = (W_r + \tfrac{1}{3} W_a) \frac{v_2^2 - v_1^2}{2g} \quad . \qquad (1a)^{1}$$

Hence, to reduce the fluctuation of speed for any given excess or defect of energy, ΔE, as much as possible, it is necessary to put as much weight as possible into the rim of the flywheel, and to make its radius as large as possible.

In the older steam engines the flywheel radius was generally 1·7 time the stroke. It is now sometimes 2 to 2½ times the stroke. But there are limits both of convenience and safety which restrict the increase of flywheel radius. The centrifugal force, due to rotation, produces a centrifugal tension in the rim, and if the peripheral velocity exceeds certain limits the wheel becomes unsafe. Of all machinery accidents, the bursting of flywheels, or, as such accidents are termed in Germany, the 'explosion of flywheels,' is among the most destructive. Anyone who will consult the list of such accidents given in a paper by Herr Köchy in the 'Verhand. des Vereins zur Beförderung des Gewerbfleisses' for 1886, will see that such accidents are not infrequent.

Let N be the revolutions per minute of the flywheel, reckoned at the centre of the rim, then $v = \pi R N/30$ is the velocity of the rim in feet per sec. For cast-iron pulleys and wheels, with solid rims, that is, rims without any joints, a peripheral speed $v = 100$ feet per sec. is the limit found safe in practice. But when flywheels exceed 10 feet in diameter it is generally necessary to construct them of segments bolted together, and then so high a velocity is not safe. For a long time it has been usual in this country to use a spur flywheel for factory engines working on to a mortice spur pinion on the first-motion shaft. The necessary speed for the shafting is thus gained in the most simple and direct way. Such spur flywheels are most commonly built of segments, and for safety the

[1] Units, lbs., feet, foot lbs., and feet per sec.

limiting speed was taken at 30 to 35 feet per sec. for cast iron, and 45 or 50 feet per sec. for cast steel. Wheels with plain rims (without teeth) were run at somewhat higher speeds. Recently more attention has been given to the design of rim joints so as to secure greater strength at the points where the rim is weakened, and in consequence flywheels are now sometimes run at greater speeds. Radinger ('Proc. Inst. Civil Engineers,' lv. 404) has given particulars of some American high-speed flywheels :

	Corliss Engine	Amoskeag Mfg. Co.	Bay State Mill
H.P. transmitted	1,400	570	400
Diameter, feet	30	28	20·5
Revs. per min.	36	49	48
Velocity of pitch line, feet per sec.	56·5	72	51·5
Pitch	5·23	5·18	4·02
Breadth	24	18	18
Pressure per in. of breadth	567	242	237

Radinger attributes the high values of speed and pressure found in these wheels to their superior workmanship; obviously in such cases the strength of the joints of the arms and rim should be very carefully studied. A large flywheel for the Crossley 400 H.P. gas-engine, 13 feet diameter and weighing about 35 tons, is described in Mr. Humphrey's paper ('Proc. Inst. Mech. Eng.,' 1901). In this case, in which great care was taken to make the rim joints as strong as possible, the speed at the surface of the rim is 102 feet per sec. Mr. Barr found the average rim velocity in a number of modern American engines to be 70 feet per sec. Flywheels are commonly used for engines driving rolling mills, which store an amount of energy when the rolls are running empty, which is afterwards restored in forcing the heated bar through the rolls. Such wheels are exposed to excessively severe straining action in consequence of the

great and almost sudden changes of velocity they undergo. Nevertheless, they are not rarely run at speeds of 90 feet per sec. or even 100 feet per sec. With such wheels, however, it is also true that most of the accidents occur.

Mr. Halpin has proposed to construct a flywheel rim of great strength by putting a worm and wheel to drive the crank shaft, and winding on to the rim steel wire of great tenacity. A flywheel of this type has actually been used for the engines driving the machinery for rolling tubes by the Mannesmann process, where an enormous power is required for a very short time. The Landore flywheel is composed of two steel discs bolted to a cast-iron nave. The discs are 20 feet in diameter and form a trough-shaped groove at the circumference, into which 70 tons of steel wire were wound with a tension of about 50 lbs. The section of the wire was about 0·05 sq. in. The speed of the rim is about 250 feet per sec. The possibility of running at this high speed is due to the great tensile strength of the steel wire.

185. *Centrifugal tension in flywheel rim.*—The centrifugal tension in the rim (I. § 278, p. 482) is $w\,v^2/g$ lbs. per sq. in., where $w = 3{\cdot}36$ lbs. is the weight of a bar of iron 12 ins. long and 1 in. square, and v is the velocity of the rim in feet per sec. Taking the ultimate tenacity of cast iron at 18,000 lbs. per sq. in., the bursting speed of the rim, if there were no stress other than the centrifugal tension, would be $v = \sqrt{180{,}000} = 425$ feet per sec. Taking the working stress of cast iron in tension at 2,800 lbs. per sq. in. (Table II. Case B, p. 4), the greatest safe speed would be $v = \sqrt{28{,}000} = 167$ feet per sec. To allow for the other stresses additional to the centrifugal tension, and for the weakening of the rim by joints, the maximum speed must be less than this, as the practice of engineers described above shows to be the case.

186. *Mr. Benjamin's experiments on the bursting speed of models of flywheels.*—When the stresses additional to the centrifugal tension are taken into the reckoning, the theory of the strength of flywheels is complicated and its application

is uncertain, because of the indefiniteness of some of the data. Hence, direct experiment may be expected to be useful in checking in a general way the conclusions of theory. Mr. Benjamin has made very interesting tests of the bursting speed of models of flywheels.[1]

Mr. Benjamin found that model wheels 15 ins. diameter, with six arms and solid rims about 2 ins. by 0·7 in. in section, burst with speeds of 430 feet per sec. They were therefore practically as strong as if there were no other stress than the centrifugal tension. With similar wheels, the rim of which had been turned down to about 0·55 in. thick, the bursting speed was 380 to 395 feet per sec. When the rim was only about 0·4 in. thick, the bursting speed was 363 feet per sec. Obviously with the thinner rims the bending was greater, and this reduced the speed at which the wheels burst. Similarly with three arms instead of six the bursting speed was somewhat reduced.

Models having two joints in the rim, with internal flanges and bolts, were very much weaker. They burst at speeds of 184 to 196 feet per sec., so that the rim had only about one-fourth the strength of a solid rim. Such wheels would be no safer at 46 feet per sec. than solid rim wheels at 100 feet per sec.

Next models with rims jointed midway between the arms, having steel hoop links shrunk over lugs cast on the rim, were tried. With three links at each joint, one on each side of the rim and one inside the rim, the bursting speed was 320 feet per sec. and with two links only 290 feet per sec. Hence such wheels are about as safe at 70 feet per sec. as solid rims at 100 feet per sec. Such joints may probably be made two-thirds to three-quarters as strong as the solid rim. Mr. Benjamin concludes that joints in the rim should be placed over the arms and not midway between them, and this construction is now often adopted.

Some later experiments have been made by Mr. Ben-

[1] 'Trans. Am. Soc. Mech. Eng.,' xx. p. 209. Also 'Cassier's Magazine,' xviii. p. 248.

jamin. In two experiments with solid rims of the Allis pattern, the rims failed at a rim speed of 395 feet per sec., the centrifugal tension in the rim being 7 tons per sq. in. Two models with solid rims and steel spokes tightened by nuts failed at 424 feet per sec., the centrifugal tension being 8 tons per sq. in. Four models with flanged joints in the rim at one-fourth the distance between the arms, instead of in the centre between them, were no stronger than models previously tested. They failed at 194 feet per sec., the centrifugal tension being 1·7 tons per sq. in. A model of a blowing engine flywheel with links shrunk into the rim failed at 256 feet per sec., the centrifugal tension being 3 tons per sq. in. Two models with solid rims, but having the arms cast separately from the rim and bolted between ears cast on the rim, burst at 392 feet per sec. Two models had the rim in halves bolted together with a flanged joint, the arms cast separately and bolted with a flanged joint. The average bursting speed was 224 feet per sec.

Determination of Weight of Flywheel for given Fluctuation of Speed

187. *Determination of the coefficient of fluctuation of energy.*—The first problem to be dealt with is the determination of the weight of flywheel necessary to secure a limitation of the fluctuation of speed within assigned limits. No doubt very rough rules of thumb are commonly used in assigning the flywheel weight, and these ensure a practically sufficient regularity. But they do not secure that the flywheel has only sufficient weight for the purpose in view, and excessive flywheel weight not only causes loss of work in friction, but in some cases, such as engines driving large pumps, may be dangerous if there is a sudden variation of load—as, for instance, when a main bursts.

188. *Variation of resistance overcome, or load.*—The steam effort on the piston gives a varying tangential effort

on the crank pin, and the resistances overcome, reckoned at the crank pin, vary in general according to a different law. Hence, in successive parts of a revolution there are alternately excesses of effort and resistance which would produce great variations of speed but for the energy stored or restored by the flywheel. As to the variations of resistance the following cases may be distinguished :—(1) An engine drives a centrifugal pump or dynamo. Then the resistance during a revolution, reckoned at the crank pin, is practically constant; (2) An engine drives a great number of smaller machines. Then the variations of resistance of different machines in any short period practically balance, and the total resistance at the crank pin is constant; (3) An engine drives a reciprocating pump, working on a constant lift. Then the resistance at the pump plunger is practically constant, but the resistance at the crank pin varies like the effort curve of an engine working without expansion (§ 89); (4) An engine drives an air compressor. Then the resistance on the compressor piston is least at the beginning of the stroke—and greatest at the end. The resistance at the crank pin varies like the effort curve of an expansive engine taken reversely; (5) An engine drives a single machine by gearing, in which the resistance varies in each revolution, but in a period different from that of the engine.

In Chapter IV. methods have been given for finding the crank-pin effort from the indicator diagram, and these are also applicable in finding the curve of resistance at the crank pin from the indicator diagram of a pump or compressor. In treating flywheels it is generally accurate enough to neglect the effect of the obliquity of the connecting rod. In no case, however, should the inertia forces due to the reciprocating parts be left out of account. They modify the effort curve very much. It is convenient in flywheel questions to plot the effort curve to a base representing the travel of the crank pin.

189. *Load and effort curves.*—In fig. 165 the curve marked 'effort curve' represents the effective (tangential) effort at the crank pin of an engine due to the steam pressure on the piston, inertia deducted, found by the methods in Chapter IV., on a base the length of which is the circumference of the crank-pin circle. If the resistance at the crank pin is constant, it may be represented by the straight line marked 'load curve.' Both the effort and the load may conveniently be reckoned either as total effort and load, or as effort and load per sq. in. of piston area. Obviously the whole area of the effort curve and the whole area of the load curve for one revolution must be equal when conditions

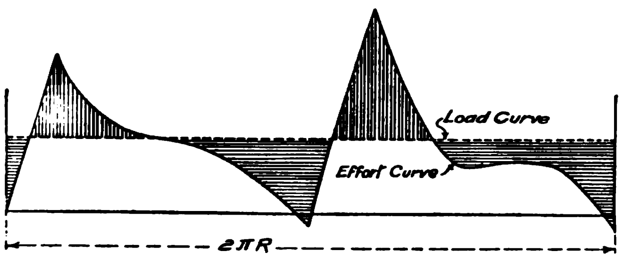


Fig. 165

of driving are steady. But the distribution of effort and resistance vary so that most commonly there are two parts of the revolution where the effort exceeds the resistance, and two parts where the resistance exceeds the effort. The two areas between load and effort curve where the effort is in excess must be equal to the two areas where the resistance is in excess. These areas represent the quantities of energy which must be alternately stored or restored by the flywheel. The greatest of the four areas is the quantity which will be termed the *fluctuation of energy* and denoted by Δ E, when the area of the effort curve for one stroke or half a revolution is denoted by E. The ratio Δ E/E is the *coefficient of fluctuation* of energy.

Fig. 166 shows the effort and load curve in the case where a steam piston and pump plunger are directly connected and the pressure against the plunger is practically constant. The resistance at the crank pin then varies as shown by the ordinates of the load curve The load curve is found by the same construction as that for find-

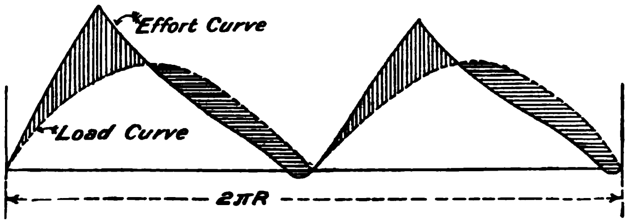


Fig. 166

ing the effort curve of an engine working with constant steam pressure. The successive areas of excess of motive and resisting work are shaded as in the previous diagram.

If there are two engines at right angles, two effort curves must be drawn (fig. 167), differing in phase by 90°. Adding the ordinates of the two effort curves, a curve of resultant effort curve is obtained. The mean ordinate A C of this

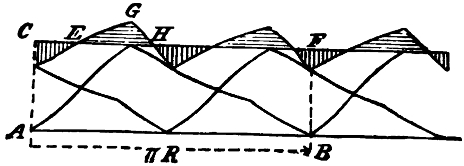


Fig. 167

must be equal to the mean or constant resistance. There will now be two minimum and two maximum velocities in a semi-revolution. The coefficient of fluctuation of energy is now the ratio of one shaded area, such as E G H to the area of the rectangle A C F B. In compound engines with cylinders of different sizes the effective pressure in the HP

cylinder must be reduced to the equivalent pressure on the low-pressure piston before proceeding to construct the effort curves, which are to be combined.

190. *Periodical excess or deficiency of energy in an engine driving a pump or air-compressor direct.*—In many pumping engines and air-compressors, the pump-plunger or air-compressor piston is on the same piston rod as the steam piston. Let A B C D be the diagram of steam piston effort corrected, if necessary, for the inertia of the reciprocating parts. Let A C E F be the corresponding diagram of effective resistance at the plunger or compressor piston. If the steam diagram is drawn for pressures in lbs. per sq. in. of the steam piston, the resisting pressures must be reduced to equivalent pressures on a piston of the same area. Then the shaded areas, fig. 168, represent the work alternately stored and restored:

$$k = \frac{\Delta \mathrm{E}}{\mathrm{E}} = \frac{\mathrm{ABC}}{\mathrm{ABCDF}} = \frac{\mathrm{CED}}{\mathrm{AEF}}.$$

An interesting case arises in the High Duty Worthington direct acting pumping engine. In this there is no flywheel. Two small oscillating cylinders are placed at right angles to the line of stroke. During the first half of the stroke the plungers of these are driven in and force back a column of water into a reservoir containing air under pressure. During the second half of the stroke the plungers move out again, giving out just as much work as was previously absorbed. By adjusting the air pressure, an almost exact compensation of the variation of effective steam pressure is obtained.

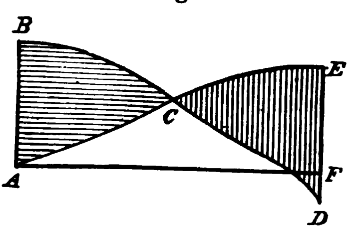


Fig. 168

Fig. 169 shows the steam-effort, pump-resistance, and

compensating cylinder effort curves for a large Worthington compound engine at Hampton. The resultant thrust curve due to combining the steam effort and compensating cylinder curves approximates to a horizontal line The

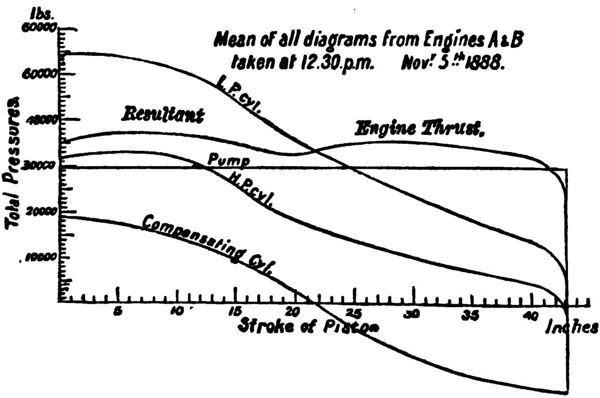


Fig 169

pump resistance curve is assumed to be a straight line. The excess of the resultant effort over the pump resistance is no doubt mainly the friction of mechanism.

191. *Coefficient of fluctuation of energy.*—The ratio of the largest of the areas representing excess or deficiency of energy to the mean energy in a semi-revolution, that is the fraction $k = \Delta E / E$, is called the coefficient of fluctuation of energy. It depends as is shown above on many circumstances, and can only be determined in any given case with approximate accuracy by drawing the effort and resistance curves by the methods described. In steam engines it is largely dependent on the ratio of expansion r, or what is nearly the same thing, the ratio of the initial absolute pressure to the pressure at release. Calculations have been made of the value of $\Delta E / E$ for various ratios of expansion, and the results may be used in rough determinations of

the weight of flywheel necessary to secure a given degree of uniformity of speed.

The following values given by Rankine are for the case of engines driving against a constant resistance at the crank pin, and E is the energy in a semi-revolution.

NON-EXPANSIVE ENGINES

$\frac{\text{Connecting-rod length}}{\text{Crank radius}}$	=	∞	6	5	4
$\Delta E/E$	=	·210	·236	·250	·264

EXPANSIVE CONDENSING ENGINES

Connecting-rod length/crank radius = 5

Ratio of expansion r	=	3	4	5	6	7	8
$\Delta E/E$	=	·326	·346	·356	·368	·378	·382

EXPANSIVE NON-CONDENSING ENGINES

Connecting-rod length/crank radius = 5

Ratio of expansion	=	2	3	4	5
$\Delta E/E$	=	0·320	·372	·418	·464

These values are valid, so far as they are trustworthy, only for engines with one cylinder. For a compound engine, both cylinders driving one crank, the variation of crank-pin effort is less than for a single cylinder, and for engines with two cylinders acting on cranks at right angles the variation is still less. On the average it would appear that $\Delta E/E$ may be taken at 0·25 for simple single-cylinder engines, 0·18 for compound engines, and 0·12 for two-cylinder engines with cranks at right angles. In gas engines the fluctuation of energy is greater and must be reckoned for two revolutions. In pumps driven by a crank shaft the fluctuation of resisting work is about the same as for a non-expansive engine.

192. *Coefficient of fluctuation of speed.*—Let v_1, v_2 be the least and greatest speeds of the flywheel rim at radius R in a period, for instance, the speeds at the points E and F, fig. 167. Then, since $v_2 - v_1$ is small, it is accurate enough to take the mean speed $v = \frac{1}{2}(v_1 + v_2)$. The ratio $(v_2 - v_1)/v$ or approximately $2(v_2 - v_1)/(v_1 + v_2)$ will be called the coefficient of fluctuation of speed, and will be denoted by n. This ratio varies for different kinds of work. The following values correspond with ordinary practice:

			$n =$
Engines doing pumping			1/20
"	driving machine tools		1/35
"	"	textile machines	1/40
"	"	spinning machinery	1/50 to 1/100
"	"	electric machinery	1/150
"	"	" " direct driven	1/300

193. *Weight of flywheel when the coefficients of fluctuation of energy and speed are given.*—Suppose an engine is doing E foot lbs. of work per stroke and the coefficient of fluctuation of energy, k, for the given conditions of working has been determined. Then $\Delta E = kE$ is known. But from Eq. 1*a*, § 184:

$$\Delta E = \left(W_r + \tfrac{1}{3} W_a\right) \frac{v_2^2 - v_1^2}{2g}$$

$$= \left(W_r + \tfrac{1}{3} W_a\right) \frac{(v_2 - v_1)\, v}{g} \qquad v\left\{\frac{v_2 + v_1}{2}\right.$$

$$= \left(W_r + \tfrac{1}{3} W_a\right) \frac{n v^2}{g}$$

$$W_r + \tfrac{1}{3} W_a = \frac{g \Delta E}{n v^2} = \frac{g k E}{n v^2} \qquad (2)$$

An equation from which the necessary weight of the flywheel can be determined. If a is the area of the rim in sq. ft., then $W_r = 2\pi R a \times 450$ lbs. nearly, from which a provisional estimate of the section of the rim can be made, if the weight of the arms is neglected.

Let H.P. be the horse-power of the engine, N the revolutions per minute, R the fly-wheel radius in feet. Then

$$E = (33{,}000 \text{ H.P.})/2\,N,$$
$$v = 2\,\pi\,R\,N/60.$$

Inserting these values—

$$\left.\begin{aligned} W_r + \tfrac{1}{3} W_a &= 48{,}420{,}000 \frac{k}{n} \frac{\text{H.P.}}{R^2 N^3} \text{ lbs.} \\ &= 21{,}620 \frac{k}{n} \frac{\text{H.P.}}{R^2 N^3} \text{ tons} \end{aligned}\right\} \quad . \qquad (3)$$

Professor Barr has examined a number of engines constructed by different makers ('Trans. Am. Soc. Mech. Eng.,' xviii. p. 737), and, taking for H.P. the rated horse-power, has found for the factor $21{,}620\,k/n$ values ranging from 502,300 to 1,549,000, the mean value in the cases examined being 927,000. Hence k/n must have had values in these engines ranging from 23 to 285, and a mean value 43. It is instructive to know that there is so great a variation in practice, but so great a range of values of k/n must be largely due to caprice of manufacturers.

Ratio of energy of flywheel rim to work done per stroke.—The following is a very convenient, practical way of stating the flywheel weight. The energy of the rim in foot lbs. is

$$\frac{W_r}{2g} \cdot \frac{\pi^2 R^2 N^2}{900}.$$

Hence the ratio of the energy of the rim to the work done per stroke is

$$\frac{W_r}{2g} \frac{\pi^2 R^2 N^2}{900} \cdot \frac{2\,N}{33{,}000 \text{ H.P.}} = 0{\cdot}0000000 1032 \frac{W_r R^2 N^3}{\text{H.P.}}.$$

Now taking Professor Barr's mean figure and neglecting the arms, $W_r = 2{,}080{,}000{,}000 \dfrac{\text{H.P.}}{R^2 N^3}$ lbs. Putting in this value, the ratio above is $21\frac{1}{2}$ nearly, or the energy of the rim is $21\frac{1}{2}$ times the work done per stroke by the engine.

In any case, this ratio is a very convenient way of comparing the relative flywheel effect in different engines similar in other respects.

194. *Flywheel weight for electric driving.*—Electric machinery requires to be driven with great uniformity, especially in the case of alternators running in parallel. The following examples are taken from data in Mr. A. Morcom's paper on 'High Speed Engines' ('Proc. Inst. Mech. Eng.,' 1897, p. 329). It will be seen that the energy of flywheel per effective horse-power is very variable; but as would be expected the ratio of the energy stored in the flywheel to the effective work per stroke is more nearly constant, allowing for the greater constancy of speed required with alternators.

Energy in flywheel and revolving parts of direct-coupled dynamo at full load.

	Energy of flywheel per electric H.P. in foot tons.	Revolutions per min.	Ratio of energy in flywheel to effective work per stroke
High-speed engines for electric lighting, with continuous current	0·2 to 0·3	350	9 to 14
Ditto, with heavy alternator	1·5 to 3·5	360	73 to 171
Electric traction, continental example	2·0	150	41
Electric traction, Montreal street railway	4·0	70	38
Electric traction, Brooklyn electric tramway	4·7	75	48

Construction of Flywheels

195. *General construction of flywheels.*—Flywheels up to 10 feet in diameter are sometimes cast solid in one casting. But sometimes the nave is split to relieve the

stresses due to contraction in casting. Then rings shrunk on or bolts must be used to connect the two parts of the nave (fig. 170). For flywheels from 10 to 15 feet in diameter a safer construction is to cast the wheel in two halves and connect the portions of the rim by dowels and cotters, and the parts of the nave by bolts or rings shrunk on. Larger wheels are cast with nave, arms, and segments separate. The segments of the rim are connected in various ways by flanges and bolts, dowels and cotters, or by straps and bolts. The arms are sometimes halved on to the rim and bolted, but this cuts away half the section of the rim. More commonly the arms are attached by flanges and bolts inside the rim. The arms are attached to the nave in wedge-shaped recesses in which they are bolted, or, better, the end of the arm is turned taper and fits into a taper hole bored in the nave.

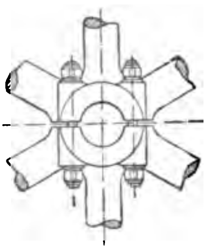


Fig. 170

Fig. 171 shows one of the best constructions for built-up flywheels. The nave in this case is keyed on the shaft with four keys, two driven one way and two the other. There are flats on the shaft and key ways in the nave. With four keys some adjustment of the wheel, to centre it, is possible, but it is also usual to bore the nave and fix it by two keys in positions at right angles.[1] The larger ends of the arms are turned and fit in bored recesses in the nave, to which they are cottered. The small end of the arms has a flange to receive the segments, the joints in the segments being over the arms. The segments are connected partly

[1] Mr. Longridge states ('Proc. Mech. Eng.,' 1896, p. 569) that flywheels fixed by two keys generally work loose. He advises that four or more broad keys should be used, bedded on flats on the shaft, and if possible driven, two and two, from opposite sides of the wheel. The corners of the key beds in the nave should be rounded.

by bolting to the arms, partly by a dowel and cotters. The outer surface of the segments is turned to receive toothed segments, the joints of which come half-way between the

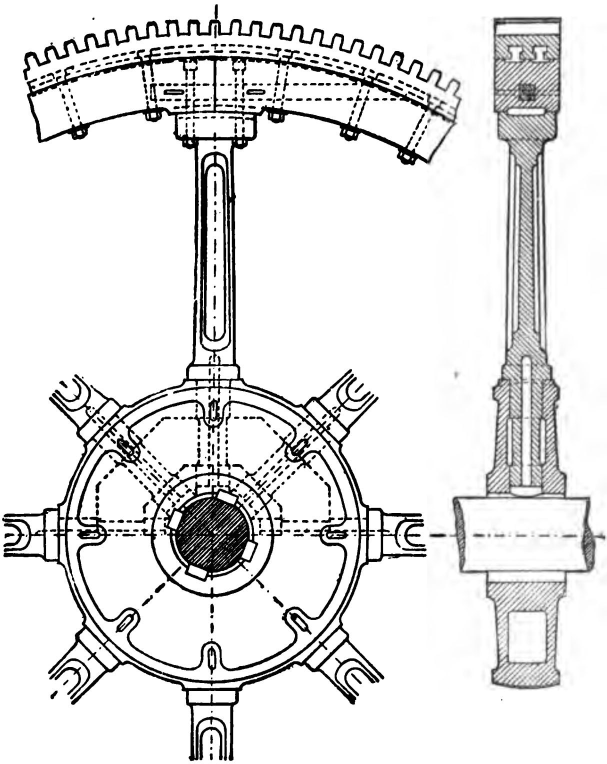


Fig. 171

joints of the rim segments. Sometimes the teeth are cast on the rim segments, but this is not a good plan. It is very difficult in that case to secure even pitch of the teeth, and the teeth are liable to be weak and spongy at the roots from

contraction of the large mass of metal in the rim in casting. The spur segments in this case have two recesses cast round their inner surface to receive T-headed bolts, by which they are fixed to the rim segments.

To secure uniformity of pitch, the teeth are now often cut to exact form after casting, even in the case of very large wheels. In very important cases the spur segments are cast of steel, but in that case cutting the teeth after casting is always necessary, as the contraction in casting is large and irregular.

196. *Nave with rings shrunk on.*—Fig. 172 shows a fly-wheel nave split into three parts and connected by two wrought-iron rings shrunk on. The stress in the rings due to shrinking on and to keying is almost indeterminable; consequently a somewhat rough calculation of the stress on the rings is the only one possible, and this matters less, as it is easy to give the rings ample section. Let W be the total weight of the flywheel in lbs., R the radius in feet to the centre of rim, ω the angular velocity per sec., and N the revolutions per minute. The centre of gravity of half the flywheel will be very nearly at the distance $2\,R/\pi$ from the centre of the wheel. Supposing each half of the wheel revolving at its centre of gravity, and neglecting the connection of the parts of the rim, there will be two radial centrifugal forces—

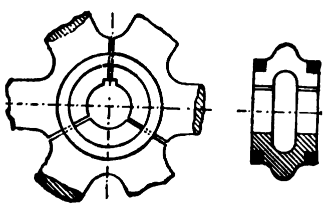


Fig. 172

$$\frac{W}{2g}\,\frac{2\,R}{\pi}\,\omega^2 = \frac{W}{2g}.\,\frac{2\,R}{\pi}\left(\frac{2\,\pi\,N}{60}\right)^2$$

$$= \frac{W}{g}\,R\,\frac{\pi\,N^2}{900}\text{ lbs.,}$$

tending to tear the two rings apart at a diametral section. Let a be the section of a ring, and f the working safe stress

at normal speed. Then, since there are four sections of the two rings to carry the tearing force,

$$4\,af = \frac{W}{g} R \frac{\pi N^2}{900}$$

$$a = \frac{W}{g}\frac{R}{f}\frac{\pi N^2}{3{,}600}$$

It appears that in actual cases in practice, even on the extreme supposition that the rings carry all the tension due to the centrifugal force of half the flywheel, f does not exceed 300 lbs. per sq. in., and is often only half this.

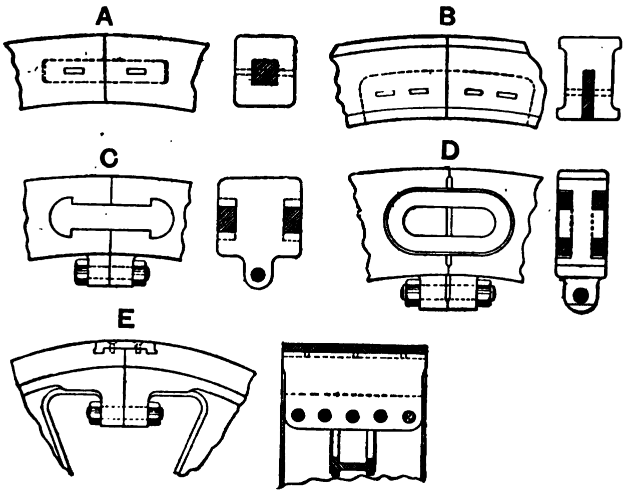


Fig. 173

Where there are bolts instead of rings shrunk on, the stress on the net section of the bolts at the bottom of the thread is sometimes 2,500 lbs. per sq. in.

197. *Fastenings of rim segments.*—Fig. 173 shows various methods of connecting the rim segments. At A

and B wrought-iron dowels are used cottered into the rim. Taking wrought iron to be four times as strong as cast iron, the section of the dowel through the cotter-hole should be one-fourth of the area of the rim through one of the cotter-holes, that is, if the stress on the sections is assumed uniform. At C a strap bolted to the rim on each side is used, and a bolt in addition inside the rim. At D rings shrunk on projections on each side and a bolt inside the rim. The only difficulty with this fixing is that the stress in the wrought-iron straps is not determinable with any accuracy. A very good fastening used in a large flywheel by Messrs. Crossley is shown at E. There are bolts in flanges inside the rim and a large dovetailed taper key in a recess on the outside of the rim.

Fig. 174 shows a joint directly over an arm, the joint

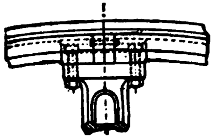


Fig. 174

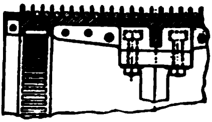


Fig. 175

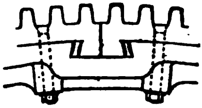


Fig. 176

being made by bolts and flanges. Fig. 175 shows a portion of the rim of a very wide flywheel carrying rope grooves. The arm is attached by T-headed bolts, and other bolts in flanges connect the rim segments.

Fig. 176 shows a method of attaching spur segments to the rim segments different from that in fig. 171.

In speaking of large spur flywheels, Mr. Longridge points out ('Proc. Inst. Mec. Eng.,' 1896, p. 544) that to place the spur segments whose velocity should be low on the outside of the rim, the velocity of which should be high, is contrary to common sense. He prefers to carry the spur segments not on the rim but on the arms, as shown in fig. 177. The rim may then have the greatest diameter permitted by the

position of the second motion shaft. The spur segments

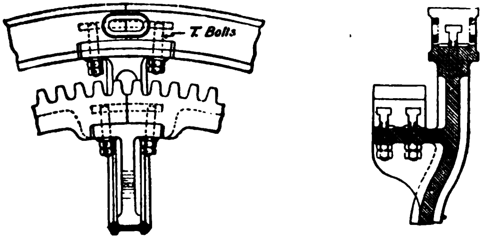


Fig. 177

also may be of steel, and of finer pitch than would be possible with cast iron.

Strength of Flywheels

198. *Approximate calculation.*—Flywheels have, no doubt, commonly been designed on a very rough theory of the straining actions, and to balance the imperfection of the theory very low working stresses have been assumed. That this rough theory so modified does not always secure safety, the accidents which occur sufficiently prove. Except where exceedingly low working stresses are allowed, it can hardly be assumed on grounds of experience that safety is assured. But the adoption of very low working stresses involves making wheels less efficient as stores of energy than they might be if the theory were more perfect and trustworthy. In this as in so many other cases in machinery, safety is often imperfectly secured by unnecessary waste of material, which with more intelligence might be economised without any greater danger, or rather with an increase of safety.

However, it may be useful to give the rough theory first, because any exact theory is complicated, and then to examine more carefully the real straining actions. In the rough theory of flywheels it is assumed that the rim is subjected to a tension due to its centrifugal force, and the

arms to a bending moment due to the greatest acceleration or retardation of the rim likely to occur. The bending of the rim and the mutual action of the arms and rim are neglected.

Let R be the radius to the centre of the rim in feet.
" N be the number of revolutions per minute.
" ω the angular velocity per sec. $= \pi$ N/30.
" v the linear velocity at radius R in feet per sec. $= \omega$ R.
" a be the section of the rim in sq. ins.
" $w = 3{\cdot}125$ lbs. = the weight of a square bar of 1 in. section 1 foot long.

The total weight of the rim is

$$W_r = 2\ \pi\ R\ a\ w = 19{\cdot}63\ R\ a \text{ lbs.}$$

The radial centrifugal force acting uniformly round the rim like a fluid pressure is for each sq. in. of section

$$C = \frac{w}{g}\frac{v^2}{R} = \frac{w}{g}\omega^2 R$$

per foot of arc. The resultant centrifugal force of a semicircle of the rim is per in. of section 2 C R. Hence the tension in the rim is

$$f = C\,R = \frac{w}{g}v^2 = \frac{w}{g}\omega^2 R^2 \text{ lbs. per sq. in.}$$

In a flywheel rim of large section the stress will be distributed like that in a thick tube subjected to internal fluid pressure, and therefore will be a little greater than f at the inside and a little less at the outside. Neglecting this, f is the stress in the rim due to centrifugal force, and it depends only on the radius and angular velocity. For cast iron $f = 0{\cdot}0971\ v^2$.

Stress in flywheel rim due to centrifugal force .

$v =$	30	40	50	75	100	ft. per sec.
$f =$	87·4	155·3	242·8	546·3	971·0	lbs. per sq. in.

The bending stresses in the rim, due to the centrifugal force of the portions between two arms, may very probably increase these stresses 50 per cent. Some wheels have teeth or toothed segments added to the rim, which increase the centrifugal force without adding to the section which resists it. In some wheels portions of the rim are cut away for cotter or bolt holes. But, allowing for all these causes of weakness, the stresses found in actual wheels are low compared with the safe stress allowed in other parts of machines.

The arms have still to be considered. The arms are principally strained by the bending moment due to variations of velocity of the wheel. Let T be the greatest twisting moment transmitted from the flywheel shaft to the flywheel. In starting, and in cases where steam is suddenly cut off and the motion is continued by the energy of the flywheel alone, T may have a value equal to the mean twisting moment transmitted in ordinary driving (Part I. § 22, p. 40). Let n be the number of arms, Z the modulus of the arm section (Part I. Table V.). Then

$$T = n f_a Z$$

where f_a is the stress due to bending in the arm, greatest near the nave. At the rim the section is often only two-thirds that at the nave. It is commonly stated that f_a should not exceed 1,000 to 1,400 lbs. per sq. in. for cast-iron arms. But with the uncertainty as to the magnitude of the moment transmitted through the arms, the calculation is too crude to be very useful.

If d is the diameter of the flywheel shaft at the bearings where the straining action is chiefly a twisting moment, and f the stress in the shaft, then the flywheel arms will be as strong as the shaft if

$$0{\cdot}2 f d^3 = n f_a Z\,;$$

if the shaft is of wrought iron and the arms of cast iron, $f = 4 f_a$. Then

$$z = \frac{4}{5} \frac{d^3}{n}.$$

The dimensions of the arm can be determined from the section modulus z (see Table V. Part I.).

199. *Stresses in a homogeneous flywheel running at uniform speed.*—By restricting the case to that of a homogeneous flywheel, it is meant to exclude the case in which the rim and arms are of different materials, and also to exclude the complications arising out of joints in the rim of strength or stiffness less than that of the rim.

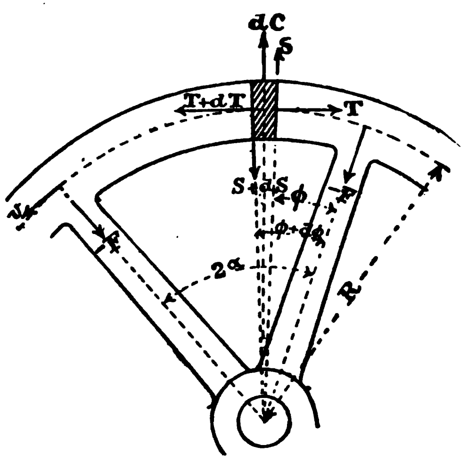


Fig. 178

Let fig. 178 represent a portion of such a flywheel in uniform rotation. Under the action of the centrifugal force there will be a tension in the rim which, considered alone, would cause an increase of radius uniform all round. This uniform expansion of the rim is, however, hindered at the junctions of the arms. The arms themselves lengthen partly under the action of their own centrifugal force, partly from the pull exerted on them by the rim ; but between

the arms the rim bends outwards more than at the arms. The result is, the rim takes a form convex to the centre of the wheel at the arms and concave between. The action is similar between each pair of arms.

Let 2α be the angle between the centre lines of two arms. This angle, from conditions of symmetry, remains constant when the wheel is deformed by the stresses acting on it. Let R be the radius to the circle through the centres of figure of the radial sections of the rim, R and all the other dimensions being in feet for simplicity. Let the angular velocity of the wheel be ω, so that the linear velocity at the radius R is $v = \omega R$. Let A be the section (in sq. feet) of the rim; A_1, the mean section of an arm. Let G be the weight of the material in lbs. per cubic foot so that G A is the weight of the rim in lbs. per foot of length.

Consider a slice of the rim between two radial sections inclined at an angle $d\phi$. The centrifugal force of this slice is

$$dC = \frac{G}{g} A R \, d\phi \frac{v^2}{R}$$

$$= \frac{G}{g} A v^2 \, d\phi \quad . \quad . \quad . \quad . \quad . \quad (3)$$

At the end faces of the slice act the radial shearing forces S and S + dS and the normal tensions T and T + dT. Further, we must suppose at these faces couples of force producing bending moments M and M + dM.

For equilibrium we must have

$$\frac{G}{g} A v^2 \, d\phi - T \, d\phi - dS = 0 \quad . \quad . \quad (4)$$

$$dT - S \, d\phi = 0 \quad . \quad . \quad . \quad . \quad (5)$$

$$dM - R \, dT = 0 \quad . \quad . \quad . \quad . \quad (6)$$

The last equation is obtained by taking moments about the centre of the wheel. From (5) and (6) we get

$$dM - R S \, d\phi = 0 \quad . \quad . \quad . \quad (7)$$

Differentiating (4) with respect to ϕ, we get

$$\frac{d_2 S}{d\phi^2} = -\frac{dT}{d\phi},$$

or with the relation in (5)

$$\frac{d_2 S}{d\phi^2} + S = 0,$$

whence by integration

$$\frac{dS}{d\phi} = \pm\sqrt{c - S^2}$$

$$\sin^{-1}\frac{S}{\sqrt{c}} = c_1 \pm \phi$$

$$S = \sin(c_1 \pm \phi)\sqrt{c}.$$

Consequently from a known relation

$$S = a\sin\phi + b\cos\phi \quad . \quad . \quad . \quad . \qquad (8)$$

where a and b are new constants requiring to be determined. Using this value in (4)

$$T = \frac{G}{g}A\,v^2 - a\cos\phi + b\sin\phi \quad . \quad . \qquad (9)$$

and from (6)

$$M = \frac{G}{g}A\,R\,v^2 - aR\cos\phi + bR\sin\phi + c\ . \qquad (10)$$

where c is a new constant.

To determine these constants a b and c, consider the values of S, T and M for $\phi = 0$, and $\phi = 2a$,

For $\phi = 0$, $S_0 = b$.

For $\phi = 2a$, $S = a\sin 2a + b\cos 2a$, and since $S_0 = -S_a$, because at all the arms S has the same value,

$$S_0 - S_a = 2S_0 = -2S = -F$$

if by F we understand the tension in the arm due to the centrifugal force. Hence

$$b = -\frac{F}{2} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (11)$$

$$\frac{F}{2} = a \sin 2a - \frac{F}{2} \cos 2a$$

$$a = \frac{F}{2} \frac{1 + \cos 2a}{\sin 2a} = F/2 \tan a \quad . \quad . \quad (12)$$

Introducing these values of the constants

$$S = -\frac{F}{2} \frac{\sin (a - \phi)}{\sin a} \quad . \quad . \quad . \quad . \quad (13)$$

$$T = \frac{G}{g} A v^2 - \frac{F}{2} \frac{\cos (a - \phi)}{\sin a} \quad . \quad . \quad . \quad (14)$$

$$M = \frac{G}{g} A R v^2 - \frac{F R}{2} \frac{\cos (a - \varphi)}{\sin a} + c \quad . \quad (15)$$

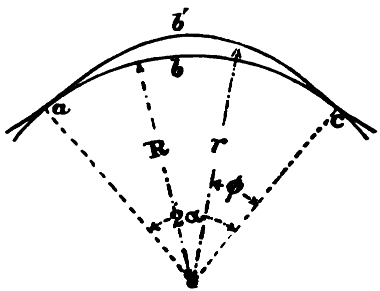


Fig. 179

It still remains to determine the constant *c*. Under the action of the bending moments the centre line of the rim, originally circular, as at *a b c*, takes a form *a b′ c*, fig. 179. The angle *a* must remain unchanged from conditions of symmetry. Using the ordinary equation to the elastic line and remembering that *r* and R differ only by a very small quantity,

$$M = \pm E I \frac{d_2 r}{(R\, d \varphi)^2} \quad . \quad . \quad . \quad . \quad (16)$$

where E is the coefficient of elasticity of the material and I the moment of inertia of the rim section about an axis

through its centre of figure. Using the value of M in (15) and integrating

$$\pm \frac{dr}{R\,d\varphi} EI = \frac{G}{g} A v^2 R^2 \phi + \frac{F}{2} R^2 \frac{\sin(\alpha - \phi)}{\sin \alpha} + c R \phi + \text{const.}$$

But for $\phi = 0$

$$\frac{dr}{R\,d\varphi} = 0$$

$$\text{const.} = -\frac{F}{2} R^2$$

Also for $\phi = 2\alpha$, $\dfrac{dr}{R\,d\phi} = 0$, and hence, inserting the value just found and reducing

$$c = -\frac{G}{g} A R v^2 + \frac{FR}{2\alpha} \quad . \quad . \quad . \quad (17)$$

Putting this value in (15) we get finally

$$M = \frac{FR}{2\alpha} - \frac{FR}{2} \frac{\cos(\alpha - \phi)}{\sin \alpha} \quad . \quad . \quad (18)$$

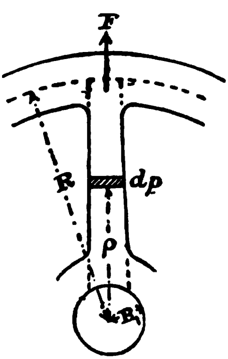


Fig. 180

Equations 13, 14, and 18 now give the shearing force S, the tension T, and the bending moment M, at any section of the rim, provided that the value of F is determined. To find this the stretching of the arm must be determined. Consider a slice of the arm between sections at radii ρ and $\rho + d\rho$, fig. 180.

Let A_1 be the mean section of the arm. Then the weight of the slice is $G A_1 d\rho$; its velocity is $v \rho / R$; and its centrifugal force is

$$dC_1 = -\frac{G}{g} A_1 d\rho \frac{v^2 \rho}{R} \quad . \quad . \quad . \quad (19)$$

The total stress on the section is therefore

$$C_1 + F = -\int_R^\rho \frac{G}{g} A_1 \frac{v^2 \rho}{R^2} d\rho + F$$
$$= \frac{G}{g} A_1 \frac{v^2}{R^2} \frac{R^2 - \rho^2}{2} + F.$$

The extension of the arm by this varying tension, which may be denoted by ΔR, is

$$\Delta R = \int_{R_1}^{R} \frac{G}{g} A_1 \frac{v^2}{R^2} \frac{R^2 - \rho^2}{2} \frac{d\rho}{A_1 E} + \int_{R_1}^{R} \frac{F\, d\rho}{A_1 E},$$

where for simplicity the arm is supposed to extend from the eye of the nave to the centre of the rim. Integrating and reducing

$$\Delta R = \frac{G}{gE} \frac{v^2 (R - R_1)}{3} \left[1 - \tfrac{1}{2}\frac{R_1}{R} - \tfrac{1}{2}\left(\frac{R_1}{R}\right)^2\right]$$
$$+ \frac{F(R - R_1)}{A_1 E} \quad . \quad . \quad . \quad . \quad . \quad (20)$$

As R_1 is small compared with R, this may be taken with accuracy enough to be

$$\Delta R = \frac{G}{gE} \frac{v^2 R}{3} + \frac{FR}{A_1 E} \quad . \quad . \quad . \quad (21)$$

But from the extension of the rim by the tension T a second equation can be obtained. The length of rim between two arms is $2 R \alpha$, and putting $\Delta (2 R \alpha)$ for the extension of this,

$$\Delta (2 R \alpha) \quad \int_o^\alpha \frac{T R\, d\phi}{A E}.$$

But 2α remains unchanged, so that $\Delta (2 R \alpha) = 2 \alpha \Delta R$. Hence, using the value of T in (12), and integrating

$$\Delta R = \frac{R}{E} \left\{ \frac{G}{g} v^2 - \frac{F}{2 A \alpha} \right\} \quad . \quad . \quad (22)$$

By elimination in Equations 22 and 20 we get

$$F = \frac{2}{3} \frac{G}{g} v^2 \Big/ \left(\frac{1}{A_1} + \frac{1}{2 A \alpha}\right)$$
$$= \frac{2}{3} \frac{G}{g} v^2 \frac{2 A_1 A \alpha}{A_1 + 2 A \alpha} \quad . \quad . \quad (23)$$

CHAPTER XII

VALVES, COCKS, AND SLIDING VALVES

VALVES

200. In all machinery put in motion by the action of a fluid (water or steam) or employed in pumping fluids, valves are required to regulate the admission and discharge of the fluid. With reference to the mode in which the motion of valves is obtained, they may be divided into four classes: (1) Valves opened and closed by hand; (2) Valves opened and closed by independent mechanism; (3) Valves opened and closed by mechanism so connected with the machine as to render the times of opening and closing synchronous with the motions of the machine; (4) Valves opened and closed by the action of the fluid.

In the case of valves opened and closed by the action of the fluid, the valve opens if the fluid moves in one direction and closes if it moves in the other direction. Independently closed valves, or valves controlled by mechanism, are more generally arranged to close a passage to the fluid, in whichever direction it tends to move.

Another convenient classification depends on the way in which the valve moves relatively to its seat. Thus we have: (1) Flap or butterfly valves, which rotate in opening; (2) Lift valves, or puppet valves, which rise perpendicularly to the seat; (3) Sliding valves, which open by moving parallel to the seat.

AUTOMATIC VALVES

201. The conditions to be fulfilled by a good automatic valve are these: (1) accurately timed opening and closing; (2) small resistance to the flow of the fluid; (3) tight closing. The tightness of the valve is mainly a question of fitting and the selection of suitable material for the valve seat. To secure small resistance to flow the valve must open fully, and this is a requirement somewhat in conflict with the condition of closing accurately at the proper time. The importance of the resistance at the valve varies very much in different cases, and a good deal of resistance may be allowed in some cases rather than incurring the risk of delayed closing of the valve, and consequent shocks. The question of the closing of the valve will be dealt with later. Very often, in quick-acting pumps, springs are added to hasten the closing of the valve at the change of stroke of the pump.

Fig. 181 gives a general view of the types of automatic valves. At B is the simplest form of leather-flap valve, the leather being stiffened with metal plates. Such valves work very well under low pressures, and when the number of beats per minute is not too great. A similar valve of brass in shown at C hinged to a metal seating *a*, which is fixed by wood wedges *b*, and can be lifted out through the aperture covered by the plate *c*. This is a common form of condenser foot valve. At A is an indiarubber valve which acts in a similar way. The indiarubber valve is in the form of a circular disc covering a gridiron seating. The bars of the gridiron support the flexible indiarubber, and should be placed closer the greater the pressure on the valve. The valve is held by a bolt at the centre, and takes a dished form in rising. A guard plate prevents the valve from rising too high. At D are two flap valves placed back to back, and then termed 'butterfly valves.' They beat on a brass seating, and have a guard to limit the lift. G is a mushroom lift

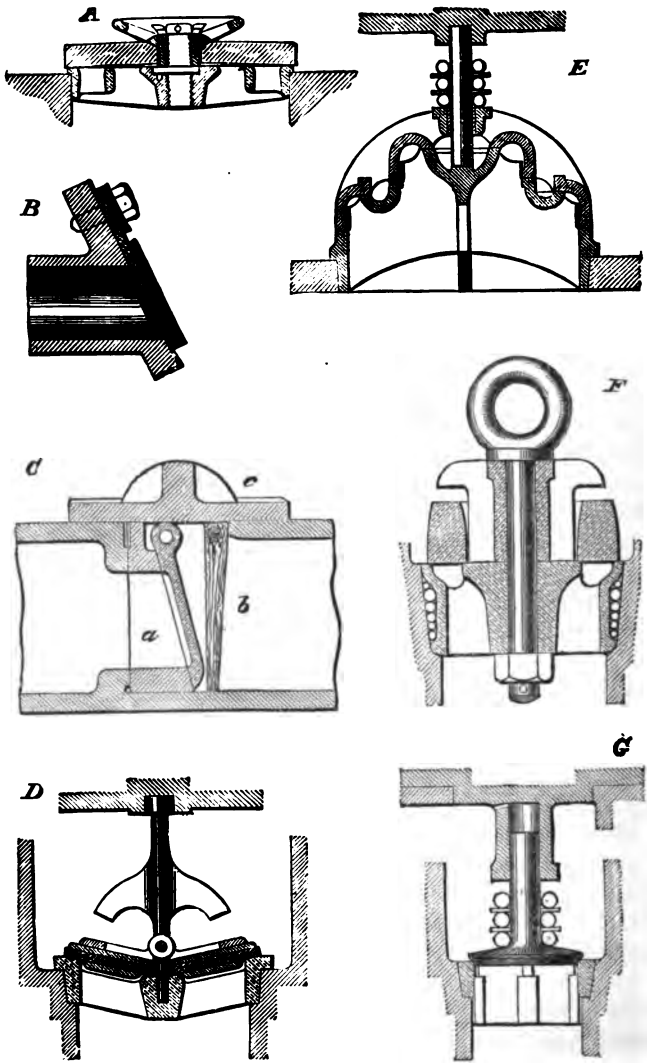


Fig 181

or puppet valve; it consists of a metal disc fitting accurately a flat, or more commonly, a conical seat. It is guided, in rising, either by a spindle or by three feathers underneath the valve, sliding in the valve seat. The bearing surface of the seat must be made narrow (about $\frac{1}{16}$ to $\frac{3}{16}$ in.), or it will not be tight. Consequently, as the valve is made larger, the intensity of the pressure on the seat increases. Hence there is a difficulty in using valves of this kind of large size. Further, the lift of such a valve should be proportional to its diameter, to give adequate waterway when open. But when the valve is large and the lift great, the shock of the valve in closing becomes severe and damages the valve and seating faces. At F is a ring valve, in which these objections are to some extent obviated. When open the water escapes at both the inside and outside edges of the valve, and hence for a given waterway only half as much lift is necessary as would be required by a mushroom valve of the same diameter. At E is a valve used for large pumping engines, which consists of two ring valves, and gives four edges for the escape of the water. At G and E the valves are provided with an arrangement to accelerate the closing of the valve, introduced by Mr. H. Davey. This consists of indiarubber washers, forming a spring. The valve has a certain amount of free lift, and the rest is obtained by compression of the indiarubber. In G the spindle often has three flats planed, so as to form passages for the fluid when the valve lifts.

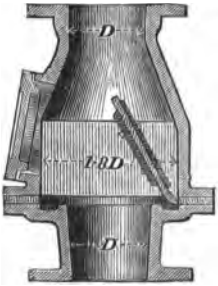


Fig. 182

202. *Flap or butterfly valves.*—Fig. 182 shows the simplest form of flap valve, formed of a leather disc, strengthened and stiffened by two plates of iron, of brass, or of lead, which at the same time give weight enough to the valve to close rapidly, when the

pressure beneath it ceases. A butterfly valve consists of two flap valves placed hinge to hinge, or sometimes edge to edge. In the latter position the direction of motion of the fluid is less interfered with. The lifting of the valve is usually restricted to an angle of 30° or 40°; width of seat $\frac{1}{8}\sqrt{D} + \frac{1}{8}$ or in some cases 0·1 D. Valves of this kind are most commonly lifted by the fluid.

203. *Indiarubber disc valve.*—A form of valve very extensively used for condensers and pumps consists of a

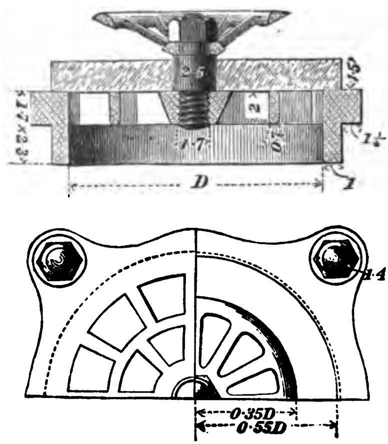


Fig. 183

circular disc of indiarubber, secured by a bolt at the centre, and resting on a brass grid which forms the seating. The indiarubber, being flexible, lifts easily from the grating, when any fluid pressure is applied beneath it, and closes again readily, and without violent shock, when the reflux begins. To prevent the indiarubber rising too high, a perforated guard plate is placed over the valve. Figs. 183 and 184 show two of these valves. In one the valve seat is attached to the cast-iron casing of the condenser by bolts, and the

indiarubber and guard plate are attached to it by a stud. In the other the seating, indiarubber, and guard plate are all secured by the same central bolt, which bears against a cross bar on the other side of the casing to that on which the valve is placed. In each of these figures the valve and guard plate are removed from one half of the plan, in order

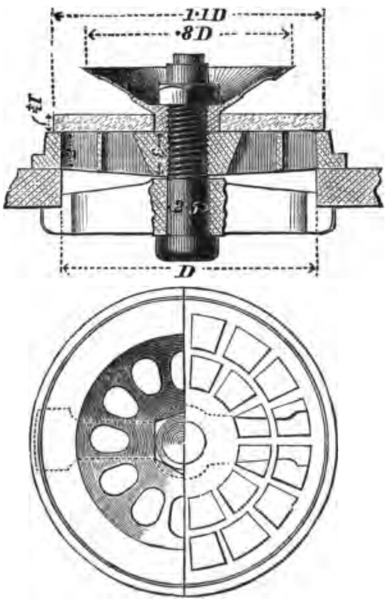


Fig. 184

to show the grating on which the valve rests. The indiarubber should not be too thin, $\frac{3}{4}$ in. to $\frac{7}{8}$ in. thickness is sufficient. The apertures of the grating should be so small that there is no great flexure of the indiarubber, and the grating area should be so large that the pressure with valve closed does not exceed 40 lbs. per sq. in.

It is more satisfactory to use several valves of 7 ins. to 9 ins. diameter than to use a single large one. Indiarubber valves should not be used for pressures above 100 lbs. per sq. in.

The guard plate is often of spherical form of a radius equal to the diameter of the valve. In other cases the guard plate is flat, and the indiarubber disc lifts bodily on the central stud. It is well to give a small lift in all cases before the valve reaches the guard plate. It is now common to place a spiral spring above the valve to insure more rapid closing.

204. *Throttle valve.*—The throttle valve used on many engines, which consists of a circular or square metal disc, capable of turning about an axis passing through it in the

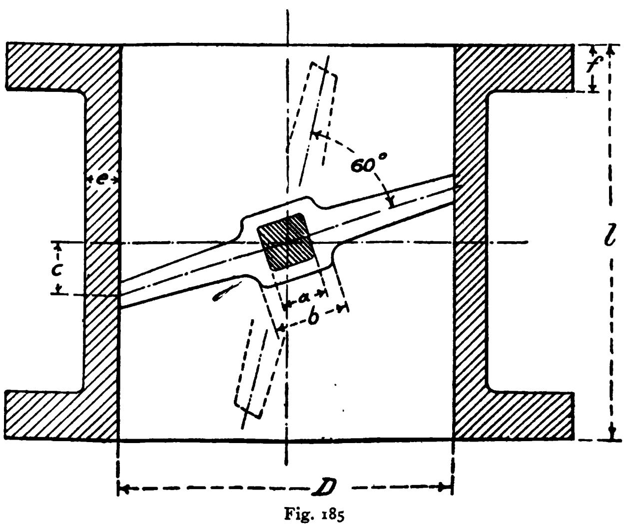


Fig. 185

direction of a diameter, is a kind of double-flap valve. The disc is placed in a pipe, and closes the passage-way when placed across the pipe, whilst it offers little resistance

when parallel to the axis of the pipe. This valve is an imperfect equilibrium valve, the pressure on one half partly balancing the pressure on the other, so that the force required to move the valve is only equal to the difference of these two pressures. The equilibrium is exact, however, only while the valve is shut or so long as there is no sensible current passing it. If a rapid current is established, the pressure on that half of the valve which first deflects the current is greater than on the other half, thus tending to close the valve.

Fig. 185 shows an ordinary steam throttle valve, in closed and open position. The diameter D, is sometimes a little larger than the steam-pipe diameter. The other dimensions may be as follows:

$$\text{Diameter of steam pipe} = a$$
$$D = 1{\cdot}1\,d + \tfrac{1}{2}$$
$$a = 0{\cdot}16\,d + \tfrac{1}{16}$$
$$b = 0{\cdot}25\,d + \tfrac{1}{8}$$
$$c = 0{\cdot}2\,d$$
$$e = \tfrac{1}{16}\,d + \tfrac{3}{8}$$
$$f = \tfrac{1}{16}\,d + \tfrac{5}{8}$$
$$l = 1\tfrac{1}{4}\,d + 1.$$

205. *Lift or puppet valves.*—These are very various in form, the simplest being a circular disc, usually of metal, with a flat or bevelled edge, which fits a circular metal seating (fig. 186). These valves are generally placed with the axis of the valve vertical, so that their weight tends to keep them closed; but they may be otherwise placed if springs or rods are used to close them.

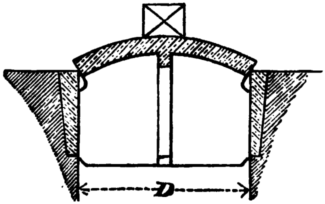


Fig. 186

Let D be the diameter of the valve seating, then the waterway through the valve seating is $\frac{\pi}{4}$ D². If h is the height of lift, the waterway round the edge of the valve when open is π D h. In order that these two may be equal, the lift h must be one-fourth of the diameter D. It is only possible to give so great a lift in small valves, because with a great lift the valve acquires too much velocity in closing, and there is a violent shock, causing vibration and damage to the valve seat. It is necessary often to restrict the lift to a less amount than would otherwise be desirable, and then the resistance to the passage of the fluid is increased. Hence it is better to use two valves of diameter 0·707 D than a simple valve of diameter D. For pumps the mean velocity of water through the seating is generally limited to 3 feet per sec. Let s be the width of the bearing surface of the valve and seat, measured on a plane at right angles to the direction of lift of the valve. Then π D s is nearly the effective area of the bearing surface. If p is the greatest difference of pressure on the two sides of the valve, then p/π D s is the crushing pressure on the narrow valve seating. Experience shows that it is not desirable that this pressure should exceed certain limits.

Greatest pressure on surface of valve seat

Gunmetal	2,000 lbs. per sq. in.
Phosphor bronze . .	3,000 ,, ,, ,,
Cast iron	1,000 ,, ,, ,,

These values determine the minimum width of valve seat. In valves for water the part of the seat on which the valve beats is sometimes of lignum vitæ, sometimes of leather, sometimes of an alloy of lead and tin. For air-compressor valves tin is used, for steam gunmetal.

Fig. 187 shows a conical disc valve and casing. The valve is guided in rising and falling by three feathers, which fit the cylindrical part of the seating, and are shown in the plan of the valve. The lift of the valve is limited by a pro-

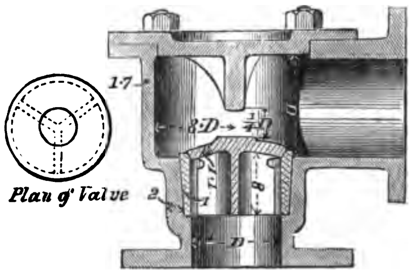


Fig. 187

jection on the cover of the casing. The fitting part, or face of the valve, should be narrow, as it is then more easy to make it tight. It must, however, present area enough to resist deformation by the hammering action of the valve. The inclination of the face of the valve is usually 45° with the axis of the valve. The horizontal projection of the bevil may have a width $\frac{1}{6}\sqrt{D} + \frac{1}{16}$, or may be determined for the pressure. Conical disc valves may either be actuated by the fluid pressure or by hand. In the latter case they are opened and closed by a screwed rod.

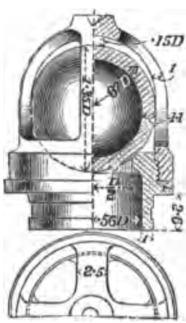


Fig. 188

Fig. 188 shows a ball valve, which acts in precisely the same way as a disc valve, except that, as the surface of the ball is accurately spherical, it fits the seating in every position. The only guide required is, therefore, an open cage, which limits the play of the valve. Such valves are often used for small

fast-running pumps. To lighten the ball it is often made hollow.

The proportions of valves depend partly on the diameter. Thus the area of the waterway must be constant, and the linear dimensions of the casing are proportional to the valve's diameter. But the thicknesses are in most cases excessive as regards strength, especially in small valves, and do not increase in the same proportion as the diameter. For these the empirical proportional unit

$$t = \tfrac{1}{5}\sqrt{D}$$

will be adopted, where D is the diameter of the valve.

Mechanically Controlled Valves

206. *Screw-down valve.*—Fig. 189 shows a lift valve arranged to be worked by hand instead of automatically. Such screw-down valves are now largely used in place of cocks, being tighter and free from liability to stick. The valve rod passes through a stuffing-box, and is made so long that the screwed part does not come in contact with the stuffing-box packing. The valve is fixed on the end of the rod by a pin in such a way that it can turn round. It does not then grind on the seat when screwed down. For steam the valve faces are of gunmetal; for water one of the valve faces may be of leather, of indiarubber, of vulcanised fibre, or, in large valves, of hippopotamus hide.

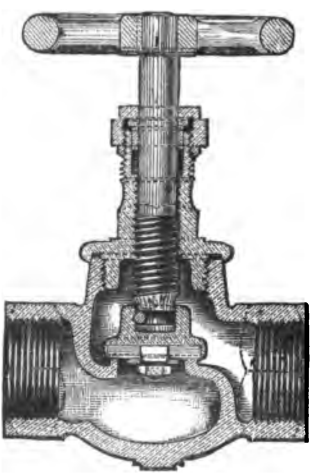


Fig. 189

When used for water, it is convenient that the pressure should be on the under side of the valve when closed; then there is less danger of leakage at the stuffing-box.

207. *Double-beat or Cornish valve.*—The objection to a great lift in metal valves has already been mentioned. In the double-beat valve, two valve faces are obtained in the same valve, and two annular areas are opened when the valve lifts. For a given area of opening, the lift is only about one-half that of a simple lift valve of the same diameter. Fig. 190 shows a Cornish valve for a pumping engine. This valve is raised and lowered by a cam acting on an arrangement of levers. The lower seating is carried directly by the steam-chest. The upper seating is carried by four feathers or radiating plates cast with the lower seating. The valve itself is ring-shaped. Since the two valve faces are nearly of the same diameter, another subsidiary advantage is gained in this form of valve. The valve is pressed down on its seat, partly by its weight, partly by the steam pressure acting on one side of it. If the valve were a simple disc valve, the steam pressure would act on an area $\frac{1}{4}\pi D^2$, where D is the diameter of the valve. As the valve is annular, however, the steam presses only on the area $\frac{1}{4}\pi(D_1^2 - D_2^2)$ where D_1 and D_2 are the diameters of the two faces. Hence the valve is easily lifted against the steam pressure. If the valve is to lift so that the area through

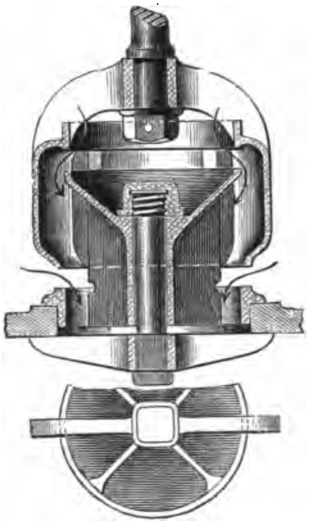


Fig. 190

the valve is equal to the area through the lower seating, the height of lift must be determined by the equation

$$\pi (D_1 + D_2) h = \frac{\pi}{4} D_1^2$$

$$h = \frac{1}{4} \frac{D_1^2}{D_1 + D_2}.$$

208. *Lord Kelvin's valves.*—In the case of most valves closed by a screw, the valve comes down on the seating always in the same position, and having once touched the seating any further motion is arrested. If there is any obstruction on the seating it at once becomes jambed between valve and seat, and if the valve and seating are indented, the indents always come together again whenever the valve closes. In some automatic pump valves arrangements are made causing the valve to twist a little in rising, so that it does not always beat on the same place. It occurred to Lord Kelvin that a valve might be arranged so that there was a definite grinding action of the valve on its seating every time it was closed, this grinding tending to keep the valve and seating true, and to obliterate any accidental damage.

Fig. 191 shows a 2-in. check valve arranged in this way, which is manufactured by the Palatine Engineering Company, Liverpool. The stuffing-box is also got rid of, so that there is no perishable material in the valve. The valve is shown at *v*, and is quite loose from the screw plug above. Two lugs on the valve project up into a slot between cheeks, *a*, in the lower part of the screw plug, so that as the screw plug turns the valve must turn with it. A pin *b* with a spiral spring resting on a shoulder presses the valve down perfectly centrally. As the screw plug *s* is turned, the valve is forced down by the spring, and after it reaches its seat it still goes on turning till the central pin *b* jambs between the valve and screw plug. During this turning the self-grinding of the valve faces goes on. In the upper part of the valve

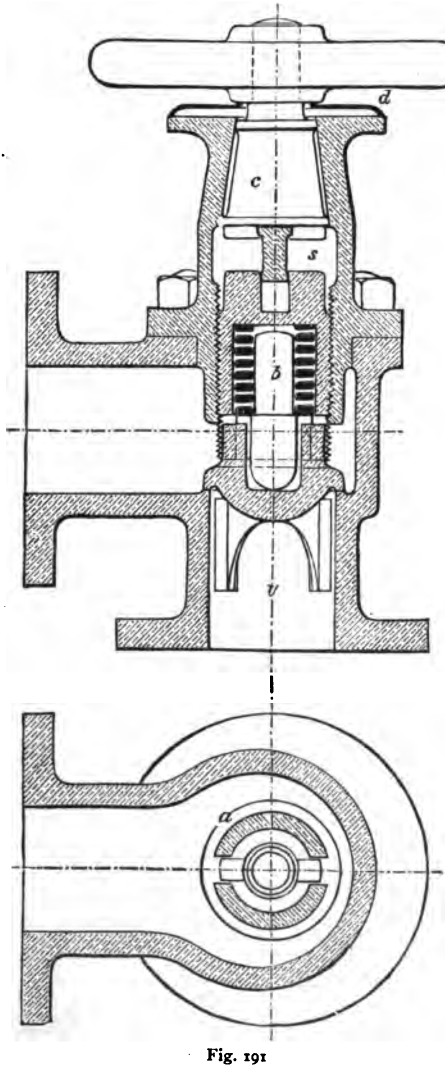


Fig. 191

casing is a conical plug *c* ground accurately into the casing. This replaces the stuffing-box, and is held up by a phosphor-bronze disc spring *d*. A loose link between the conical and screw plugs transmits the twisting effort from the hand-wheel to the screw plug.

It will be seen that the valve is free from any packing needing to be replaced, the parts are all strong and free from any liability to be bent by rough usage, and they can all be completely fitted by machinery.

The same principle has been applied for ordinary draw-off cocks for houses. One of these is shown in fig. 192. The valve is separate from the handle and screw plug, but at the same time is forced to rotate with it, having two lugs which slide in slots in the screw plug. A phosphor-bronze spiral spring in the cylindrical recess presses down the valve to the bottom of its range. In closing the draw-off cock, the loose valve first comes down on the seat. Continuing to turn the handle, the spiral spring is compressed and at the same time the valve is rotated on its seating while subject to the pressure of the spiral spring. In this way a slight grinding action occurs, tending to keep the valve and seat true, every time the valve is closed. In use these valves are found to keep in perfect order, and similar draw-off cocks have been opened and closed by mechanical means at a rate and for a time which would represent fifty years of ordinary wear. At

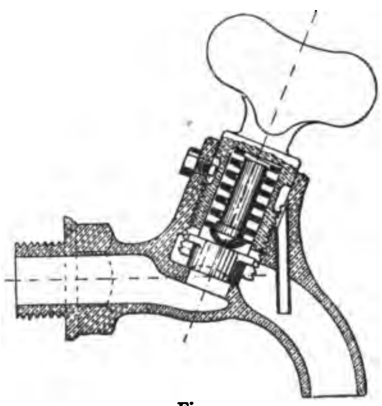


Fig. 192

the end of this trial both valve and seating were in perfect order.

Besides the self-grinding action, the stuffing-box is got rid of in this draw-off cock. A small pipe leads any leakage which passes the screw thread back into the current of water flowing out of the cock. Thus there is no perishable material or packing in the cock, and there is no reason it should not remain permanently in perfect order without any regrinding or attention.

209. *Controlled hydraulically-moved valves.*—In pressure engines and some other cases it is convenient to use valves moved by the action of the fluid, but under conditions which place under control the times at which the valves open and shut. The general principle is to connect the valve to a piston in the valve chamber, and to use a small subsidiary slide valve to admit or release the pressure on the back of the piston.

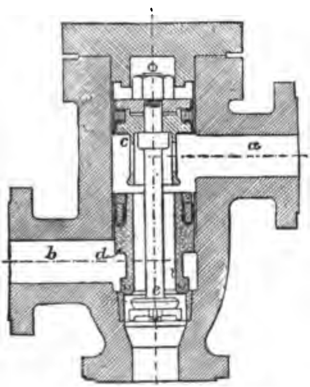


Fig. 193

Fig. 193 shows a valve of this type designed by Mr. Henry Davey ('Proc. Inst. Mech. Eng.,' 1880, p. 245). The water is admitted to the pressure engine through the passage *a*, and is exhausted through the passage *b*. The valve chamber contains an ordinary puppet valve *e*, rigidly attached to the piston *c*, and a second valve *d* which has two conical seatings, an inner and outer, on its bottom surface, and which itself forms a piston or plunger. The under surface of the piston *c* and the upper surface of the valve *d* are always subjected to the water pressure. To the upper surface of the piston *c* the pressure is alternately admitted and released by a small

subsidiary slide valve. In the position shown the pressure is on the top of piston *c*, and water is being admitted through the centre of valve *d* to the engine cylinder. If now the pressure on the top of *c* is released, it will rise, lifting the puppet valve *e* so as to close the admission of water to the engine cylinder, and afterwards lifting *d* and *e* together so as to open the exhaust. On readmitting water to the top of *c*, the reverse action takes place. As *c* descends the exhaust first closes, from *d* coming down on its outer seating, and then *e* opens. The areas of *d* are so arranged that there is always an unbalanced pressure acting downwards.

Sliding Valves and Cocks

210. *Sliding valves.*—Sliding valves are more commonly used than any others for stop valves, which are opened and closed by hand; they may be divided into two classes: (1) those with plain faces and seats; (2) those with cylindrical or slightly conical faces and seats. The former class includes engine slide valves and the sluices, often very large, which are used as stop valves on water mains. The latter class includes the hand-worked valves commonly known as 'cocks.'

Sliding sluice valves.—Fig. 194 shows a form of sliding sluice valve often used on water mains and often constructed of very large size. At first these valves were made with only one face, and were kept tight by the water pressure on the back. Now they are almost always double faced, as in fig. 194, and close the pipe against flow in either direction. The double-faced valve is due to Mr. Nasmyth. The valve is wedge-shaped, so as to lift without much friction when once started. It is desirable to make the screw of gunmetal to prevent oxidation. In large valves the screw and its nut are outside the valve casing, where they are more accessible and more easily lubricated.

211. The term 'cock' is sometimes used for any valve opened or closed by hand, but it is more properly restricted to valves which are nearly cylindrical, and which rotate in

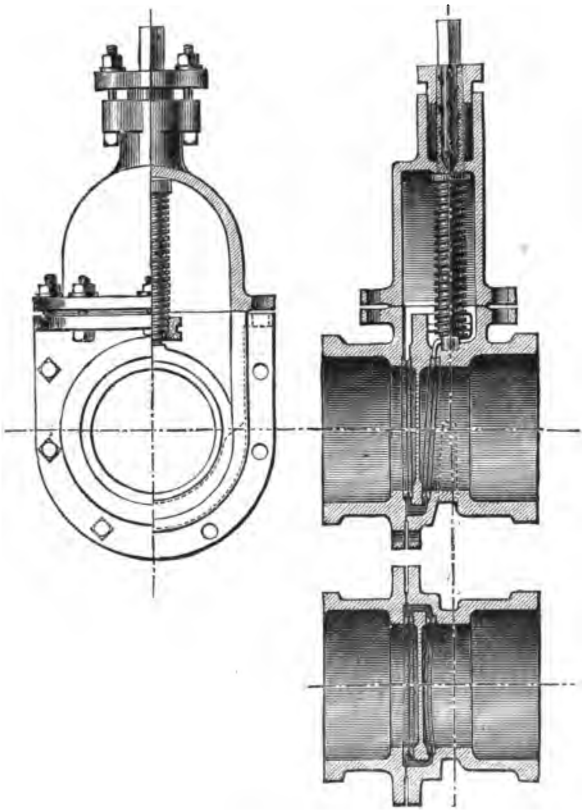


Fig. 194

seatings of the same figure. In ordinary cocks, the seating is a hollow, slightly conical casing, and the valve, which is termed a 'plug,' fits accurately in the seating. The passage-

way for the fluid is formed through the plug (fig. 195). By rotating the plug in one direction its apertures are made to coincide with the entrance and discharge orifices of the casing. The cock is then open. By rotating it in the other

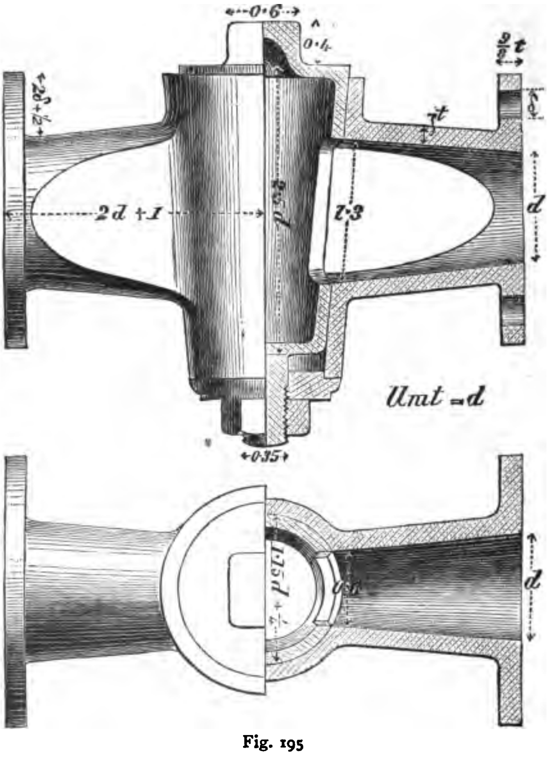


Fig. 195

direction the holes in the plug are brought over blank parts of the casing and the cock is closed. The slight taper given to the plug enables it to be accurately fitted, by turning and grinding, to its seating, and it can from time to time be refitted. Each time it is refitted the plug sinks a little lower

in the casing. If the plug were cylindrical this refitting would be impossible. The objection to the use of cocks in many cases, especially for pipes of large size, is that a good deal of power is required to move them, and this is partly due to the conical form, which increases the friction.

The simplest cocks have a solid plug, which is kept in place by a screwed end. When the cock is small, the casing has a screwed socket on one side and a screwed end on the other, for the attachment of the cock to the pipes with which it is connected. But in larger cocks, the inflow and outflow orifices are provided with flanges (figs. 195, 196). In the cock shown in fig. 195, there is one defect pointed out by Mr. Druitt Halpin—viz. that the plug cannot conveniently be ground after wear, as shoulders would form on the plug at the top and on the shell at the bottom in grinding. By undercutting, as shown in fig. 196, this defect is obviated.

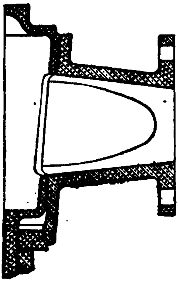


Fig. 196

For small brass cocks, with socket and spigot ends, the following proportions may be adopted:—

Diameter of waterway of cock $= d$
Diameter of plug at centre $= 1{\cdot}15\,d + \frac{1}{4}$
Height of hole in plug $= 1{\cdot}3\,d$
Width of hole in plug $= 0{\cdot}6\,d$
Total length of tapered part of plug $= 2{\cdot}5\,d$ to $3\,d$
Side of square for handle $= 0{\cdot}7\,d$
Height of square for handle $= 0{\cdot}4\,d$
Thickness of metal $= 0{\cdot}2\,d + \frac{1}{16}$
Diameter of plug screw $= 0{\cdot}35\,d$
,, screwed end $= d + \frac{5}{16}$
Internal diameter of socket end $= d + \frac{3}{16}$
Total length $= 3{\cdot}3\,d$
Taper of plug = 1 in 12 to 1 in 9 on each side.

For cocks with flanged ends, like that shown in fig. 195, the proportions are the same. When the cock is not very small the thickness is best obtained from the rule—

$$t = \tfrac{1}{8}\sqrt{d} + \tfrac{3}{16} \text{ for cast iron}$$
$$= \tfrac{1}{12}\sqrt{d} + \tfrac{3}{16} \text{ for brass.}$$

Some proportions are marked on the figure.

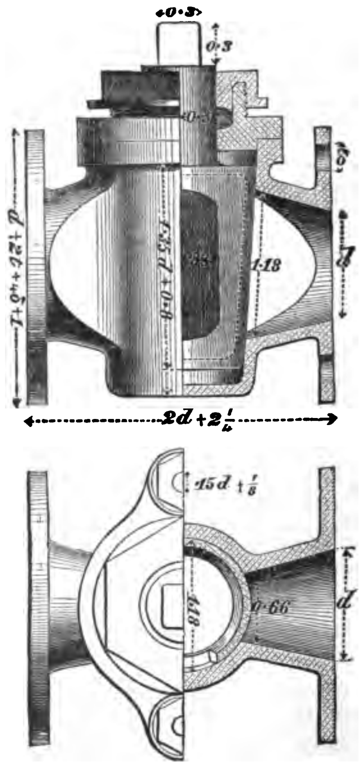


Fig. 197

212. Large cocks connected with boilers, and in situations where failure would be dangerous, are best made with

closed ends, as shown in fig. 197. The proportions of cocks of this description are a little different.

Diameter of waterway $= d$

Thickness of plug (brass) $= 0{\cdot}12 \sqrt{d} + \frac{1}{8}$

" " (cast iron) $= 0{\cdot}18 \sqrt{d} + \frac{1}{4}$

" shell (brass) $= 0{\cdot}18 \sqrt{d} + \frac{1}{8}$

" " (cast iron) $= 0{\cdot}25 \sqrt{d} + \frac{1}{4}$.

The shell may be reduced to the same thickness as the plug in parts which do not require to be turned.

Diameter of plug at centre $= 1{\cdot}18\, d$
Size of openings in plug $= 1{\cdot}18\, d \times 0{\cdot}66\, d$
Overlap of plug at top and bottom $= 0{\cdot}08\, d + 0{\cdot}4$
Depth of stuffing-box $= \frac{1}{8} d + \frac{1}{2}$
Depth of gland $= \frac{1}{20} d + \frac{1}{4}$
Diameter of studs in cover $\frac{1}{8} d + \frac{1}{8}$
Taper of plug = 1 in 12 on each side.

Some other proportions are marked on the figure.

213. *Dewrance's asbestos packed cocks.*—Ordinary taper plug cocks, when used for steam purposes, often give trouble by sticking fast or leaking. The expansion of the plug causes sticking, and grit between the surfaces causes abrasion and leakage. Hence Messrs. Dewrance have introduced cocks packed with asbestos like a stuffing-box. The plug is smaller than the shell, so that there is room for expansion. The plug is packed with strips of asbestos in recesses running down the shell. There are four grooves down the shell, and a recess also at the bottom under the plug.

Theory of the Action of Automatic Valves

214. An automatic valve is one which is opened by the action of the fluid pressure and closed by the action of its own weight or by a spring. To open the valve there must be an excess of fluid pressure below it. While open, the

effective closing force, consisting of the weight of the valve (immersed in water) together with any applied force, such as a spring pressure, must be balanced by the forces due to the deviation of the water round the valve, producing an excess of pressure below, and probably, in consequence of the curvature of the stream lines, a diminished pressure at the back. Lastly, as the flow through the valve diminishes, the effective closing pressure should be sufficient to bring it to its seat before any reflux can take place.

Taking the case of a pump valve, to be more definite, the valve ought to open and close immediately at the turn of the stroke of the pump bucket or plunger. It should cause as little loss of head as possible while open; it should be tight against leakage when closed. The last condition depends chiefly on accurate workmanship. If the velocity through the valve is not excessive (usually 2 to 6 feet per sec.), the resistance of the valve is not generally a serious item of loss of efficiency. Consequently the accurately timed opening and closing of the valve are the considerations practically most important.

For the valve to open easily and fully the effective closing force must be as small as possible. On the other hand, if the closing force is too small, the valve closes late, and thence occur shocks, the most serious evil arising in the use of valves. Further, if the valve is of a given weight and lifts during flow to a given height, its closing, due to the acceleration caused by the effective closing force, requires a definite time. Consequently for any valve there will be a limit to the number of strokes per minute of the pump, beyond which the time allowed for closing will be insufficient, the valve will close late, and shocks will arise. According to experiments of Bach, it appears that it is prejudicial to try and get rid of shock by limiting the lift of the valve by fixed stops. Quicker closing can only be secured by increasing the effective closing force.

A remarkable series of observations has been made by

Prof. Riedler[1] on the action of pump valves, of very various kinds. He used an indicator somewhat like an engine indicator to obtain curves of pressure during the action of the valves. It is a result of these observations that he has traced shocks, caused by the action of valves in pumping engines, chiefly to the suction valves, and he regards the accurate action of these as of the highest importance. Suppose the suction valve closes late, that is, after the pump plunger has returned though part of its stroke and acquired some velocity; then, when at last the suction valve closes, and the delivery valve lifts, the entire column of water above the delivery valve has to be suddenly accelerated. The shock is greater, the greater the velocity of the plunger at the moment and the greater the length of the column. Very heavy valves begin to close before the turn of the stroke, and then shocks are less possible. Quietness of action is secured, though at the expense of some additional resistance at the valve.

215. *The excess of pressure due to the area of valve seat.*—If we suppose a valve, fig. 198, to fit its seat so tightly that there is no pressure between the faces of valve and seat, the pressure p_1 above the valve acts on a larger area than the pressure p below the valve. Neglecting the weight of the valve, and the fluid pressure between the seating and valve at the moment when the valve opens,—

$$p\,d^2 = p_1\,d_1^{\,2}$$

$$p - p_1 = p_1 \frac{d_1^{\,2} - d^2}{d^2},$$

where $p - p_1$ may be termed the excess pressure due to the area of valve seat. If the assumption that there is no pressure between the valve and seat were true, this excess pressure would be in many cases important, and, in fact, it

[1] 'Indicator-Versuche an Pumpen,' von A. Riedler, München, 1881; Schröder, 'Zeitsch. des Vereines Deutscher Ingen.' May 1902.

has been supposed to have a great influence on the action of the valve. Experiments with indicators, especially those of Prof. Riedler, show that this excess pressure does not exist, or at all events is much smaller than it would be if the above assumption were true. Suppose, however, there is a fluid layer between the faces of valve and seat. Then the pressure must vary from p at the inner to p_1 at the outer edge, and its mean value cannot be very different from $\frac{1}{2}(p+p_1)$. Then, still neglecting the weight of the valve, when the valve opens,

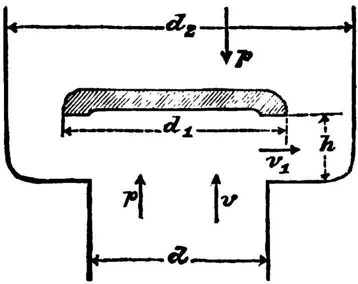


Fig. 198

$$p\,d^2 + \tfrac{1}{2}(p + p_1)(d_1^{\,2} - d^2) = p_1 d_1^{\,2}$$

which is only satisfied if

$$p = p_1,$$

so that there is no excess pressure due to the seating, a result which agrees much better with experience.

Effective closing force of valves in terms of the velocity of the water.—Consider the conditions of equilibrium when the valve is open. Let the dimensions be taken in feet the pressure in lbs. per sq. foot, the velocities in feet per sec. Let G be the weight of a cubic foot of water (62·4 lbs.). Let q be the effective force tending to close the valve in lbs. per sq. foot of its projected area. This consists of the weight of the valve in water (about seven-eighths of its weight in air) together with any applied closing force, divided by the area of the valve.

Since the volume of flow upwards through the passage

under the valve at velocity v must be equal to that horizontally between valve and seat,

$$\pi d_1 h v_1 = \frac{\pi}{4} d^2 v \quad . \quad . \quad . \quad (1)$$

$$\frac{v_1}{v} = \frac{d^2}{4 d_1 h}$$

But

$$v_1 = \sqrt{2g\frac{p-p_1}{G}}$$

$$= \sqrt{p-p_1} \text{ nearly} \quad . \quad . \quad (2)$$

Putting this value in (1),

$$d_1 h \sqrt{p-p_1} = \tfrac{1}{4} d^2 v.$$

But since there is equilibrium

$$p - p_1 = q \text{ lbs. per sq. ft.}$$

$$4 d_1 h \sqrt{q} = d^2 v$$

$$v = 4 \frac{d_1 h}{d^2} \sqrt{q}.$$

Let $d_1 h/d^2 = k$, the ratio of area of opening of valve to area of passage below valve. Then

$$q = \frac{v^2}{16 k^2}$$

$k =$	1	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
$q =$	$\cdot 062 v^2$	$\cdot 111 v^2$	$\cdot 25 v^2$	v^2 lbs. per sq. ft.

Taking $v = 3$ and 6 feet per sec., and reducing to lbs. per sq. in.

$k =$	1	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$		
$v = 3$; $q =$	·0039	·0069	·0156	·0625	lbs. per sq. in.	
$v = 6$; $q =$	·0156	·0276	·0724	·2500	"	"

which shows that the valve must be of small weight to give an area of lift equal to the passage under the valve.

This theory must, however, be taken only as a rough approximation. The form of the valve, affecting the curvature of the stream lines in the valve-box, may greatly influence the lift for any given velocity of flow.

Some experiments by Prof. Bach[1] on small valves show that if Ω is the area through the valve seat, ω the area between the valve and seat, v the velocity through the valve seat, then the effective closing force is given by an equation of the form

$$q = \frac{v^2}{2g}\left[\alpha + \beta\left(\frac{\Omega}{\omega}\right)^2\right].$$

Let s be the length of stroke of the pump and n the number of strokes per minute; then for any given description of valve the limit of speed at which the valve closes quietly is given by an equation of the form

$$n^2 s = \text{constant},$$

and at that limit the effective closing force is given by the equation

$$q = n^2 s \times \text{constant}.$$

216. *Prof. Riedler's valves.*—As accurate closing of valves is so important, and as the conditions to secure this conflict with the conditions of easy and full opening, it has occurred to Prof. Riedler to arrange valves so that they open very easily and automatically, and then to secure the closing at the proper moment by mechanism. He has applied such valves with very great success, both in pumping engines and air compressors.

The valves of a ram pump, for instance, are closed by mechanically moved fingers. These stand clear of the valves on the suction and delivery strokes respectively, but positively close the valves soon enough to prevent

[1] 'Versuche über Ventilbelastung,' Berlin, 1884; 'Zeitsch. d. Ingenieure,' 1886.

pounding on the seat when the motion of the ram is reversed. As the pounding action is prevented, comparatively large valves of a simple kind can be used, in

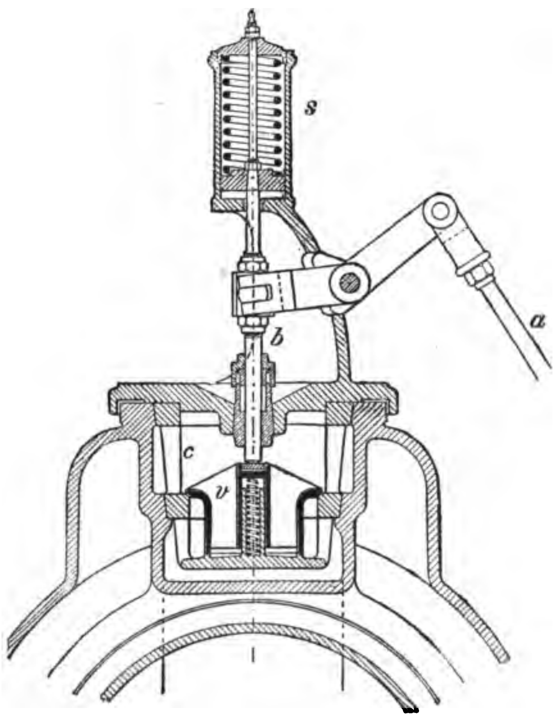


Fig. 199

place of the multiple valves which are necessary when the closing of the valve is automatic.

Fig. 199 shows a valve for an air compressor. The valve *v* is a double-beat valve, and *c* is the valve seating. In the centre of the valve is a helical spring which presses the valve upwards, almost but not quite balancing its weight.

Above the valve is a rod *b*, not attached to the valve, and driven by an eccentric or cam connected with the lever and rod *d*. This rod, except when lifted by the eccentric, is strongly pressed down by the spring *s*. The eccentric lifts the rod *b* before the time of opening of the valve, so that it opens by the action of the fluid pressure automatically. When the valve is to be closed, the eccentric allows *b* to force the valve down by the action of the spring *s*.

CHAPTER XIII

VALVE GEARS

217. The proper action of a steam engine depends on the exactness with which the operations of admitting steam to the cylinder and exhausting it are timed to accord with the positions of the mechanism. The economy of the engine depends greatly on the suppression of leakage and the arrangements for reducing clearance and wire-drawing, and for obtaining the best ratio of expansion. Consequently the design of the distributing valves on the cylinder, and of the mechanism for actuating them or the valve gear, is of the highest importance.

218. *Methods of varying the engine power to meet varying demand.*—In most cases the work to be done by an engine varies and arrangements must be made to adjust the energy developed in the engine to the work to be done. There are three ways of varying the engine power:—(1) The speed of the engine may be varied according as more work or less work has to be done. This method is used in engines which pump water against a fixed head; (2) The initial pressure in the cylinder is varied, partly by adjusting the boiler pressure and more directly by a throttle valve which introduces a resistance to the flow of steam and acts as a pressure-reducing valve. Generally the throttle valve is controlled by a governor which keeps the engine speed approximately constant; (3) The initial pressure and

speed are kept constant, and the point of cut-off is varied, so that the amount of steam admitted to the cylinder is reduced as the work to be done diminishes. If it were not for cylinder condensation this would be much the most economical method of regulating engines. But as the loss due to cylinder condensation increases as the ratio of expansion increases, part of the advantage of varying the cut-off is neutralised; and in some cases regulation by throttling is not only simpler but is also more economical than regulation by varying the cut-off. In some cases the valve gear is arranged so that the cut-off can be varied from time to time by hand adjustment. In others the gear is so arranged that the governor controls the point of cut-off of steam.

219. *Types of steam-distributing valves.*—There are, broadly, two classes of steam-distributing valves, sliding valves and lifting or mushroom valves. (1) Sliding valves include ordinary slide valves with flat faces, which reciprocate over a seating in which the steam ports are formed; Corliss valves which have cylindric faces and oscillate about the cylinder axis and piston valves, which are cylindric also, but which reciprocate in the direction of the axis of the cylinder. (2) Lifting valves are usually double-beat valves of the form shown in fig. 190, p. 313. It is now known that with all forms of sliding valve there is some leakage, however well they are fitted, and the leakage is greater with wet or saturated steam than it is with superheated steam. Double-beat valves if well fitted are probably generally tighter than slide valves. Simple slide valves are not well adapted to regulating by varying expansion. In engines working with varying expansion double slide valves are used, the ordinary slide completely controlling exhaust and a second expansion slide regulating the point of cut-off.

220. *Types of valve gear.*—Valve gears are either positive gears definitely controlling the distributing valve in all positions, or are trip gears which open the steam port

positively, but at the point of cut-off release the valve so that it closes the port by the action of a spring, the rapidity of closing being usually governed by a dashpot or air cushion. Trip gears are specially suitable when the point of cut-off is to be controlled by the speed governor. The scope of this treatise only permits an examination of the simpler valves and gears.

221. *General considerations on valves and gears.*—The ordinary slide valve is the simplest of distributing valves and the most generally used, but it has defects. The friction of the valve is great, especially with high steam pressures, and this necessitates heavy gear and causes wear and waste of energy. In most forms of slide valve the clearance between valve and cylinder is large, and this involves waste of steam. The *Corliss valve* fits well on its seating and is specially convenient when there are separate steam and exhaust valves, whether driven positively or by trip gear. With four Corliss valves to the cylinder the clearances may be very small.

Piston valves are balanced as regards steam pressure, and move with little friction. But there are difficulties in making them as steamtight as other valves, and except when placed in the cylinder ends they have large clearances. As they cannot lift from the seatings like ordinary slide valves, they sometimes have given trouble when there is water in the cylinder which is unable to escape. Double-beat valves are nearly frictionless and can be designed with small clearances. Being balanced, or very nearly so, they can be driven by light gear. They are sometimes driven positively, with some form of link to vary the cut-off. More commonly they are actuated by a cam motion, which permits very wide variation of cut-off, secures rapid opening and closing, and is easily put under control of the speed governor. Double-beat valves are, however, rather expensive to construct, and are liable to hammer on the seatings unless controlled by well-adjusted air cushions. To reduce clearance, piston and

Corliss valves are now often placed in the cylinder ends. The Willans central valve is a piston valve placed in the centre of the cylinder in a hollow piston rod, and it has very small clearances.

222. *Ordinary or locomotive slide valve.*—In its simplest form the slide valve consists of a dish-shaped rectangular piece (fig. 200), the face of which is accurately planed and sometimes scraped to a true plane surface. The valve slides upon a seating, also accurately planed, in the steam chest, and termed the *cylinder face* (§ 37). In the cylinder face are formed three *ports*, two communicating with passages leading to the ends of the cylinder, and termed *steam ports*, the third communicating with the atmosphere or condenser, and termed the *exhaust port*. The outside edges of the valve which control the steam admission are called the *steam edges*; and the inside edges, which control the exhaust, are termed the *exhaust edges*. The slide valve is pressed down on its seating by the excess of steam pressure on its back over that on its face (§ 123), and leakage is prevented by the accuracy of fitting of the valve and its seating.

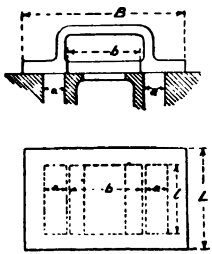


Fig. 200

The ordinary form of the valve in longitudinal section is D-shaped, as shown in fig. 200. It has two flat faces which, when the valve is in mid position, cover the steam ports, the arched part of the valve covering at the same time the exhaust port. If the valve moves in either direction from mid position it uncovers one steam port and admits steam to one end of the cylinder. At the same time the other steam port is put in communication with the exhaust passage. The reciprocating motion of the valve which opens the ports alternately is effected by an eccentric (p. 183), which may be regarded as a very short

crank, keyed on the same shaft as the engine crank. It is obvious that the greatest travel of the valve each way from mid position is equal to the radius of the eccentric, except when modifying levers are interposed.

Slide valves have been made of cast iron, of gunmetal, and of phosphor bronze. The wear of cast-iron valves appears to be one-third less than that of ordinary gunmetal valves, and the friction is less also. The faces of the valve and seating are sometimes scraped to true planes. It appears to be sufficient, however, in valves of moderate size, to plane the surfaces so that when in position the direction of planing for the valve is at right angles to that for the seating. They then wear to good surfaces.

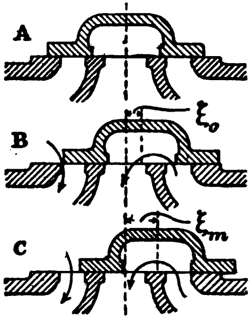


Fig. 201

Fig. 201 shows a slide valve in three positions. At A the valve is in mid position, the centre line of valve coinciding with the centre line of the cylinder face. At B the valve has already opened one port a little to steam and the other to exhaust. The valve has travelled a distance ξ_0 from mid position. This is the position when the crank is at the dead point and the piston stroke beginning. The amount of the opening of the port at the moment the stroke of the engine begins is called the *lead* of the valve. At C the valve is at the end of its travel towards the right, and the valve has travelled a distance ξ_m from mid position equal to the radius of the eccentric.

Neglecting the small disturbance introduced by the obliquity of the eccentric rod, the valve is at the ends of its travel when the eccentric is at its dead points on the line of stroke, and the valve is in mid position when the eccentric is 90° from its dead points, or at right angles to the line of stroke.

223. *Disturbance introduced by obliquity of eccentric rod. Setting valve to equalise lead.*—With an indefinitely long eccentric rod the travel of the valve would be exactly symmetrical on either side of mid position, equal travel on either side from mid position corresponding to equal angles of rotation of the crank and eccentric radius. Then, to secure a similar distribution of steam to both ends of the cylinder, the centre of the valve in mid position must coincide with the centre of the cylinder face. The obliquity of the eccentric rod introduces a small amount of unsymmetry in the forward and backward travel of the valve.

It is customary, in setting the valve in the workshop, to adjust the length of the valve rod so that the lead of the steam edge of the valve is the same for both steam ports—that is, the valve opening is made the same, with the crank at both dead points. It will then be found that the centre of the valve's travel does not exactly coincide with the centre of the cylinder face, being a little on the side of the interior dead point, or dead point nearest the cylinder.

224. *Lap and lead of slide valve.*—In the earliest slide valves the width of the faces of the valve was sensibly equal to the width of the steam ports. Then, the moment the valve passed its mid position, it began to open one port to steam and the other to exhaust. Apart from a circumstance to be mentioned presently, the steam piston should be at the end of its stroke at the moment steam begins to be admitted on one side and exhausted from the other. It follows that, with this form of valve, the valve is at mid stroke when the steam piston is at the end of its stroke; consequently the eccentric must be at right angles to the crank.

It was discovered, however, that with this arrangement the steam entered and left the cylinder with difficulty at the beginning of each stroke in consequence of the very gradual opening of the slide valve. To afford a wider opening to the steam, it was found necessary that the valve should be

already a little open at the beginning of a stroke. To secure this it is only necessary to fix the eccentric a little more than 90° in advance of the crank. The width of port open at the beginning of the stroke, at the steam edge of the valve, is termed the *lead*, and will be denoted by e.

Next it was found desirable to make the faces of the valve wider than the steam ports, so that when the valve is in mid position (fig. 202) the valve faces overlap the edges of the ports. The width of overlap on the steam edge of the valve is called the *outside lap*, o, and that on the exhaust edge of the valve the *inside lap*, i. Generally the former is greater than the latter. Then one port opens sooner or more widely to exhaust than the other to steam,

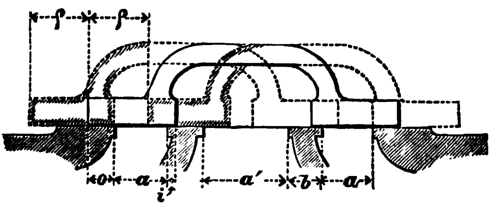


Fig. 202

and this diminishes the back pressure without sensibly diminishing the work done by the steam in the forward stroke.

The valve must be open at the beginning of the stroke on the steam edge by the amount e fixed for the lead. Hence at the beginning of a stroke the valve must have already travelled from mid position a distance $\xi_0 = o + e$. The eccentric must therefore have also passed its mid position at right angles to the crank by an angle necessary to move the valve through a distance ξ_0. This angle, which will be denoted by θ, is called the *angle of advance*. The angle of advance may be regarded as made up of two angles, one corresponding to the movement of the valve through the distance o, which may be called the 'lap angle,'

and one necessary to move it the further distance e, which is the opening of the port at the beginning of the stroke, and which may be termed the 'lead angle.' The whole angle between the crank and eccentric radius, or $90° + \theta$, which is a fixed angle for simple valve gears, may be called the *angle of keying*.

One effect of giving outside lap to the valve and advance to the eccentric is that the valve closes the steam port before the end of the forward stroke. Then the steam is cut off before the end of the stroke, and the valve acts as an expansion valve. There is a limit to the amount of lap which can be used with an ordinary slide valve. To whatever extent the opening of the exhaust is made earlier, to the same extent the closing of the exhaust is made earlier also. If the exhaust is closed too soon, steam is retained in the cylinder and compressed, as the piston returns, into the small clearance space at the end of the cylinder. This action is termed 'cushioning.' A moderate amount of cushioning is useful, but excessive cushioning would be prejudicial. To prevent this the outside lap is usually not greater than is sufficient to close the steam port at $\frac{5}{8}$ths of the stroke. When more expansion is wanted, a double-slide valve or some other arrangement is used.

Fig. 202 shows a section of a slide valve and of the steam ports, taken parallel to the direction in which the valve moves. The dotted lines show the positions of the valve at the ends of its stroke, in either position completely uncovering one port to exhaust, and partially uncovering the other to steam. In this figure a is the width of steam port, o the outside lap, i the inside lap, and ρ the half travel of the valve or eccentric radius.

225. *Construction of slide valves.*—Fig. 203 shows a locomotive slide valve of the simplest form; the valve rod passes through the valve, the position of which is fixed in setting the valve by a pair of lock nuts at each end. The chief fault of the short D slide valve here shown is that the

steam passages to the cylinder ends are necessarily long, and this increases considerably the clearance in the cylin-

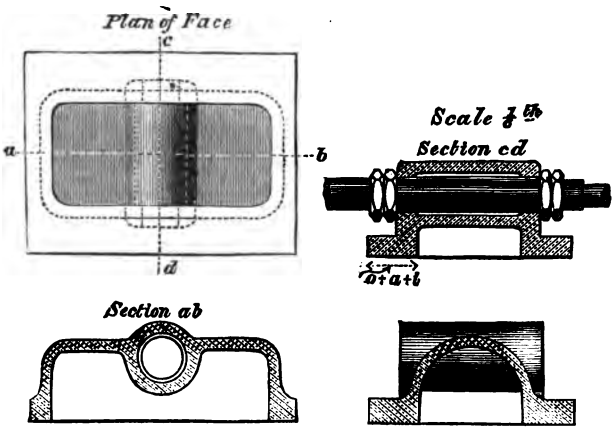


Fig. 203

ders. The valve chest and cylinder face for such a valve are shown in figs. 30 and 33, pp. 51, 53.

226. *Modifications of the ordinary slide valve.*—The fault of the ordinary slide valve is the large clearance space between valve and cylinder. The clearance is much smaller

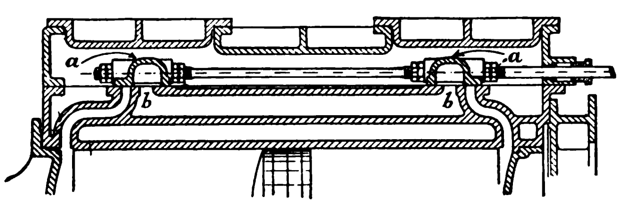


Fig. 204

if the valve is in two parts placed near the ends of the cylinder (fig. 204). The figure shows a section of a jacketed cylinder ; *a a* are the two parts of the valve, and *b b* are the

two exhaust ports. Sometimes, also, it is divided into four separate slides, two on steam ports controlling steam admission, and two on exhaust ports controlling steam exhaust. The great advantage of this is that the steam valves can be connected to a gear which varies the ratio of expansion without affecting in any way the operation of exhaust. The simple slide valve is not well adapted to cut off steam earlier than five-eighths stroke, because if more lap is used exhaust is interfered with. It can be used as a variable expansion valve, with a link motion or other suitable gear, but it does not cut off steam at various points of the stroke quite satisfactorily. When a variable expansion is required it is usual to have a second expansion slide, generally working on the back of the ordinary slide, and controlling the point of cut-off.

227. *Slide valves with pressure-relief rings.*—The friction of the slide valve (§ 123, p. 184) involves a considerable waste of energy, and this in very large valves becomes a serious loss. To reduce it double-ported valves are used which, having only half the travel, involve only about half the loss of work in friction. This, however, is only a partial remedy. It is, perhaps, an even more prejudicial effect that the faces of the valve and valve seating wear away. The slide valve can be refaced when worn and the seating can be remade by adding a false seating. Nothing wears better than a cast-iron valve on a cast-iron seating, but then the wear is equal on both and both require refacing. Sometimes the valve is of gunmetal, or a gunmetal seating is fixed initially on the cylinder face, so that, the wear being chiefly concentrated on one face, the readjustment is easier.

A more complete remedy for the evils of valve friction is to use a valve so arranged that the pressure on the back is diminished. A rectangular or circular steam-tight space is formed between the back of the valve and the steam-chest cover, and this is put in communication with the

exhaust. This relieves the valve from a great part of the pressure on its back.

Fig. 205 shows one form of balanced valve used in locomotives. A rectangular space on the back of the valve is enclosed by a metal wall. In a groove in this wall are four cast-iron packing strips pressed upwards by springs. These strips slide on the planed face of a plate attached to the valve-chest cover. A small hole in the valve puts the space thus enclosed in communication with the exhaust. Hence on the enclosed area of the back of the valve the steam pressure acts only on the fixed plate and the valve carries only the exhaust pressure.

Great care must be taken that there is no leakage of

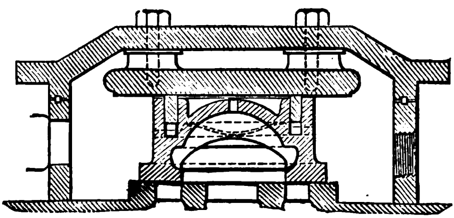


Fig. 205

steam past the packing strips direct to the exhaust space. If this occurs the waste of steam may be very serious, and in the form of relief ring just described, steam-tightness depends on the accuracy of fitting of the strips. Valves with relief rings are sometimes termed balanced valves.

If instead of four packing strips a circular relief ring is used, leakage is more easily prevented. Fig. 206 A shows an ordinary form of circular relief ring. The ring is of gunmetal and is pressed upwards by coiled springs in the groove. Two rings of asbestos packing are inserted between the relief ring and a lower ring which takes the thrust of the springs. Fig. 206 B shows a relief ring made steam-tight by a Ramsbottom spring ring, like those used in

pistons (§ 158, p. 238). Fig. 206 C, shows a relief ring made steam-tight by metallic packing consisting of wedge-shaped

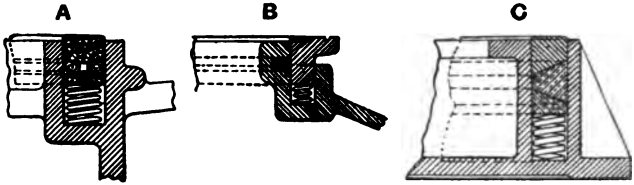


Fig. 206

split spring-rings, pressed upwards by coiled springs. The wedge-shaped rings make a steam-tight joint with both the outside and inside faces of the groove.

228. *Corliss valves.*—Corliss valves are slide valves with a cylindrical face, oscillating about their cylindric axis. They are generally used as four valves, two for admission

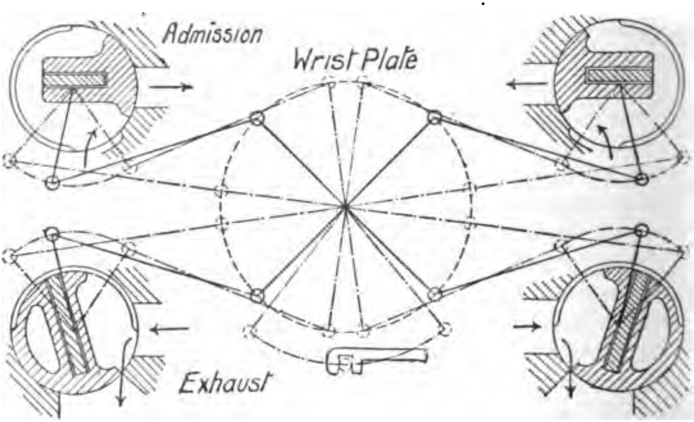


Fig. 207

and two for exhaust, as shown in the section of the cylinder, fig. 34, p. 54. They lie loose on the seatings and are driven by a rectangular rod in a slot. In the original Corliss engine (fig. 207) the four valves were driven by rods

attached to an oscillating plate, termed a wrist plate, actuated by a single eccentric. The exhaust valves were positively attached to the wrist plate, but the steam valves were controlled by a trip gear which disconnected them from the wrist plate at cut-off, allowing them to be closed very rapidly by springs. The trip gear was arranged to be under the control of the engine governor, so that the moment at which steam was cut off was varied to suit the demand on the energy of the engine. The exhaust valves were permanently connected to the wrist plate, so that the timing of the operation of exhaust was not affected by the variation of cut-off. Many varieties of trip gear have been used with Corliss valves, and sometimes all the valves are driven positively and variation of expansion is obtained by hand adjustment.

Fig. 208 shows the mode of driving Corliss valves adopted by Bollinckx. It will be seen that the valve lies closely on its seating, even if the driving rod is displaced (as shown in the figure in an exaggerated way), by the wear of the stuffing-box in which it is supported.

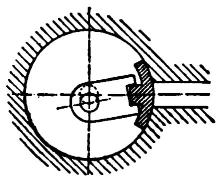


Fig. 208

229. *Piston valves.*—Piston valves (fig. 209) are cylindrical valves which reciprocate axially. They are perfectly balanced as regards steam pressure, and are nearly frictionless. They are increasingly used in engines working at high pressures, and sometimes in locomotives. A comparison of fig. 209 with the figures of ordinary slide valves shows that they open and close the ports in precisely the same way. In A the outer edges of the pistons are the steam edges, and the inner are the exhaust edges, as in the ordinary slide valve. In B the outer edges are exhaust edges, and the inner are steam edges. The chamber round the pistons, of the same width as the port, secures perfect balance of the pressure on the cylindrical

surface of the pistons. In fig. 209 the piston valves are without packing rings, and the cylindrical face in which they slide is not bushed. Sometimes spring packing rings

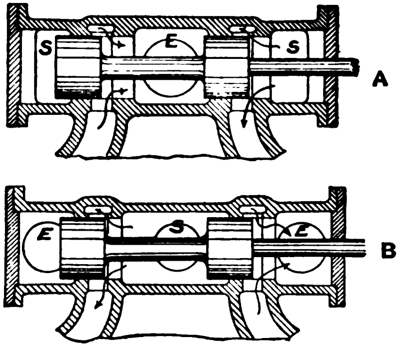


Fig. 209

are used in the pistons, and generally they work in a bush, which can be removed and replaced if worn. Fig. 210 shows a piston valve with spring packing and its bush. In this form (Smith's piston valve) the principal packing ring

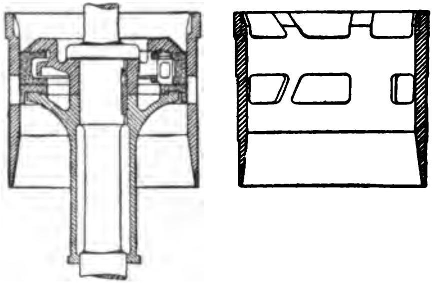


Fig. 210

is divided into three segments, each guided by a rib. The segments are pressed out by steam admitted into the piston through a small hole. They give way a little if there is

water in the cylinder when the port is covered, and prevent damage to the cylinder. The bridges across the ports in the bush cover the joints in the packing ring. The smaller packing ring, which has only one joint, covers the ends of the joints in the larger packing ring. The covers of the piston are screwed hard together, leaving just clearance enough to permit the action of the spring packing rings.

Fig. 211 shows a piston valve for a compound engine which controls the steam distribution in both high-pressure and low-pressure cylinders. The steam passes through the interior of the valve from the high to the low-pressure cylinder. The inner edges of the piston regulate the exhaust from the low-pressure cylinder and the outer edges the admission to the high-pressure cylinder. The exhaust edges for the high pressure and steam edges for the low pressure are formed by ports in the valve leading to the tubular cavity in its interior.

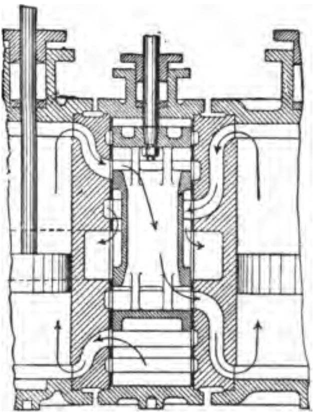


Fig. 211

230. *Area of steam ports.*—The area of the steam ports must be so arranged that the mean velocity of the steam does not exceed about 50 to 100 feet per sec. Particulars of the velocity in various steam passages in different cases have been given in § 39, p. 55. If v is the velocity of steam piston and v the permitted velocity of steam through the port, ω the area of the port and Ω the area of the piston, then on the average and subject to the reservation mentioned in § 39,

$$\mathrm{v}\,\Omega = v\,\omega,$$

and the ratio ω/Ω of port area to piston area is as follows :—

Piston velocity. Ft. per min.	Piston velocity. Ft. per sec.	Velocity of steam through port. Feet per sec.			
		50	75	100	125
240	4	·080	·053	·040	·032
300	5	·100	·068	·050	·040
360	6	·120	·080	·060	·048
420	7	·140	·093	·070	·056
480	8	·160	·107	·080	·064
540	9	·180	·120	·090	·072
600	10	·200	·134	·100	·080
720	12	·240	·160	·120	·096

In quick-rotation engines a larger port area and lower steam velocity is desirable than in long-stroke low-speed engines. The steam ports in quick-rotation engines have most commonly the ratio $\omega/\Omega = 0{\cdot}10$, and in low-speed engines $\omega/\Omega = 0{\cdot}09$. The exhaust ports are usually larger than the steam ports and ω/Ω is from 0·08 to 0·09. For locomotives which run at a high but variable speed, $\omega = {\cdot}07\,\Omega$.

Let l be the length and a the width of each steam port. Then $a\,l = \omega = \left(\frac{\omega}{\Omega}\right)\Omega$. The proportions of the port are variable; the length l may be from 0·5 to 0·8 of the cylinder diameter, and the ratio $\frac{l}{a}$ is 6 in small engines, 7 in medium engines, and 9 in large engines.

It may be noted that the frictional loss of pressure of steam in steam passages is proportional to its density, and its density increases almost directly as its absolute pressure. Hence, if the loss in the passages is to be restricted to a given number of lbs. pressure, the ports must be larger the greater the pressure of the steam. Some remarks on the unexpectedly large loss of pressure in passages and ports when high-pressure steam is used will be found in Willans

on Non-condensing Steam Engine Trials, 'Proc. Inst. Civil Eng.,' vol. xciii. Some constructors allow about 90 sq. ins. of port opening to steam and 140 sq. ins. to exhaust, per lb. of steam used in the cylinder per stroke. This rule allows an area increasing directly as the density of the steam at release.

The whole width of the steam port is opened to exhaust, but often only 0·7 to 0·9 of the width to steam.

231. *Proportions of the slide valve*:—

Let a = width of steam port.

na = greatest width opened to steam.

o = outside lap.

i = inside lap.

e = lead; e' = inside lead.

b = width of bar between steam and exhaust ports.

a' = width of exhaust port.

ρ = half travel of valve, or radius of eccentric.

r = radius of crank or half stroke of engine.

ϵ = ratio of eccentric radius to length of eccentric rod.

ξ = distance valve has travelled from its mid position when the crank has moved through an angle ϕ from the dead point.

l = distance piston has travelled from beginning of stroke at the same moment.

θ = angle of advance of eccentric, so that the eccentric is $90° + \theta$ in advance of the crank.

The width b of the bars is fixed empirically. In small engines $b = 0·5\ a + \frac{3}{8}$. In large engines b is fixed with regard to convenience in casting. The proportion of port opened to steam n is 0·7 to 1·0. The lead e is fixed by experience. In simple slide-valve arrangements $e = 0·1\ a$ to $0·25\ a$. The outside lap o is fixed with reference to the point at which steam is to be cut off. In simple slide-valve arrangements it may be $0·5\ a$ to a; but when there is a special expansion valve a smaller outside lap is sufficient.

If the valve opens $n a$ of the port to steam and a to exhaust, the eccentric radius is

$$\rho = n a + o = a + i \quad . \quad . \quad (1)$$

so that the inside lap is

$$i = o + (n - 1) a \quad . \quad . \quad (2)$$

But the inside lap is sometimes less than this and then the exhaust edge of the valve travels beyond the port edge. The angular advance of the eccentric is given by the relation,

$$\sin \theta = \frac{o + e}{\rho} = \frac{o + e}{o + n a} \quad . \quad . \quad . \quad (3)$$

The inside lead is $e' = \rho \sin \theta - i$. The width of the exhaust port must be equal to or greater than $a' = \rho + a + i - b$, and the hollow under the valve $a' + 2 b - 2 i$

Example.—In a simple slide valve $a = 1\frac{1}{2}$, $e = \frac{3}{8}$, $o = 1\frac{1}{4}$, and $n = 1$. Then the eccentric radius is $\rho = 1\frac{1}{2} + 1\frac{1}{4} = 2\frac{3}{4}$. If the port just opens to exhaust the inside lap $i = 1\frac{1}{4}$, but it might be less than this. Sin $\theta = (1\frac{1}{4} + \frac{3}{8}) / (1\frac{1}{2} + 1\frac{1}{4}) = 0{\cdot}59$. Hence the angle of advance is $\theta = 36°$. For a slide valve provided with a separate expansion valve let $a = 1\frac{5}{8}$, $o = 1\frac{1}{4}$, $e = \frac{1}{4}$, and the opening to steam $= n a = 1\frac{1}{2}$. Then $\rho = 1\frac{1}{2} + 1\frac{1}{4} = 2\frac{3}{4}$, $i = 1\frac{1}{4} - \frac{1}{8} = 1\frac{1}{8}$, or better say, 1 or $\frac{7}{8}$. Sin $\theta = (1\frac{1}{4} + \frac{1}{4})/2\frac{3}{4} = 0{\cdot}545$. Hence $\theta = 31°$.

232. *Travel of valve and corresponding crank angle when the influence of the obliquity of the eccentric rod is neglected.*—Let a line through the two dead centres of the crank-pin circle be termed the 'line of stroke.' Generally this line is also parallel to the axis of the cylinder. If the obliquity of the eccentric rod is neglected, the valve is in its mid position when the eccentric radius is at right angles to the line of stroke. Let that position be termed, for shortness, the mid

position of the eccentric. As the eccentric moves through an angle α from its mid position, the valve travels a distance

$$\xi = \rho \sin \alpha \quad . \quad . \quad . \quad (4)$$

which will be + or — according as α lies between 0° and 180°, or between 180° and 360°, α being measured in the direction of motion of the crank. Since the eccentric is 90° + θ in advance of the crank,

$$\alpha = \phi + \theta,$$

where ϕ is the angle through which the crank has moved from its position at the beginning of the stroke. Hence

$$\xi = \rho \sin (\phi + \theta). \quad . \quad . \quad (5)$$

The opening of the port to steam is

$$w = \xi - o = \rho \sin (\phi + \theta) - o \quad . \quad (6)$$

and the opening of the port to exhaust is

$$w' = -\xi - i = -\rho \sin (\phi + \theta) - i \quad . \quad (7)$$

When admission begins and when steam is cut off, $w = 0$; and when exhaust or compression begins, $w' = 0$. Inserting these values, we obtain four values of the crank angle for each edge of the valve and for one revolution of the engine.

For			$(\phi + \theta)$ lies between
Admission	$w = 0$	$\sin (\phi_1 + \theta) = \frac{o}{\rho}$	0° and 90°
Cut-off		$\sin (\phi_2 + \theta) = \frac{o}{\rho}$	90° and 180°
Release	$w' = 0$	$\sin (\phi_3 + \theta) = -\frac{i}{\rho}$	180° and 270°
Compression		$\sin (\phi_4 + \theta = -\frac{i}{\rho}$	270° and 360°

From these equations the values of $\phi + \theta$, and therefore of ϕ, can be obtained. The angles are connected by the relations $\phi_2 = 180° - \phi_1 - 2\theta$; $\phi_4 = 180° - \phi_3 - 2\theta$.

The following form of the same equations is sometimes more convenient :—

Since

$$\sin\theta = \frac{o + e}{\rho}, \quad o = \rho \sin\theta - e.$$

Hence,

$$w = \xi - o = e + \rho\left\{\sin(\phi + \theta) - \sin\theta\right\} \qquad (8)$$

For admission and cut-off, $w = 0$, and we get

$$\sin(\phi + \theta) = \sin\theta - \frac{e}{\rho};$$

$$\text{For admission, } \phi_1 = 2\pi - \frac{e}{\rho\cos\theta} \quad . \quad . \quad . \qquad (9)$$

$$\text{For cut-off, } \phi_2 = \pi - 2\theta + \frac{e}{\rho\cos\theta} \quad . \quad . \quad . \qquad (10)$$

The angles are in circular measure in these equations, and can be reduced to degrees by multiplying by $\frac{180}{\pi}$ or by 57·3.

Similarly, since $i + e' = o + e$,

$$i = \rho\sin\theta - e$$

$$w' = -\xi - i = e - \rho\left\{\sin(\phi + \theta) + \sin\theta\right\} \qquad (11)$$

For release and compression, $w' = 0$; then

$$\sin(\phi + \theta) = -\sin\theta + \frac{e'}{\rho}$$

$$\text{For release, } \phi_3 = \pi - \frac{e'}{\rho\cos\theta} \quad . \quad . \quad . \quad . \qquad (12)$$

$$\text{For compression, } \phi_4 = 2\pi - 2\theta + \frac{e'}{\rho\cos\theta}. \quad . \qquad (13)$$

233. *Position of piston for given crank angles, when the obliquity of the connecting rod is neglected.*—If l is the distance the piston has travelled from the beginning of its stroke, when the crank has revolved through the angle ϕ, measured from the dead point, then if the obliquity of the connecting rod is neglected

$$l = r\,(1 - \cos\phi) \quad . \quad . \quad . \quad (14)$$

where $\cos\phi$ is negative, if ϕ lies between 90° and 270°. By inserting the values of ϕ, obtained above, we obtain approximately the position of the piston for admission, cut-off, release, and expansion. As, however, the obliquity of the connecting rod sensibly affects the position of the piston, it is better to set off the positions of the crank corresponding to the above values of ϕ on a diagram drawn to scale, and then by laying off the connecting-rod length, the position of the piston is found exactly; or the graphic constructions given below may be used.

234. *Crank angles corresponding to given ratios of expansion.*—Let l_2 be the travel of the piston corresponding to the crank angle ϕ_2 at cut-off. Then $l_2/2r$ is the ratio of cut-off. The following table gives the relation between these quantities, when the obliquity of the connecting rod is neglected:—

$\frac{l_2}{2r}$ =	0·4	0·45	0·5	0·55	0·6	0·65
ϕ_2 =	78½°	83	90	96	101½	107½
$\frac{l_2}{2r}$ =	0·7	0·75	0·8	0·85	0·9	0·95
ϕ_2 =	113½°	120	127	134½	143	154

The ratio $2r/l_2$ is the ratio of expansion or number of times the steam is expanded.

235. *The Trick valve.*—Fig. 212 shows a modified slide valve, giving a quicker and fuller opening of the steam port. A passage-way is formed through the valve itself. This

passage-way overlaps the ends of the cylinder face at the ends of the travel, so that steam is admitted not only at the steam edge of the valve but also through the valve passage.

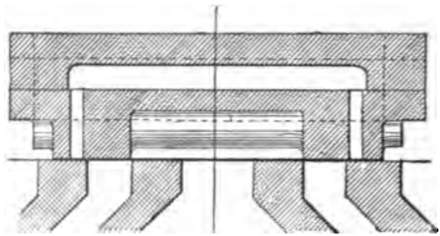


Fig. 212

Supposing the valve in the figure to move to the right, steam will enter the left port, both at the left steam edge of the valve and through the passage which overlaps the right-hand end of the cylinder face. The pressure on the back of the

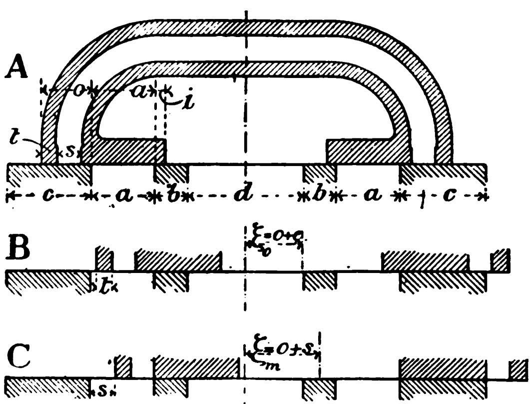


Fig. 213

valve, when there is steam pressure inside the passage, is diminished and the friction of the valve is reduced.

Fig. 213 shows a Trick valve at A in mid position; at B

open to the extent of the lead at the beginning of a stroke, steam entering the left-hand port; at C at the end of its travel towards the right, the left-hand port being fully open to steam and the right-hand to exhaust.

Proportions of a Trick valve.—Let a be the width of port, o the outside and i the inside, lap as indicated in fig. 213 A. The width t of the wall of steam passage must be taken arbitrarily. It is often $\frac{1}{2}$ to 1 in. The width of bar between steam and exhaust port must also be taken arbitrarily; it is often taken $a/2$, or in large valves of the dimension easiest to cast.

It is convenient in a Trick valve that the passages should just completely open to steam and exhaust, as shown in fig. 213 C, where the left-hand port is just full open to steam and the right-hand port to exhaust. The width of steam passage through the valve is taken

$$s = \tfrac{1}{2}(a - t).$$

Then the eccentric radius or half travel is

$$\rho = a + i = o + s,$$

and o and i are not independent. Suppose

$$o = a + i - s.$$

The whole width of valve face from the exhaust edge to the extreme steam edge is $a + o + i$.

When the valve is at the end of its travel towards the right, there must still be an opening at least equal to a into the exhaust port. Hence, looking at fig. 213 C,

$$d + b + a - s = a + (a + o + i)$$

$$d = a + o + i + s - b.$$

Lastly, the width of cylinder face c beyond the steam port must be

$$c = 2o - t.$$

The width of the central aperture in the valve for exhaust is then

$$d + 2b - 2i$$

The figure shows at A the valve in mid position. At B it has travelled to the right a distance $\xi_0 = o + e$, and the steam port on the left and the steam passage through the valve on the right are open to steam by the amount e, which in this case is therefore half the effective lead. At C the valve has travelled $\xi_m = a + i = o + s$, and both ports are open fully. The valve diagram is drawn precisely as for a simple valve, merely remembering that the effective width of port is a to exhaust and $2s$ to steam.

CHAPTER XIV

VALVE DIAGRAMS

236. *Graphic methods of determining the relation between piston travel and crank angle. Müller circles.*—The alteration of the position of the piston due to the obliquity of the connecting rod cannot be neglected in studying valve gears. The algebraical expression for the piston travel, when the obliquity of the connecting rod is taken into account, is complicated, but there are easy ways of finding it graphically. Fig. 214 shows a construction due to Prof. Müller. Let A B be the cylinder, the piston being initially at A, P e the connecting rod, and e C the crank. For simplicity the piston is reduced to a line, and the connecting rod supposed attached directly to the piston, but this in no way alters the motion.

Let the crank length e C = R, the connecting-rod length e P = L. From centre C with radius L + R describe the circle X and with radius L − R the circle Y. From centre I with radius L describe the circle Z. The points I and E are the interior and exterior dead points. As the crank travels from C I to C e, the piston travels a distance A P, which may be found by taking e P = L. It is more easy in many cases to produce C e to cut the circles Z X in m n. Then m n = A P. Join m I, m P, e I. In the triangles I e m, I e P, m I = e P, being radii of the circle Z; and e I is common to the two triangles. Also since C e = C I, the exterior angle m e I = P I e. Hence the triangles are superposable and e m = I P. But e n = I A. Therefore m n = A P, the piston travel.

The Zeuner valve diagram (§ 244) gives directly the

crank positions for given positions of the slide valve. If Müller circles are drawn to any convenient scale outside the valve diagram, the lengths of piston travel for any crank position are easily scaled off. Neglecting the eccentric obliquity, which produces generally only a small effect, it will be

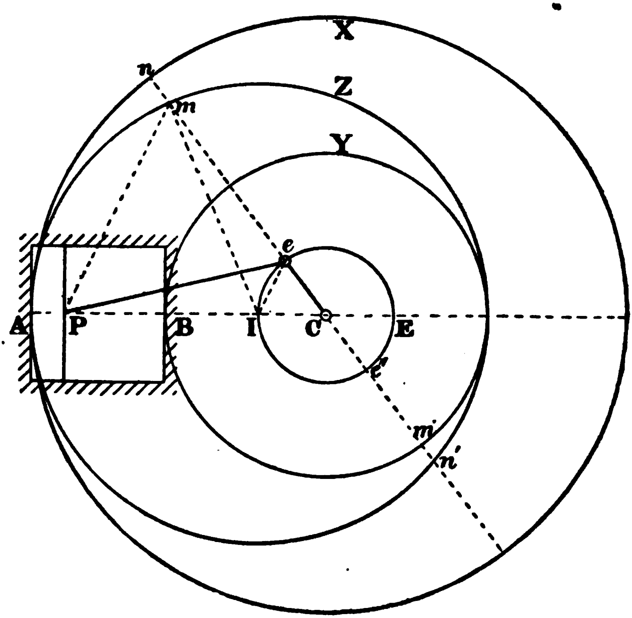


Fig. 214

found that if C e is a position of the crank (say at cut-off) when the action of the steam on the left of the piston is considered, then C e' is the corresponding position of the crank when the action of the steam on the right of the piston is considered. If m n is, for instance, the distance of the piston from A when cut-off takes place on the left of the piston, m' n' is the distance of the piston from B when cut-off takes place on the right of the piston. It is due to

the different obliquities of the connecting rod for these two positions that *m n* is not generally equal to *m′ n′*.

A still easier construction for finding the piston travel is the following :[1] Let C *e*, fig. 215, be the crank of radius R, and *e* P the connecting rod of length L as before. With centres A and B and radius equal to L, draw the arcs X I X, Y E Y, which may be termed the 'inner and outer dead-point arcs.' If I E is taken to represent the stroke, we can find the point *f* corresponding to P by drawing the arc *c e f* with

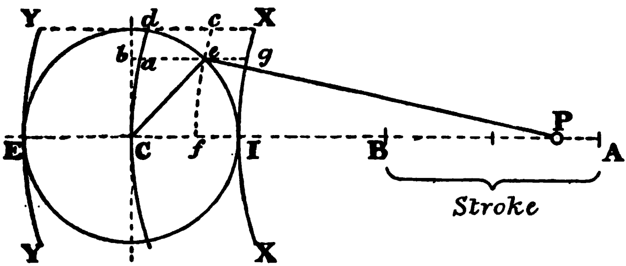


Fig. 215

centre P and radius P *e* = L. Since A I = L, then I *f* = A P the piston travel from the end of the stroke. If through *e* a line *b g* is drawn parallel to E I, then it is easy to see that *f* I = *e g*. For the crank-pin position *e*, the distance of the piston from mid-stroke is C *f* = *e a*. If the connecting rod were infinitely long, *e b* would be the distance from mid-stroke. Hence *a b* is the deviation of the piston position due to obliquity of connecting rod.

It is convenient to notice that if any line Y X is taken parallel to the line of stroke, an arc through *e* parallel to X X or Y Y cuts this in *c*, so that *c d* is the piston distance from mid-stroke.

237. *Graphic representation of valve displacement. Harmonic diagram.*—The direct calculation of the relative

[1] Coste and Maniquet, 'Traité des Machines à Vapeur,' 1886. Grashof, 'Maschinenlehre,' vol. iii. 1890.

position of valve and piston becomes complicated, if the obliquity of connecting rod and eccentric rod is taken into account. Graphic methods can be used which are simple. Such methods are approximate in this sense, that the results have to be measured on a drawing and their accuracy depends on the exactness obtained in making the necessary geometrical constructions. For practical designing the graphic methods are convenient and accurate enough.

One of the simplest graphic constructions is to draw curves of valve and piston displacement, which are nearly curves of sines. Then the relative position of piston and valve at any point of the stroke is easily measured.

In fig. 216 let O A be the crank which has moved through an angle ϕ from the inner dead point, and O D the

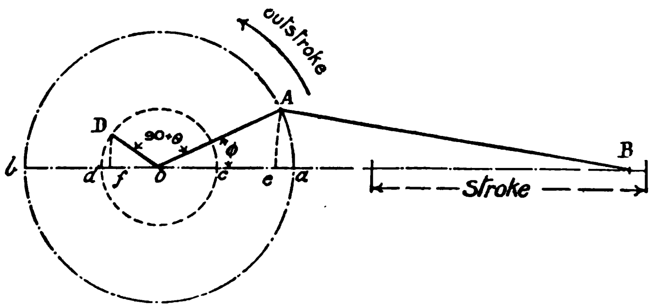


Fig. 216

corresponding position of the eccentric radius. The angle A O D is $90° + \theta$ where θ is the angle of advance. It is convenient that the eccentric radius should be drawn to a larger scale than the crank. $a\ b$ is the piston stroke on the crank scale and $c\ d$ the valve stroke on the eccentric radius scale. If from B an arc is drawn with the connecting rod as radius, $a\ e$ is the distance the piston has travelled from its inner dead point, and O e is the distance of the piston from the middle of its stroke. Similarly, if an arc is drawn through D with the eccentric rod as radius, O f is the distance

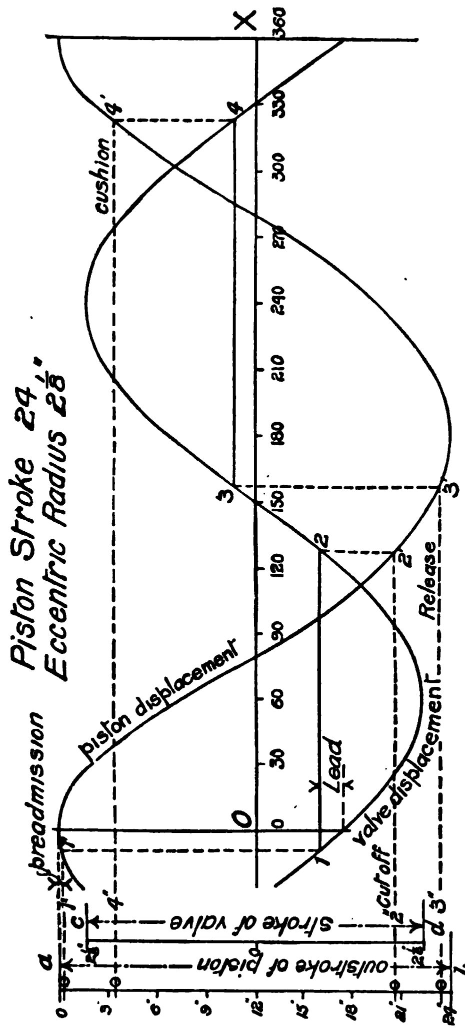

Piston Stroke 24"
Eccentric Radius 2 1/8"
preadmission
piston displacement
valve displacement
cushion
Release
cutoff
Lead
stroke of valve
outstroke of piston
0
30
60
90
120
150
180
210
240
270
300
330
360
0
3
6
9
12
15
18
21
24
X
O
1
2
3
4
1'
2'
3'
4'
a
b
c
d

Fig. 217

of the valve from the middle of its stroke. The arc D *f* will differ very little from a perpendicular on *a b*. Shortly, O *e* and O *f* are the piston and valve displacements reckoned in each case from mid-stroke.

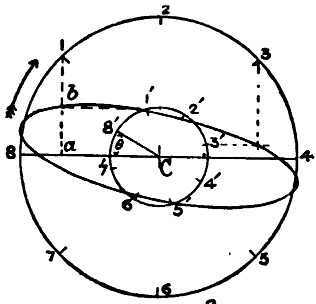
In fig. 217 take any line O X to represent the development of the crank-pin circle, and graduate it in degrees. If for each value of ϕ, fig. 216, the piston distance from mid-stroke O *e* is set up as an ordinate, the curve of piston displacement is obtained. Similarly, if the corresponding values O *f* of the valve distance from mid-stroke are set up, the curve of valve displacement is obtained. These will be nearly similar curves differing in phase by the angle $90° + \theta$. Draw 1 2 at a distance from O X equal to the outside lap, and 3 4 at a distance equal to the inside lap of the valve. The valve opens to steam when it has moved from mid-stroke a distance equal to the outside lap, that is, at the point 1. The vertical 1 1′ determines the position 1′ of the piston, and if 1′ is projected horizontally to a scale representing the piston stroke, the part of the stroke during which there is preadmission of steam is determined. When the valve has travelled back and is at a distance equal to the outside lap from mid-stroke steam is cut off, that is, at the point 2. The corresponding position of the piston is 2′, and projecting this horizontally to the scale of piston stroke, the length of stroke at cut-off is found. Similarly, exhaust begins at 3 the piston position being 3′, and compression begins at 4 the piston position being 4′.

This diagram is convenient for examining the relative positions of an ordinary slide valve and an expansion slide valve such as the Meyer valve, and then both valve curves should be drawn to the same scale.

238. *Valve ellipse. Case I. Obliquity of rods neglected.*— A very old and useful method of representing the action of a slide valve is to plot the motion of the piston as abscissa and that of the valve as ordinate. Then, if the obliquities of the rods are neglected, the curve obtained is an ellipse.

In a simple way, however, the obliquities of the rods can be taken into account, and then the curve is no longer a true ellipse. Perhaps no graphic representation is more

Fig. 218



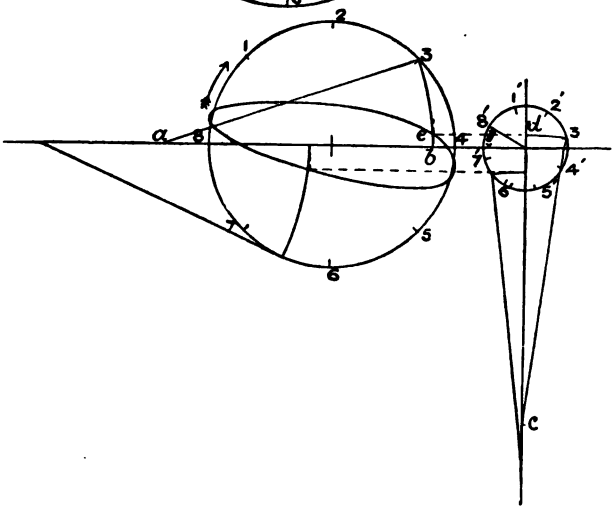


Fig. 219

convenient than this for exhibiting the action of a valve gear already designed. It is less fruitful than some other diagrams in settling the proportions of the gear beforehand.

Neglecting at first the obliquities of the rods, let 8 4, fig. 218, represent the stroke of the piston, and the circle described on 8 4 the crank-pin circle, the crank going round in the direction of the arrow. If we divide the crank-pin circle into parts 1, 2, 3 . . . the corresponding positions of the piston will be found by dropping perpendiculars on 8 4. Now let the smaller circle represent the path of the eccentric centre. In order to project the valve movement at right angles to the piston movement the eccentric must be turned back through 90°. Hence, when the crank is at C 8, the eccentric radius will be C 8′, the angle 8 C 8′ being the angle of advance θ. Now, starting from 8′, divide the eccentric circle into the same number of equal parts as the crank-pin circle, and number them to correspond. When the crank pin is at 1, the piston will have moved a distance 8 *a* from the dead point found by drawing 1 *a* vertically. At the same time the valve will be at a distance *a b* from its middle position, found by drawing 1′ *b* horizontally. *b* is a point in a curve the abscissa C *a* of which is the distance of the piston from mid-stroke, and the ordinate *a b* is the distance of the valve from mid position. This curve is the valve ellipse.

239. *Valve ellipse. Case II. Obliquity of both connecting rod and eccentric rod taken into account.*—So far the diagram gives no more than can be found by simple calculations, though, as will be seen presently, the graphic representation is a practically convenient one. It is easy, however, to take account of the obliquities of the rods, and then results are obtained which are calculated with difficulty.

In fig. 219 the larger circle is the crank-pin circle and the smaller the eccentric circle, and they are divided exactly as in the last figure. Now, to find the exact position of the piston corresponding to any crank-pin position 3, it is only necessary to take 3 *a* = the connecting-rod length and strike the arc 3 *b* with radius *a* 3. *b* is the piston position, allowing for the obliquity of the connecting rod. Similarly, to

find the true position of the valve, it is only necessary to take $3'\,c$ = the eccentric-rod length (from centre of eccentric to centre of valve-rod pin). Striking an arc $3'\,d$ with radius $c\,3'$, d is the true position of the valve, its line of stroke being perpendicular to that of the piston. Projecting b vertically and d horizontally, we get a point e on a curve which gives the true relative motion of the piston and valve, and which may still for convenience be called the 'valve ellipse,' though it is not an exact ellipse.

It will be convenient generally to take a larger scale for the eccentric radius than for the crank radius, and this in no way renders the construction inexact.

Fig. 220 shows how the action of the valve may be rendered clear by the aid of a valve ellipse. In this figure

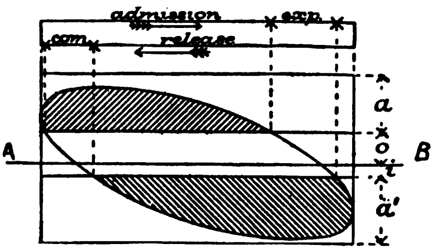


Fig. 220

a true ellipse has been drawn, but the construction is the same if an exact curve has been drawn, taking account of the obliquities of the rods. Let A B be the line of piston stroke corresponding to 8 4 in the previous figures. Draw lines parallel to A B at distances equal to the outside lap o, the inside lap i, the steam-port width a, and the exhaust-port width a' (see fig. 202). The shaded areas show at once the periods when the distance of the valve from mid position is greater than o or i, and these are the periods when the port is open to steam and exhaust to the left end of cylinder during one forward and return stroke. For the other end

of the cylinder the lines parallel to A B must be reversed in position. Vertical lines permit the marking out of the periods of admission, expansion, release, and compression.

If the eccentric rod is long compared with the eccentric radius, the piston positions may be determined as in Case II., and the valve positions more simply, as in Case I.

240. *Reuleaux, Reech, or Coste and Maniquet diagram.*—By using the same circle to represent (to different scales) the crank-pin circle and the circle described by the eccentric centre, a very simple and useful diagram is obtained. When the obliquities of the rods are neglected, the diagram has long been known in France as Reech's diagram, and in Germany as Reuleaux's diagram. Lately, Coste and Maniquet have indicated how the diagram may be improved so as to take account of the obliquities of the rods.[1] It may be used very conveniently to obtain the data necessary for plotting valve ellipses. The exact diagram will be given first, and then the modification for the case when the eccentric rod is so long that the effect of its obliquity may be disregarded.

241. *Exact determination of the valve travel, taking account of the obliquity of the eccentric rod.*—Let the circle E B I, fig. 221, represent to one scale the crank-pin circle, I E being the piston stroke, and to another scale the circle of the eccentric, I E being then the total valve travel. O E $=$ R is the crank radius to one scale, and O B $= \rho$ is the eccentric radius to the other scale Let θ be the angle of advance, so that when the crank pin is at E, the eccentric centre is at B. Bisect I E in O and draw A A at right angles. With radius equal to the length of eccentric rod, or with a templet if the radius is inconveniently long, draw arcs X X, Y Y, touching the valve circle in I and E. These may be termed the 'interior and exterior eccentric dead-centre arcs.' Through O draw a parallel arc Z Z which may be termed the 'mid-travel arc.' Also, at a distance from O equal to the

[1] 'Traité des Machines à Vapeur,' Paris, 1886.

outside lap o, draw parallel arcs, which may be termed 'lap arcs.'

When the crank is at O E the eccentric radius is at O B, and the travel of the valve from mid position is $\xi_0 = \text{B}\,b$ *exactly*. The opening of the port is less than the valve travel by the amount of the outside lap o—that is, by the quantity $b\,d$. Hence d B is the opening of the port at the beginning of the stroke or the lead e.

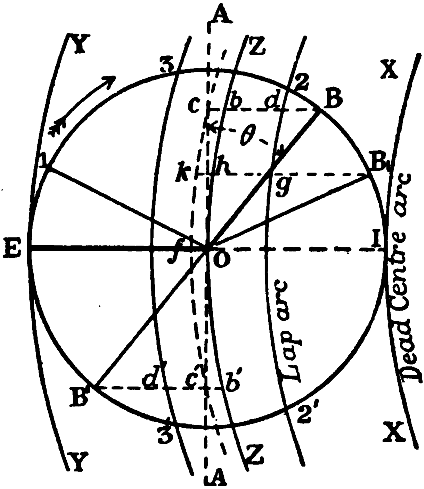


Fig. 221

When the crank has come to I, the eccentric will be at B′, the valve travel will be $\xi_0' = \text{B}'\,b'$, and the port opening B′ d'. Now, it is easy to see that, in consequence of the obliquity of the eccentric rod, B′ b' is greater than B b, and the lead B′ d' greater than B d, by the quantity $c'\,b' + c\,b$. It is customary in the workshop to adjust the valve to equal leads by lengthening the valve rod. The effect of this on the diagram is easily seen. Draw a new arc with the same radius through $c\,c'$. Then, if the valve is displaced so that

its mid position is distant $u = of$ from the centre line of the valve face, the port opening to steam will be $\xi_0 - o + u = \text{B}d + bc$, and $\xi_0' - o - u = \text{B}'d' - c'b'$ for the right and left hand ports respectively, and these quantities are now equal.

If the crank comes to I and the eccentric to B_1, turning through equal angles, the valve travel will be $\text{B}_1 h$ and the port opening will be $\text{B}_1 g$, if the valve is not adjusted, or $\text{B}_1 g + hk$, if the valve is adjusted to equal leads. The quantity u is to be added to the valve travel for all positions of the eccentric to the right of A A and deducted for all to the left. If new lap arcs are drawn at a distance u to the left of the old ones, the port opening is the horizontal distance from any eccentric position B_1 to the new lap circle.

If the eccentric rod is indefinitely long the arcs X Y Z become straight lines parallel to A A, and in that case no adjustment of the valve to equal leads is necessary.

242. *Complete valve diagram, drawn both for a long and short eccentric rod. Determination, also, of piston positions.*— Figs. 222 and 223 show complete valve diagrams, fig. 222 being drawn for the case where the eccentric rod is very long compared with the eccentric radius; fig. 223 for the case where the eccentric rod is exceptionally short.

Let E A B I be the crank-pin circle, E and I being the dead points. Through I and E draw arcs touching the circle with radius equal to the connecting-rod length. Then for any position O I of the crank the piston will be at a distance I m from one end of its stroke and I n from the other, *exactly*. The piston positions are thus given in terms of the crank positions.

Now, on this diagram the valve diagram shown in fig. 221 is to be superposed, such scales being chosen that O E represents both the crank and eccentric radius. It is convenient to turn back the eccentric, fig. 221, through an angle $90° + \theta$, so that O B may coincide with O E, and the piston

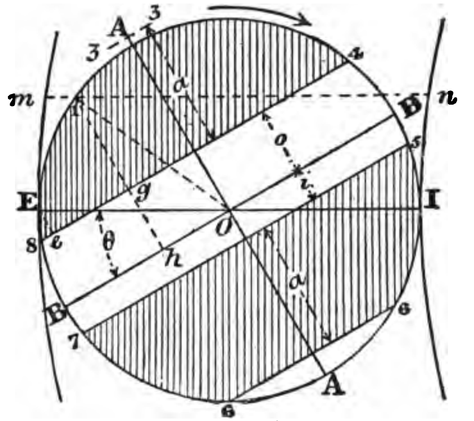


Fig. 222

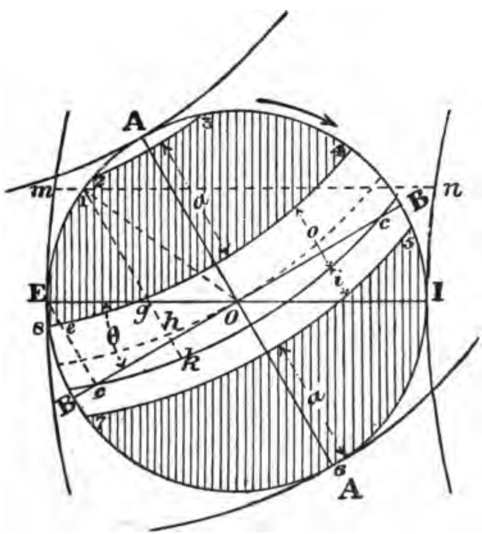


Fig. 223

and valve travel may be measured from the same point of the valve circle. In fig. 222 draw B B making an angle θ equal to the angle of advance with E I. Then the eccentric radius having been turned back $90° - \theta$ relatively to the direction of rotation coincides with the crank, and the valve travel is to be measured from the crank position at right angles to B B. Draw A O A at right angles to B B. Draw 4 8 and 5 7 parallel to B B at distances equal to the outside lap o and inside lap i. Draw 6 6 and 3 3 at a further distance equal to a, the width of port.

For any crank position, O I, the whole valve travel ξ from mid position will be I k, measured parallel to A A, and the opening of the port to steam will be $\xi - o =$ I g. If the crank position is taken in the lower semicircle we get the opening of port to exhaust in the return stroke.

The valve opens to steam at 8, and E e parallel to A A is the lead. The port remains open till the crank reaches 4, when steam is cut off. The valve, in this case, travels a little short of the port edge 3 3 at the extreme travel. At 5 the exhaust edge opens and steam is released. The port remains wide open to exhaust as the crank passes from 6 to 6, and compression begins when the crank is at 7. The corresponding positions of the piston can be found by the distances of the points on the crank-pin circle from the dead-centre arcs.

Fig. 223 is the same diagram with the correction necessary if the eccentric rod is short and if the valve is adjusted to equal leads. At A A draw arcs touching the valve circle with radii equal to the eccentric-rod length. Draw B B as before, making the angle θ with E I. From E drop a perpendicular E c on B B. Through c draw an arc parallel to the arcs at A A. This arc will cut the valve circle in those positions of the crank pin in which the centre of valve and centre of cylinder face coincide, after the valve has been adjusted to equal leads. From the arc through c c, parallel

to A A, measure distances equal to the outside and inside laps and draw the lap arcs parallel to the arcs at A A.

The port opens to steam when the crank is at 8 and E e is the lead. The port is fully open with the crank at 2, and begins to close with the crank at 3; cut-off occurs with the crank at 4; release begins with the crank at 5; the port is just fully open to exhaust with the crank at 6, and compression begins with the crank at 7. The widths of the shaded parts measured parallel to A A are the widths of port open.

To study the action of the steam in the other end of the cylinder it is only necessary to set off the laps o and i in reversed positions relatively to the arc through $c\,c$ and then to draw the lap arcs.

Problems.—Given θ, ρ, o, and i. Then the valve diagram is obtained precisely as described above.

Given θ, ρ, e and the lead to exhaust e'. From E with e as radius describe a circle. Draw 8 4 (fig. 222) or the arc 8 4 (fig. 223), touching this circle and parallel to B B or the arc through c. With centre I and radius e' describe a circle, and draw 5 7 or the arc 5 7 touching this and parallel to B B or the arc through c. The inside and outside laps are then determined.

Given ρ, e, and the fraction of the stroke at which steam is to be cut off. To find the outside lap and angle of advance. With centre E and radius e describe a circle. At the given fraction of the stroke measured from E towards I draw an arc parallel to the arc at I. This will cut the valve circle in the crank position 4, at which steam is to be cut off. Draw 4 8 parallel to B B and touching the circle described round E.

Given the fraction of the stroke at which steam is to be released, together with o and θ to find the inside lap. Along E I set off the distance the piston moves before release takes place. Through the piston position at release draw an arc parallel to the arc at I. This gives 5, the crank

position at release. Draw 5 7 parallel to B B; this determines *i*.

Given ρ, e, and the fractions of the stroke at which cut-off and compression occur. To determine θ and the inside and outside lap. With centre E and radius e describe a circle. Setting off along E I the lengths of stroke which correspond to admission and compression, draw arcs parallel to the arc through I. These determine the crank positions 4 at cut-off and 7 at compression. Draw 4 8, touching the circle round E. This determines the angle of advance and outside lap. Draw 7 5 parallel to 4 8; this determines the inside lap.

243. *Zeuner's polar valve diagram for a simple valve.*[1]—The valve diagram of Prof. Zeuner, of Dresden, is one of the simplest, and it permits the obliquity of the connecting rod to be taken into account without much complexity, though not the obliquity of the eccentric rod. As it consists entirely of circles it is easily drawn, and gives a very approximate solution of most practical problems on valve gears by easy constructions. This principle is this: The travel of the valve from the middle of its stroke is given directly for any crank position; thence the corresponding piston position can be found.

Let fig. 224 represent the mechanism of an engine, which for definiteness is supposed horizontal. O b, $b\,c$ are the positions of the crank and connecting rod, and O f, $f\,g$ the corresponding positions of the eccentric radius and eccentric rod. The crank is supposed moving in the direction of the arrow. By setting off from $a_0\,a_1$ lengths equal to $b\,c$, the piston stroke can be marked out. By setting off from $s_0\,s_1$ lengths equal to $f\,g$, the valve stroke can be marked out. Further, it may be noted that the angle b O f is equal to 90° + the angle of advance, or 90° + θ.

[1] For fuller information consult Zeuner's 'Treatise on Valve Gears,' translated by Professor Klein.

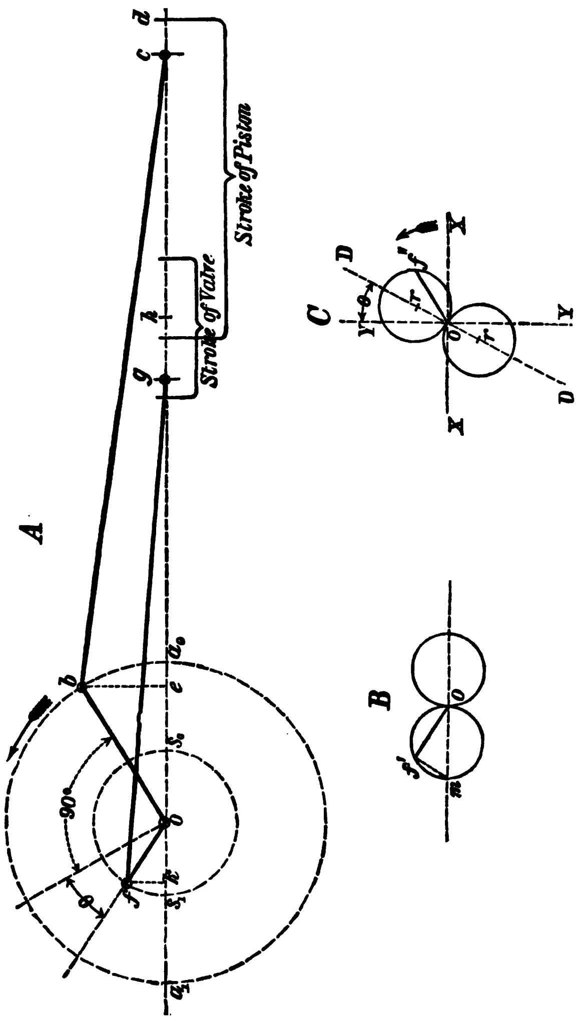


Fig. 224

244. *Theorem I. The polar locus of the valve travel reckoned from mid-stroke is a pair of circles.*—Draw $b\,e$, $f\,k$ perpendicular to the line of stroke, and bisect the valve stroke in h. Confining attention at present to the movement of the valve, the valve travel, reckoned from mid-stroke, in the position of the mechanism shown, is $\xi = h\,g$. Since $\mathrm{o}\,h = f\,g$, and (when the eccentric rod is long compared with the eccentric radius) $k\,g = f\,g$ nearly, therefore $\mathrm{o}\,k = g\,h$ nearly. In Zeuner's diagram the quantity $\mathrm{o}\,k$ is taken as a sufficiently accurate approximation to the valve travel ξ.

Now in diagram B, with centres on any horizontal line, and radii equal to half the eccentric radius, draw the two valve circles touching at O. Draw $\mathrm{o}\,f'$ parallel to $\mathrm{o}\,f$ in diagram A and join $f'\,m$. In the triangles $\mathrm{o}\,f\,k$, $\mathrm{o}\,f'\,m$, $\mathrm{o}\,m = \mathrm{o}\,f$, the angle $m\,\mathrm{o}\,f' = k\,\mathrm{o}\,f$, and the right angle $\mathrm{o}\,f'\,m = \mathrm{o}\,k\,f$. Hence $\mathrm{o}\,f' = \mathrm{o}\,k$. Therefore the diagram B will answer as a valve diagram, because the radius vector $\mathrm{o}\,f'$ drawn parallel to any position of the eccentric will be approximately enough equal to the travel of the valve from its mid position; and knowing the travel of the valve, it will be easy to infer the condition of opening of the ports to steam and exhaust. The diagram B, however, will be more convenient if it is rotated backwards through an angle $90° + \theta$, as shown at C. Then the valve travel $\mathrm{o}\,f'$, which in diagram B is parallel to the eccentric radius in A, comes to $\mathrm{o}\,f''$ in diagram C, parallel to the crank $\mathrm{o}\,b$ in diagram A.

The diagram C is the simplest form of Zeuner's diagram. Two circles are drawn which are termed *valve circles*, the diameters of which are each half the total travel of the valve, and which touch at O. The centres of the valve circles are on a line which makes an angle $90° - \theta$ with the crank at the beginning of the stroke. *Then if a line* $\mathrm{o}\,f''$ *is drawn parallel to any position of the crank of the engine, the intercept* $\mathrm{o}\,f''$ *is the travel* ξ *of the valve reckoned from the middle of its stroke, for that position of the crank.* To draw the valve

diagram C correctly proceed thus: Take any rectangular axes, X X, Y Y, the former being parallel to the line of stroke. From Y Y set off the angle of advance θ *towards the initial position of the crank.* On the line D D so obtained take O r = O r = half the eccentric radius. Then $r\,r$ are the centres of the valve circles. Lastly, the radius vector of the valve circles, drawn from O parallel to any position of the crank, is the corresponding travel ξ of the valve from its mid position.

Generally it is necessary to find the piston positions also, and this can easily be done when the crank position is known. In cases where the obliquity of the connecting rod may be neglected, it is only necessary to drop a perpendicular $b\,e$ diagram A, on the line of stroke, then $a_0 e$ will be the travel of the piston while the crank pin moves from a_0 to b. If to any scale $a_0 a_1$ represents the whole piston stroke, e will be the piston position when O b is the crank position. Neglecting the obliquity of the connecting rod, however, introduces a not inconsiderable error. The exact piston travel can be ascertained by either of the methods in § 236.[1]

245. *Zeuner's valve diagram for a simple valve with lap circles.*—In fig. 225 let I E, Y_0 Y_1 be the rectangular axes, the motion of the crank being from I to E in the direction of the arrow. Draw D_0 D_1 making the angle of advance θ with Y_0 Y_1 on the side of I. Take O D_0, O D_1, each equal to the half travel of the valve or to the eccentric radius, and on these lines as diameters describe the valve circles. Take O I equal on any scale to the crank radius, and draw the crank circle I 2 3 E. With centre O and radius equal to the outside lap o, draw the outside lap circle $a\,a$; with centre O and radius equal to the inside lap i, draw the inside lap circle $b\,b$.

The port opens to steam when the valve has travelled a

[1] The geometry of Zeuner's diagram and the working out geometrically of a number of problems is given in a treatise by Mr. Cowling Welch on 'Designing Valve Gearing.'

distance equal to the outside lap so that $\xi = o$. Hence O *a* I is the position of the crank when the valve opens. At the

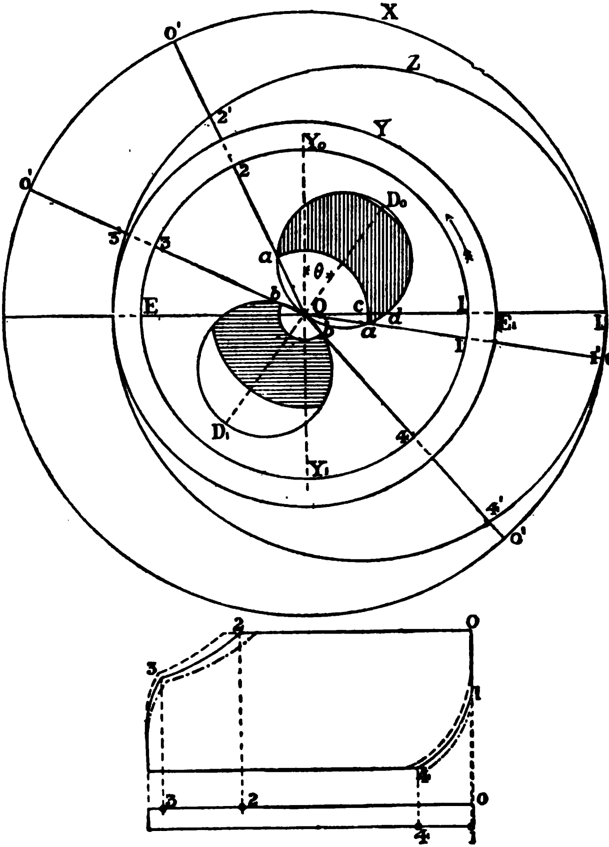


Fig. 225

beginning of the stroke, when the crank is at O I, the travel of the valve is O *d*. Hence $cd = \xi - o$ is the lead *e*. The valve closes to steam when ξ is again diminished to *o*.

Hence O a 2 is the position of the crank when the valve closes and expansion begins. Similarly O b 3 is the position of the crank when the valve opens to exhaust, and O b 4 that when the valve closes to exhaust and compression begins.

By drawing circles with centres at O and radii equal to $a + o$ and $a + i$, where a is the width of port, we mark out the periods during which the valve is fully open to steam and to exhaust. In the figure the steam-port width has purposely been taken narrower than is usual, and the opening of the port is shown by the width along any vector of the shaded areas. More commonly the width of port is at least such that the shaded area extends to D_0. It should be noted that D_0 D_1 bisects the angles a O a and b O b.

The lower figure is a horizontal projection of the points determined by the valve diagram on the crank-pin circle. It gives the proportionate lengths of stroke for each period, if the obliquity of the connecting rod is neglected. Thus, O 2 is the period of admission; 2 3 the period of expansion; 3 4 the period of exhaust; 4 1 the period of compression; and 1 O the period of preadmission.

The point 2 is the point of cut-off, 3 that of release, and 4 that of cushioning.. Round the valve diagram Müller circles have been drawn, 1′ E′ being the stroke (§ 236). Then radial distances such as 2′ O′ between Z and X give the true piston travel, taking into account the obliquity of the connecting rod. With these distances the corrected indicator diagrams for the forward and return stroke have been drawn, and are shown by dotted lines in the figure below.

Suppose that in designing a valve gear there are given the ratio of cut-off z, the eccentric radius or half valve travel ρ, and the lead e. In fig. 226 take X_0 X_1 parallel to the line of stroke. With radius ρ describe the circle D K A. Find the position O K 2 of the crank at which expansion begins. This is found approximately by taking $\frac{BC}{AC} = z$, or

more exactly by the method above, § 236. Join A K, and take K E, equal to the given lead *e*. Bisect A E in F. Then A F = F E is the necessary outside lap *o*. Take O G = F K, and draw G D perpendicular to X X, meeting the circle C D K A in D. Then D O Y is the required angle of advance. On O D describe the valve circle. Take O H = F E, and through H

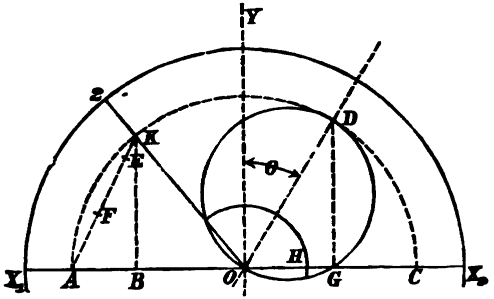


Fig. 226

draw the lap circle. The valve diagram can then be completed by drawing the other valve circle and the inside lap circle.

246. *Modifications of the ordinary valve arrangement.*—So far the line of stroke of the valve has been taken to be parallel to the line of stroke of the piston, and in the plane containing the axis of the crank shaft and the axis of the cylinder. This is the ordinary arrangement. But modifications are sometimes necessary. In the following figures the notation is the same as in fig. 224. O *b* is the crank at the inner dead centre, O *f* the eccentric radius, *fg* the eccentric rod, θ the angle of advance calculated from the lap and lead, O *d* the line of piston stroke.

Case I. The line of stroke of the valve is inclined to the line of piston stroke at an angle β, fig. 227, I, so that the valve reciprocates along the line O *g*. The valve must have precisely the same motion along O *g* as in the ordinary arrangement along O *d*. Hence the eccentric must be keyed

at $90° + \theta + \beta$ in advance of the crank. The valve diagram is to be drawn for an eccentric radius ρ and angle of advance θ as in the ordinary arrangement.

Case II. Motion is communicated from the eccentric to the valve through a lever, fig. 227, II.—The lever A B is pivoted

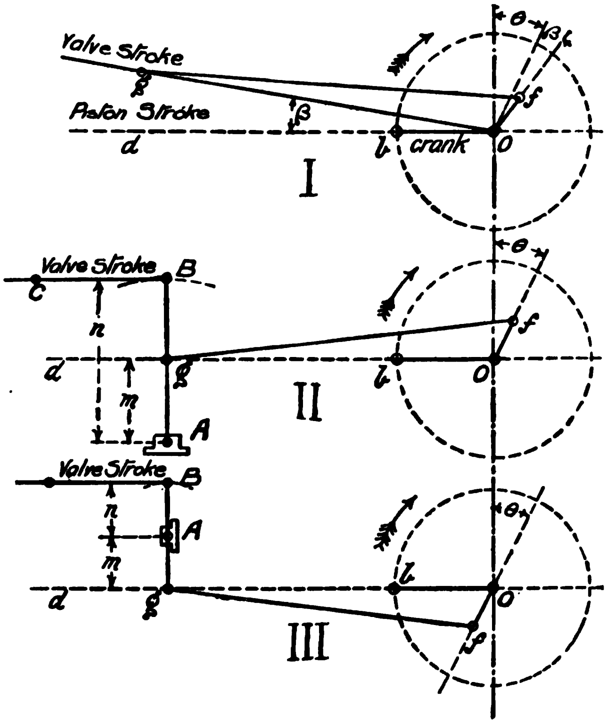


Fig. 227

at A, and the valve driven parallel to the line of piston stroke by a link B C. Neglecting the small influence of the obliquity of the link, the motion at C is similar to that at g, but increased in the ratio n/m. Hence if $\text{O}f = \rho$ is the actual eccentric radius, the valve moves as if driven by an eccentric

with the same angle of advance, and with the virtual radius $\rho' = \rho\, n/m$. The valve diagram is drawn as in the ordinary arrangement with the virtual eccentric radius ρ' and angle of advance θ. In finding θ from the lap o and lead e, $\sin\theta = (o + e)/\rho'$.

Case III. Motion communicated from the eccentric to the valve by a reversing lever, fig. 227, III.—The lever A B g is pivoted at A, and the motion of the valve is similar to that of the point g, reversed in direction and altered in the ratio n/m. As the valve must have the same motion as in the ordinary arrangement, the eccentric must be advanced through an angle of 180° to neutralise the reversal by the lever. Hence the eccentric must be keyed at $270° + \theta$ in advance of the crank. If ρ is the actual eccentric radius, the virtual eccentric radius is $\rho' = \rho\, n/m$. The valve diagram is drawn as for the ordinary arrangement, but with the virtual eccentric radius ρ' and an angle of advance θ such that $\sin\theta = (o + e)/\rho'$.

If the line of stroke of the valve is not parallel to the piston line of stroke, then the lever should be bent so that the arms A g, A B, are perpendicular to the lines of stroke in the mid position of the valve.

CHAPTER XV

EXPANSION VALVES

247. As in the case of simple slide valves it was necessary to confine attention to the most ordinary forms, so in dealing with the very extensive subject of Expansion Gears and Reversing Gears it will be necessary to select only some much-used types.

With a single slide valve the steam edges and exhaust edges are rigidly connected. Hence if any alteration of travel is made to vary the cut-off, the period of exhaust is also altered and generally in a way which is prejudicial. It was not uncommon at one time, especially in marine engines, to attempt to secure an earlier cut-off than was possible with a single slide valve, by placing a second steam chest above the ordinary steam chest, with an expansion valve moved by a separate eccentric. As this second valve only regulated the admission of steam, it could be arranged to cut off at varying points of the stroke, by varying the valve travel, without entailing the evil of excessive compression during exhaust. But with this arrangement, the whole steam chest containing the main or distributing valve formed part of the clearance volume of the cylinder, up to the moment at which the main valve closed the cylinder port. This large clearance space almost nullified the action of the separate cut-off valve.

Since the steam in the clearance expands with that in the cylinder, it is obviously important, in engines intended to work with an early cut-off, to reduce the clearance as

much as possible. This is fairly well accomplished by putting the expansion valve on the back of the main valve, so that the addition to the clearance space, between the closing of the expansion valve and the closing of the main or distributing valve, is only the volume of the short passage through the main valve.

Fig. 228 shows an ordinary slide valve, in its central position, over the cylinder face. This valve has been extended at the ends and ports formed through it, usually of the same width as the ports in the cylinder face. On the

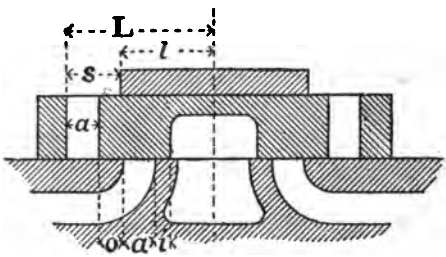


Fig. 228

back of this is an expansion plate driven by a separate eccentric. The expansion plate will cut off steam at the left port, if it travels a distance s to the left, relatively to the main valve; that is, if its centre line passes to the left of the centre line of the main valve by a distance s.

248. *Action of an expansion plate cutting off steam at its outside edges.*—Let O I, fig. 229, be the crank at its inner dead point; let O a be the main valve eccentric of radius r_1 and angle of advance θ_1; and let O b be the expansion eccentric of radius r_2 and angle of advance θ_2. Join $a\,b$, draw O c equal and parallel to $a\,b$, and drop perpendiculars $a\,d$, $b\,e$, $c\,f$ on I E.

In the position shown, neglecting the eccentric-rod obliquity, the main valve will be at a distance O $d = \xi_1$ to the left of its mid position and the expansion plate at a distance

o $e = \xi_2$. Consequently the relative travel, or distance of centre of expansion plate to the left of centre of main valve, is $\xi_2 - \xi_1 = e\,d$. This is the projection of $a\,b$ on I E. But o c is equal and parallel to $a\,b$, hence o$f = d\,e$. Now suppose o c is a third eccentric fixed on the same shaft as the others and revolving with them. In all positions, o $c = \rho$ will be equal and parallel to $a\,b$. In all positions, the projection of o c on I E will be the difference of the projections of o a and o b, that is, to the distance apart of the plate and valve centres. Hence the relative motion is the same as if the main valve were at rest and the expansion plate driven by

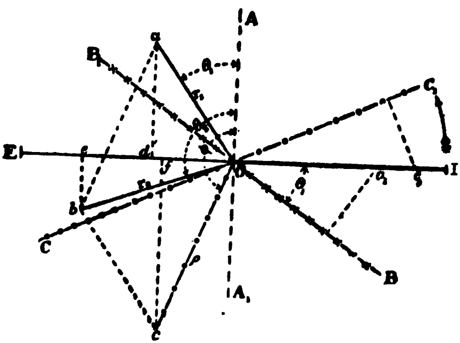


Fig. 229

an eccentric, o c. Hence o c may be termed the *relative eccentric* or virtual eccentric of the relative motion. This relative eccentric has the radius $\rho =$ o c and the angle of advance A o $c = \phi$. The relative travel $\zeta =$ of for any position will be equal to the perpendicular from c on A A_1, and will be a movement of the expansion plate to the left relatively to the main valve if c is to the left of A A_1, to the right if c is on the right of A A_1. When o c coincides with A A_1 the centre lines of the valve and plate coincide.

If, as in fig. 224, we turn backwards the valve circle for o a through an angle $90° + \theta_1$, then a will come to a_1 on the

crank and AA_1 to BB_1. The travel of main valve from mid position will now be the perpendicular from a_1 on BB_1 and will be a travel to the left if a_1 is above BB_1. Similarly, if the relative eccentric circle is turned back through an angle $90° + \phi$, c will come to the crank at c_1, and AA_1 to CC_1. The relative travel of expansion plate will be the perpendicular from c_1 on CC, and will be a travel to the left if c_1 is below CC_1.

In fig. 230, with crank radius R draw the crank circle having its internal and external dead points at I, E. At I

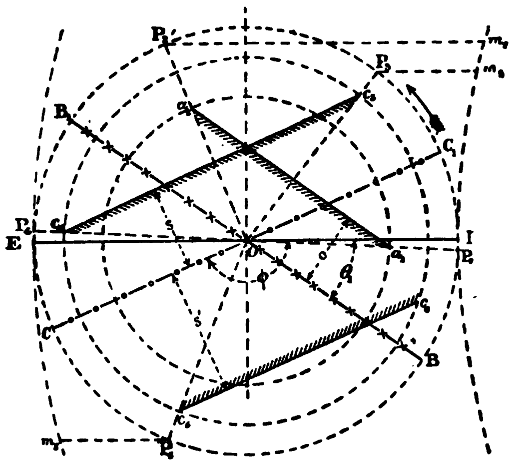


Fig. 230

and E with radius equal to the connecting-rod length draw the dead-centre arcs (see fig. 221). With radius $o\,a_1 = r_1$ draw the main-valve circle, and with radius $o\,c_3 = \rho$ draw the relative-motion circle. Suppose the directions BB_1 and CC_1 determined as in the previous figure; that is, BB_1 makes an angle θ_1, and CC_1 an angle ϕ, reckoned backwards to the direction of motion from IE. Draw $a_1\ a_2$ parallel to BB, at a distance equal to o, the outside lap of the main valve,

and draw $c_3\, c_4$ parallel to C C$_1$ at a distance equal to s in fig. 228.

The main valve will open the right-hand port when the crank is at O P$_1$, because at that moment the travel of the main valve to the left will be the perpendicular from a_1 on B B$_1$, which by construction is equal to the outside lap. It will close the port again when the crank is at O P$_2$, because then the diminishing travel from mid position, the perpendicular from a_2 on B B$_1$, is again equal to the outside lap. The piston travel when steam is cut off will be P$_2$ m_2 drawn parallel to I E.

Now consider the action of the expansion plate. It has already been shown that the passage through the main valve will be closed when the relative travel is equal to s. But when the crank is at O P$_3$ the relative travel is the perpendicular from c_3 on C C$_1$, which by construction is equal to s. The piston travel when the expansion plate cuts off will be P$_3$ m_3. The expansion plate will reopen the port through the main valve when the crank is at O P$_4$, the relative travel being again equal to s. It is obvious that P$_4$ must fall later in the stroke than P$_2$ where the main valve cuts off, or steam will be admitted to the cylinder a second time in the stroke.

Suppose it is required that steam should be cut off at the same fraction of the forward and return stroke. Take P$_5$ m_5 equal to P$_3$ m_3. Then O P$_5$ is the crank position at cut-off in the return stroke. Draw $c_5\, c_6$ parallel to C C$_1$. Then the distance s' of $c_5\, c_6$ from C C$_1$, is the value of s at the left-hand port necessary to give the same cut-off as in the forward stroke. It is obvious that, in consequence of the obliquity of the connecting rod, the centre of the expansion plate must be displaced a distance $\frac{1}{2}(s' - s)$ by lengthening the valve rod if the cut-off is to be equalised for both strokes.

249. *Meyer variable expansion gear.*—The action of the much-used Meyer expansion gear is easily derived from

that of the expansion plate, already treated. Fig. 231 shows the arrangement of this gear. There is an ordinary slide valve, which is extended in length, and two admission ports formed in it equal in width to the ports in the cylinder face. This valve is driven by an eccentric in the ordinary way and

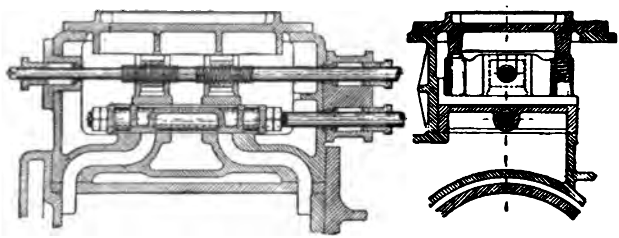


Fig. 231

completely controls the exhaust from the cylinder. The back of this valve is formed into a valve face, and a pair of expansion plates slide on this driven by a second eccentric. The function of these is to cut off the steam at any desired point of the stroke by closing the ports through the main

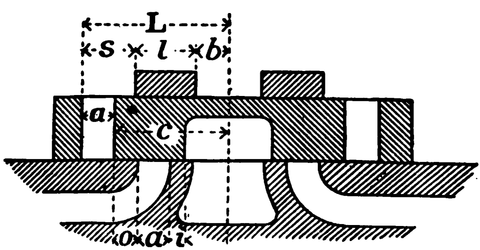


Fig. 232

valve. The expansion plates are connected by a right and left hand screw, which can be rotated by a handwheel outside the steam chest. By rotating this screw the distance between the expansion plates and the length between the cutting-off edges can be altered, the centre line between

the plates keeping the same position. As the distance between the plates is increased cut-off occurs earlier in the stroke.

Fig. 231 is a diagram of the valve and expansion plates in mid position. Let ρ be the radius of relative eccentric; then the greatest travel of the expansion plates each way from the centre of the main valve is ρ.

Let b_1 and b_2 be the greatest and least values of the half distance between the plates, and s_1, s_2 the corresponding values of s. Then b_1 and s_1 will correspond to the earliest cut-off, and b_2 and s_2 to the latest. In order that the expansion plate may not reopen the port at its inside edge, which is most likely to occur when the plates are furthest apart,

$$b_1 + \rho \leqq b_1 + l + s_1 - a$$

$$l \geqq \rho + a - s_1,$$

a relation which determines the necessary length of plate when s_1 is ascertained from the valve diagram.

On the other hand, it is unnecessary for the expansion plates to cut off later than the main valve, and for that grade of expansion they may reopen immediately, for the main valve will have closed the steam port. To secure this condition

$$s_2 = \rho$$

and the plates are then closest together. Hence, if the distance L has to be made as small as possible,

$$L \geqq l + \rho,$$

a relation which determines the minimum distance between the ports through the main valve. The ports may have to be sloped to get room enough for the expansion plates.

It is common to make the expansion eccentric radius r_2

equal to the main valve eccentric radius r_1. But a sharper cut-off is obtained and the arrangement of the valve is made easier by taking

$$r_2 = 1\tfrac{1}{4}\, r_1.$$

250. *Complete diagram for a Meyer gear.*—Fig. 233 is a complete valve diagram for a Meyer valve gear. The

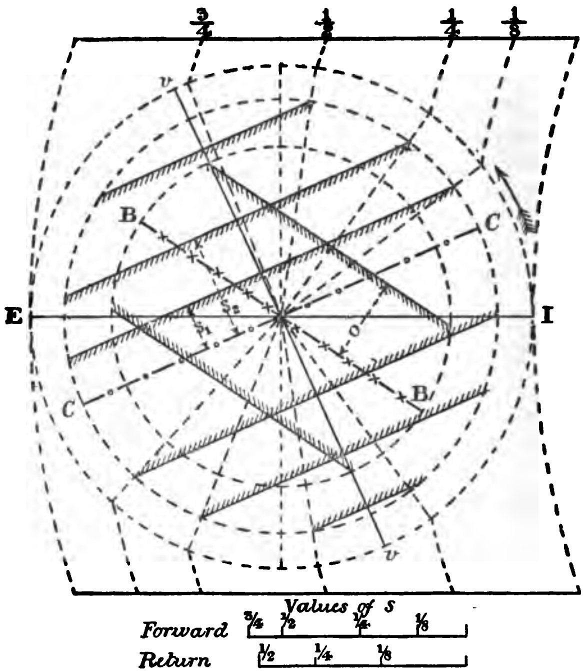


Fig. 233

three circles—crank circle, relative eccentric circle, and main valve eccentric circle—are first drawn. Then with radius equal to the connecting-rod length the dead centre arcs at I and E. On the stroke line any desired points of cut-off $\frac{1}{8}$, $\frac{1}{4}$ . . . are marked and arcs through these parallel to the

dead-centre arcs cut the crank circle in positions of the crank for those grades of expansion.

Now draw B B, making the main valve angle of advance θ_1 with I E, and parallel to it the lap lines for the main valve. The crank positions at which the main valve closes the steam ports are given at *v v*. It will be seen that the main valve cuts off a little after $\frac{3}{4}$ stroke in the forward stroke and a little before it in the return stroke. The difference due to the obliquity of the connecting rod is here marked because a short connecting rod has been chosen.

Next draw C C, making the relative eccentric angle of advance ϕ with I E, reckoning in a direction reverse to the crank's motion. If the latest cut-off of the expansion plates is to be that at which the main valve cuts off, C C should be at right angles to the position *v* of the crank at which the main valve cuts off. If the relative eccentric radius is chosen and its angle of advance fixed thus, then the radius and angle of advance of the expansion eccentric can be found, as in fig. 229.

At the points where the crank at the desired grades of cut-off cuts the relative eccentric circle, draw lines parallel to C C. These mark off the values s_1 s_2 . . . to which the valve must be set for these grades of expansion. The little figure below gives the values of *s* for the forward and return stroke, and these are necessarily unequal. By setting the valve a little out of centre, the cut-off may be equalised exactly for any given cut-off and approximately for the others.

251. *Application of Zeuner's polar diagram to Meyer's valve gear.*—Zeuner's diagram may be used very conveniently in designing a Meyer valve gear. Two theorems additional to the one above, § 224, for a single slide valve may be given first, in order to show how the valve circles for the relative eccentric are found.

252. *Theorem II. The polar locus of the relative travel of two valves moved by two eccentrics of different radii keyed*

at the same angle is a pair of circles.—Let fig. 234 represent two slide valves (the valve rods are suppressed for simplicity) driven by two eccentrics, Of_1, Of_2 keyed at the same angle $90° + \theta$ with the crank Ob. In diagram B take D D, making the angle of advance θ with Y Y, and with centres on D D and radii equal to half the eccentric radii draw the pairs of valve circles 1, 1, and 2, 2. Draw Oqp parallel to the crank Ob.

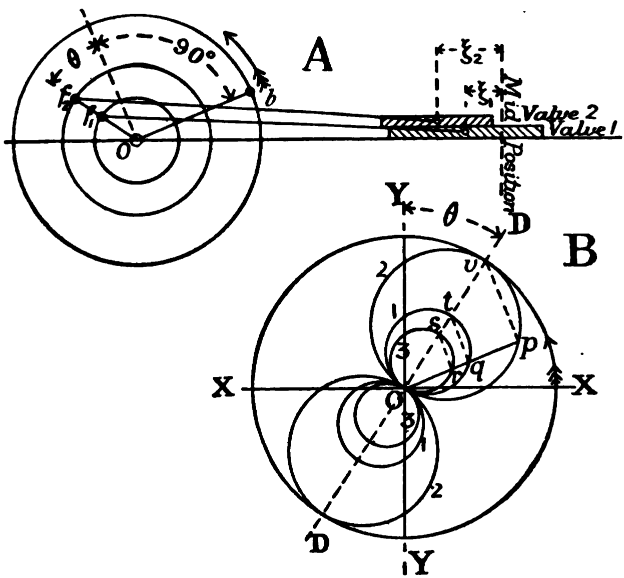


Fig. 234

In the position shown, valve 1 will have moved a distance $\xi_1 =$ Oq to the left from its mid position and valve 2 a distance $\xi_2 =$ Op. Then the relative motion or relative travel of the valves is $\xi_2 - \xi_1 = pq$. Take O$r = pq$ and join vp, tq, and draw rs perpendicular to Orp. The angles vpo, tqo are right angles, being angles in a semicircle. Hence vp, tq, sr are parallel. But Or by construction is equal to qp, and r is

always an apex of a right-angled triangle the hypothenuse of which is $o\,s$. Hence the locus of r is the pair of circles marked 3. Consequently the relative travel of two valves, reckoned from their mid position, driven by eccentrics keyed at the same angle, is given by the intercept on the crank of a pair of valve circles which touch at O, whose centres are on a line making an angle $90° - \theta$ with the initial position of the crank, and whose diameters are equal to the difference of the eccentric radii. The circles 3 3 are then the valve circles of the relative eccentric.

253. *Theorem III. The polar locus of the relative travel of two valves driven by eccentrics with different angles of advance is a pair of circles.*—In fig. 235, let 1, 1 be the valve circles for a valve having the angle of advance θ_1 and 2 2 the valve circles for a valve having the angle of advance θ_2. $D_1\,O\,D_1$ and $D_2\,O\,D_2$ are the diameters of these circles. Let $O\,p\,q$ be any position of the crank. Then one valve will have travelled $\xi_1 = O\,p$ from mid position; the other $\xi_2 = O\,q$. The relative travel is $\xi_2 - \xi_1 = p\,q$.

The three points $D_1\ a\ D_2$ are in one straight line, the angles $D_1\ a\ O$ and $D_2\ a\ O$ being angles in semicircles. Draw $D_1\,p$ and $D_2\,q$ and produce the latter to meet $O\,D$ in b. These lines are perpendicular to $O\,p\,q$ since the angles at p and q are angles in semicircles. Hence they are parallel.

Take $O\,r = p\,q$ and draw $c\,r\,d$ perpendicular to $O\,p\,q$. This meets $O\,D_1$ in c and a line $O\,d$ parallel to $D_1\,D_2$ in d. A circle 3 described on $O\,d$ as diameter will pass through r.

$O\,r$ being equal to $p\,q$, $O\,c$ is equal to $D_1\,b$. But the angle $O\,c\,r = D_1\,b\,D_2$ and the angle $c\,O\,d = b\ D_1\ D_2$. Hence the triangles $c\,O\,d$ and $b\,D_1\,D_2$ are equal, and $O\,d = D_1\,D_2$.

Hence r lies on a circle 3 described on $O\,d$ as diameter, and $O\,d$ is equal and parallel to $D_1\,D_2$, the line joining the ends of the diameters of the primary valve circles. Consequently, to draw the valve circles for the relative eccentric, take $O\,d$ equal and parallel to $D_1\,D_2$, and describe on it the circles 3 3 touching at O. $O\,a$, drawn to the intersection of

the primary valve circles, is a tangent to the valve circles for the relative eccentric; O r, the intercept on the crank, is the

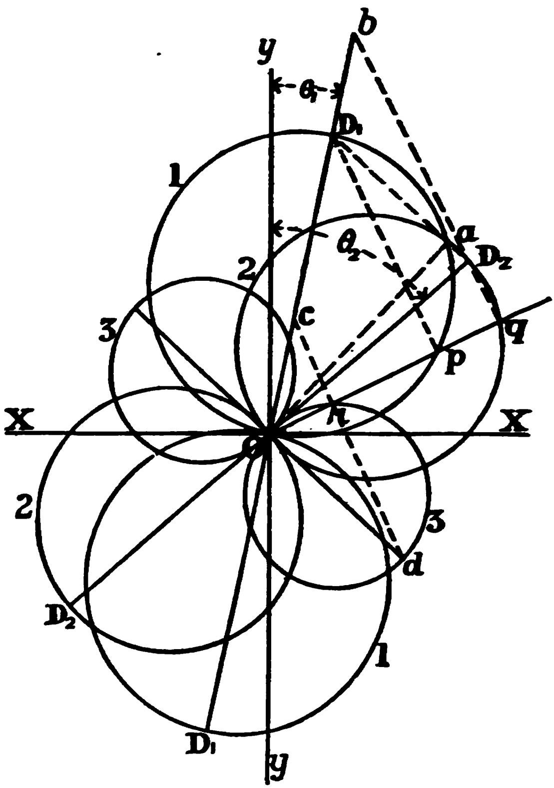


Fig. 235

relative travel of the two valves for that position of the crank.

254. *To draw a complete polar diagram for a Meyer gear.*—Take I E, fig. 236, to represent the length of stroke and draw the crank circle and dead centre arcs. On lines

a a, *b b*, which are equal to I E, set off for the forward and return strokes the points of the stroke at which the expansion valve is to cut off steam, and find the corresponding crank positions by drawing arcs to the crank circle parallel to the dead-centre arcs.

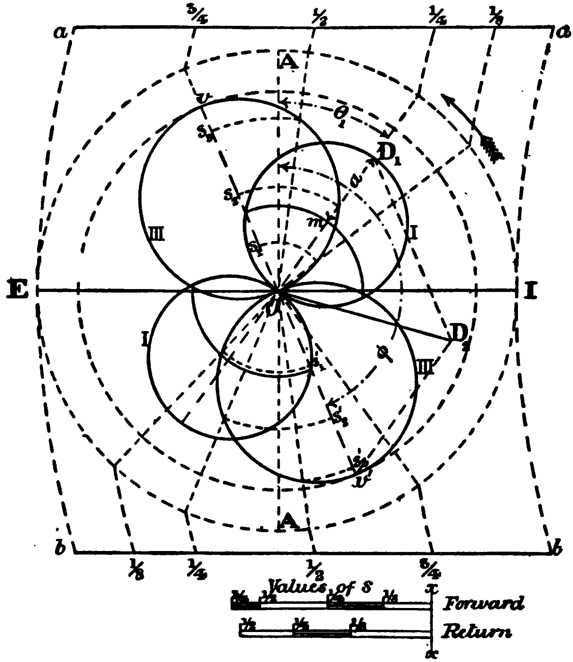


Fig. 236

Now draw the main valve circles I I, the eccentric radius being O D_1 and the angle of advance A O $D_1 = \theta_1$. With radius O $m = o$, the outside lap of main valve, draw the lap arcs. Then steam is cut off by the main valve in the crank positions O v, O v' which pass through the intersections of the valve circles and lap arcs. It will be seen that steam is cut off at

$\frac{3}{4}$ stroke in the forward stroke and at about $\frac{5}{8}$ stroke in the return stroké.

If the expansion eccentric radius O $D_2 = r_2$ and its angle of advance, $\theta_2 =$ A O D_2 are given, complete the parallelogram O D_1 D_2 v'. Then O v' will be the radius ρ of the relative eccentric, and A O v' will be its angle of advance. This will not necessarily fall, as in fig. 236, on the crank position O v', at which the main valve cuts off. But it is convenient that it should do so, for then the latest cut-off by the expansion valve coincides with that by the main valve, and the range of action of the gear is greatest. It is therefore more convenient to assume the relative eccentric O v', determining the angle of advance by the crank position O v', at which the main valve closes the steam port. Then the expansion eccentric O D_2 can be found by drawing the parallelogram O D_1 D_2 v'.

Suppose the relative eccentric radius O v found or assumed, and draw the relative eccentric valve circles III, III. A O v' reverse to the direction of motion is the angle of advance ϕ of the relative eccentric. Through the intersections of the relative valve circle and the crank positions at cut-off, draw the dotted circles. Then O s_1, O s_2, . . . are the values of s (fig. 232) for these grades of expansion. These values have been set off from $x\,x$ below. The inequalities in the values of s for any given cut-off in the forward and return stroke are obvious. By adjusting the valve so that, say at $\frac{1}{8}$ cut-off, s has the values found for both strokes, there will be less inequality of cut-off for the other grades of expansion also.

Supposing the radius of relative eccentric and its angle of advance assumed, then the expansion eccentric is found thus. Draw D_1 D_2 parallel to O v, and v D_2 parallel to O D_1. The OD_2 is the radius of expansion eccentric and A O D_2 its angle of advance.

It may be pointed out in repetition that if the main valve just fully opens the steam port, the width of port a must be the distance m D_1. O D_1 is the radius r_1 of the main

valve eccentric; O D_2 the radius r_2 of expansion eccentric; O v the radius ρ of the relative eccentric. The length l of the expansion plates must not be less than $\rho + a - s_1$, where s_1 is the value of s for the earliest cut-off. In the figure l must not be less than $s_1 v + m D_1$. The distance L from centre to edge of steam port of main valve must be equal at least to $l + \rho$ or to $s_1 v + m D_1 + O v$.

If a Meyer gear is used on an engine which reverses, the angle of advance of the expansion eccentric should be 90°, if the action is to be the same in forward and backward gear. Sometimes, however, it is necessary to be content with a good action of the expansion plates in forward gear and a more imperfect action in backward gear. Then the condition $\theta_2 = 90°$ is not imperative.

255. *Expansion by a movable eccentric.*—Suppose a

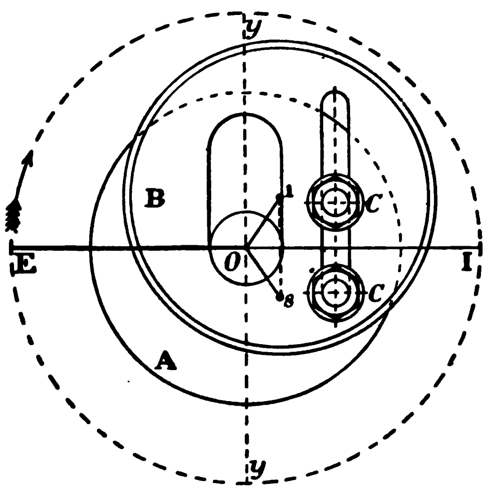


Fig. 237

plate A, fig. 237, keyed on the shaft and an eccentric cam B attached to this by bolts C C. If the cam is slotted as

shown, it may be moved by slacking the bolts so that its centre travels along the straight line 1–8. In the position drawn the eccentric cam centre is at 1, the radius of eccentricity is O 1, and the angle of advance Y O 1. But as the cam shifts the radius diminishes and the angle of advance increases.

Fig. 238 shows the valve diagrams for five positions of the cam to a larger scale. The outside lap circle has been drawn and the intersections of this with the valve circles

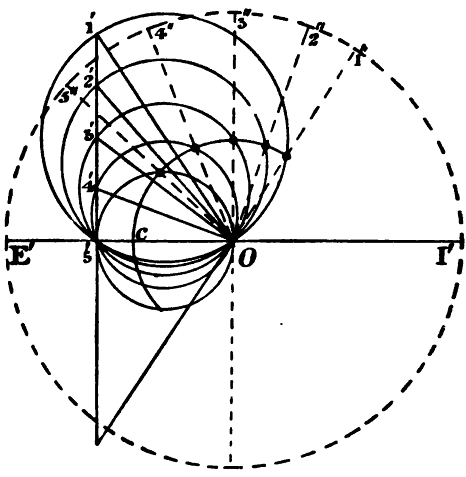


Fig. 238

determine the crank positions at cut-off 1″, 2″, 3″ . . . for the chosen positions of the cam. If the path 1–8, fig. 237, of the eccentric centre is a straight line perpendicular to the crank, the linear lead c 5′, fig. 238, is constant for all positions of the cam, though the crank angle at which steam is admitted varies. When the angle of advance is greater than 90° the engine reverses.

There are mechanical difficulties in arranging for the

shifting of the cam while the engine is running except in one important case. Of late powerful shaft governors have been introduced, keyed on and revolving with the crank shaft. It is possible to connect the shifting cam of an eccentric, such as that just described, directly with one of these governors, so that the position of the cam and consequently the point of cut-off of steam is absolutely controlled by the governor. The Turner-Hartnell, Westinghouse, and other governors are applied in this way. As the speed of the engine increases, the eccentric cam is shifted and steam cut off earlier. The path of the eccentric is not always a straight line. Then the linear amount of lead is not constant for different positions of the eccentric cam.

CHAPTER XVI

LINK MOTIONS AND REVERSING GEARS

256. *Principle of reversing gears.*—Locomotive, marine, and some other engines must have means of reversing the direction of motion of the engine, and the reversal must be effected easily and rapidly. Fig. 239 shows an engine with the crank $o\,b$ at the inner dead centre and turning in the direction of the arrow F, so that the piston is just beginning an outstroke. At that moment the eccentric will be at an angle $90° + \theta$ forward of the crank in the direction of motion, θ being the angle of advance; the valve will be in such a position that the left port is open to steam by an amount equal to the lead and the right port is open to exhaust. Suppose now it is required to move the eccentric and valve to such a position that the engine will turn in the direction of the arrow B. As in this case also the piston has to make an outstroke, the left port must be open by an amount equal to the lead, and therefore the valve position will be the same whether the engine is to turn in the direction F or the direction B. It is otherwise with the eccentric. Since the engine is to turn in the direction B, and the valve is to move to the right to continue to open the left port, the eccentric must be in a new position $o\,f'$, also $90° + \theta$ forward of the crank in the direction of B. If, therefore, in an engine turning in the direction F, the eccentric $o\,f$ is moved to $o\,f'$ the direction of the motion will be reversed. The two positions of the eccentric for forward or backward going are symmetrical to the crank, and the angle between them is $180° - 2\,\theta$, which may be called the reversing angle of the

eccentric. In certain older types of engine the eccentric was loose on the shaft and was driven by a feather fixed in the shaft. The eccentric sheave had a slot so placed that when the feather was at one end the engine drove forwards, and when at the other it drove backwards. Reversal was effected by disconnecting the eccentric rod for an instant, allowing the shaft to rotate in the eccentric till the feather was at the other end of the slot and then reconnecting the eccentric rod and valve. It is more common to have two eccentrics on the shaft, one keyed in the position of, the

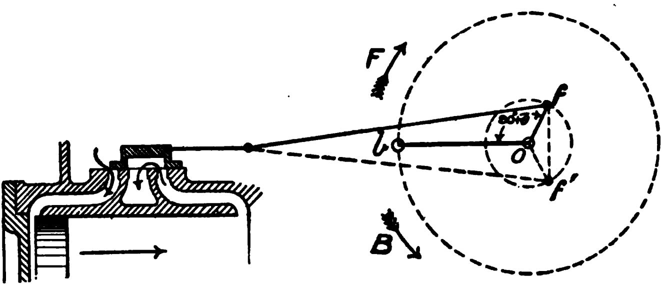


Fig. 239

other in the position of'. The ends of the eccentric rods are connected by a link, on which slides a block to which the valve rod is connected. By shifting the link, either eccentric rod can be brought opposite the slide block, and then the valve receives motion from that eccentric. Adopted at first merely for reversing, it was soon found that the link motion was a convenient expansion gear, for, if the slide block occupies a position intermediate to its extreme positions in the link, it gets a motion due to both eccentrics; its travel is reduced and cut-off occurs earlier in the stroke. Exactly at the centre of the link is a neutral position, where the motion of the valve is too small to admit steam effectively for motion in either direction.

The motion of a valve driven by a link cannot be quite

easily and simply calculated except by approximate methods. Most commonly a very beautiful extension of the polar diagram due to Zeuner is used, and Zeuner's analysis has undoubtedly shown clearly the conditions to aim at in arranging a link motion. But there is a defect in Zeuner's method for students and practical engineers. The diagram is arrived at by a somewhat tedious algebraic analysis, in the course of which, for simplicity, various approximations are adopted. No doubt, if the link motion is well designed and of normal type, the approximations are legitimate and lead to no serious practical error. But it is difficult for students or practical engineers to satisfy themselves as to the error introduced in the approximations, and if the link motion is not of normal type, if the eccentric rods are short, the centre line of the link at a distance from the eccentric-rod ends, or the mode of suspension unusual, then the error introduced may no longer be unimportant. In any case, from the difficulty of the analysis, engineers are tempted simply to adopt the final construction arrived at by Zeuner as a mere rule of thumb, and they then feel they are working in the dark as to the accuracy of the method.

It is possible to arrive at an approximate solution of the link-motion problem in a very simple way, and one involving no tedious algebraical analysis. Suppose this approximation adopted first in roughly designing the valve motion. Then, by an easily understood graphic method, the exact motion of the valve can be determined, however abnormal the proportions of the gear. If the action is found to be imperfect, modifications can be introduced and the effect of these again determined. So an engineer can satisfy himself by a method clear and familiar, and involving no suppressions or assumptions, that the valve gear will act properly.

257. *Stephenson's link motion.*—The link gear originally introduced by Stephenson is the simplest to construct and is the one most frequently used. The link (figs. 240, 241) is curved and concave to the crank shaft. o *f*, o *b* are the

forward and backward eccentrics for clockwise and counter-clockwise rotation. *a d* is the link which is curved to a radius equal to the eccentric-rod length, or, more strictly, to a radius equal to the distance from the centre of an eccentric to the centre line of the link, measured along the centre line

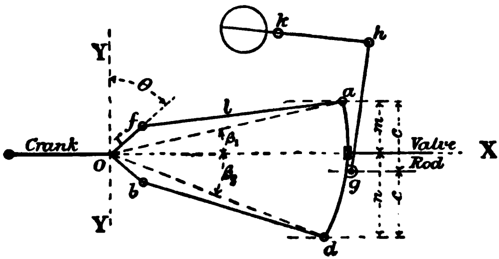


Fig. 240

of the eccentric rod. If with the crank at the outer dead point both eccentric radii are turned to the link side of the crank shaft, then the rods may have the positions shown in fig. 240, *open rods*, or that in fig. 241, *crossed rods*. Either

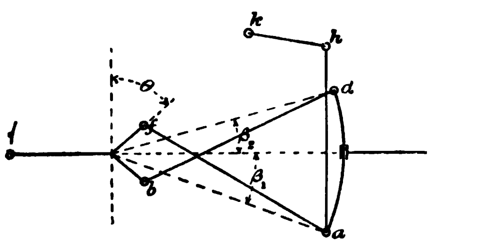


Fig. 241

arrangement can be adopted, but there are differences in the action of the gear according as the rods are open or crossed. With either arrangement the cut-off is earlier as the block is moved towards the centre of the link, by linking up or notching up as it is termed. But in other respects

the action when linking up is different. The following table shows what happens as the block is moved towards the centre of the link:

	Open rods	Crossed rods
Lead (steam edge) .	Increases	Decreases
Cut-off	Earlier	Earlier
Lead to exhaust . .	Increases	Decreases
Compression begins .	Earlier	Earlier

The excessive compression when an early cut-off is obtained by a link is an objection to the gear except with quick-rotation engines. Usually the eccentrics have equal angles of advance. By giving slightly unequal angles of advance the lead can be made nearly constant for forward running at the expense of greater variation in the backward running. If the link can be moved so that the block is precisely opposite an eccentric-rod end, the half stroke of the valve is equal to the eccentric radius, unless modified by a rocker. Then the eccentric is of the smallest size and its friction is least. On the other hand, the eccentric rod must then usually be jointed to the link at a distance from its centre line, and this introduces some irregularity in the motion of the valve. If the eccentric is jointed to the link on its centre line, then for most forms of link the block cannot be brought opposite an eccentric-rod end. Then the eccentric radius must be greater than the half travel of the valve in full gear. The eccentrics, eccentric rods, and link form a four-bar chain, which is indefinitely deformable. To give a definite motion to the valve, one point of the link must be guided. This is usually done by suspending the link by a suspending link $g\,h$, fig. 240, carried by a lever $h\,k$, which often carries a balance weight to balance the weight of the link and rods. By moving the lever $h\,k$ the link block is brought to any part of the link. The lever is held in position in working by a catch, which drops into one of a set of notches.

The mode of suspension is important. The lever *h k* should be parallel to the line of stroke of the valve when the link block is at the neutral point of the link. When the valve is in mid position, a line through *g* perpendicular to the line of stroke should bisect the versed sine of the arc in which *h* moves. The lever *h k* should be as long as possible, usually not less than a quarter of the eccentric-rod length. Lastly, the suspending link *h g* should be as long as convenient. The object to aim at is that *g* should move as nearly as may be parallel to the line of stroke. There is least slip of the link block in the link if *g* is at the middle of the link. But the suspending rod is often attached to the lower end of the link, fig. 241, to get a longer suspending link. The slip of the link block in the link is a serious practical evil, because it causes wear and consequent slackness of fit.

258. *General arrangement of Stephenson's link motion.*— Fig. 242 shows an ordinary form of link motion in full forward gear. o *f* and o *b* are the eccentric radii for forward and backward motion. The link block is attached to the lower end of a rocker or reversing lever *r*, which at its upper end drives the valve by a link. *s* is the suspending rod of the link attached at its lower end to the link by a saddle piece, and at its upper to the gear for linking up or reversing. Fig. 243 shows various forms of link. A and B are termed slotted links; C is a double solid-bar link. In B the eccentric-rod pins *a a* are on the link arc or centre line of the link; then the block cannot be brought opposite an eccentric. In A the eccentric pins *a a* are behind the link, and the block can be brought opposite each eccentric. In C the eccentric pins are on the link arc, and at the same time the block can be brought exactly opposite an eccentric. In A and C the suspending rod *s* is attached to the bottom of the link, so that it is as long as possible. In B the suspending rod is attached near the centre of the link, which secures rather more uniform

motion of the valve in forward and backward gear. The strongest link is that shown at C which is often used in

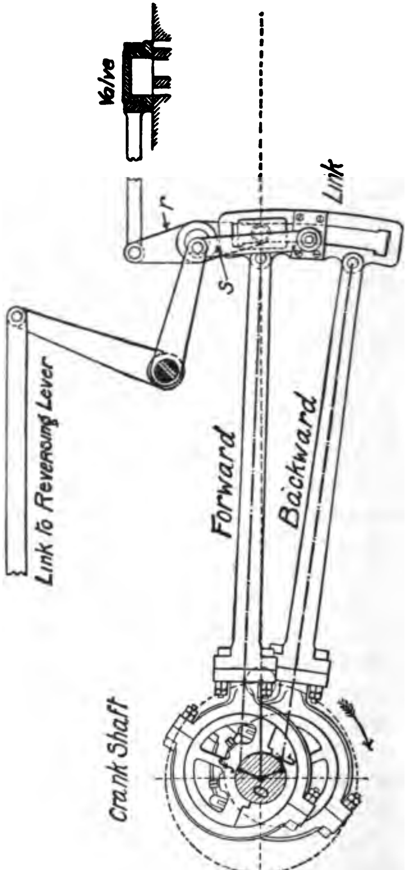


Fig. 242

marine engines. The most compact is A, which is the usual form in locomotives. When the eccentric rods are

attached as in A, then in drawing approximate valve diagrams instead of taking the real eccentric-rod length from centre of sheave to centre of pin, the virtual length

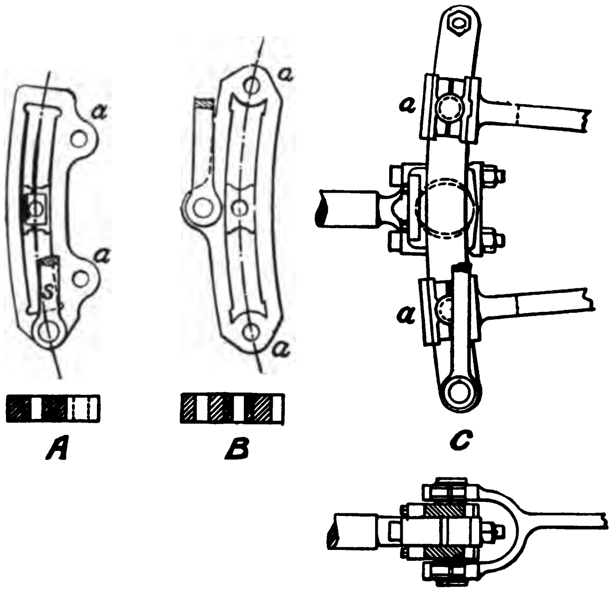


Fig. 243

from centre of sheave to centre line of link should be taken, measured along the eccentric rod.

259. *Proportions of link gear.*—The diameter of the valve spindle may be determined from the valve friction by rules already given (§ 124). Very commonly it is fixed by an empirical rule such as the following: Let A be the total area of the back of the valve in sq. ins.; p the greatest difference of pressure in steam chest and exhaust port in lbs. per sq. in. Then the diameter of the valve spindle is

$$d = c\sqrt{\mathrm{A}p},$$

where $c = 0{\cdot}009$ to $0{\cdot}01$ for iron, or $0{\cdot}009$ to $0{\cdot}0083$ for steel.

Then the diameter of block pin $= d$, if overhung, and $0{\cdot}75\,d$, if supported at each end. Diameter of eccentric-rod pins $= 0{\cdot}75\,d$; diameter of suspension-rod pins $= 0{\cdot}55\,d$; breadth of link (perpendicular to plane of oscillation) $0{\cdot}8\,d$ to $0{\cdot}9\,d$; thickness of bars of slotted link (in plane of oscillation) at centre $= 0{\cdot}7\,d$; diameter of suspension rod $= 0{\cdot}7\,d$, if single, $= 0{\cdot}55\,d$, if double; length of block $= 1{\cdot}6$ to $1{\cdot}8\,d$; length of link between eccentric-rod pins $2\frac{1}{2}$ to 3 times the travel of valve in full gear. The bearing pressure on the pins should not exceed 1,000 to 1,500 lbs. per sq. in.

260. *Travel of a slide valve driven obliquely by an eccentric.*—The action of link gear on the valve nearly approximates to the conditions shown in fig. 244. Let X O X be the line of piston stroke and O b d the line of valve stroke. Y O Y is at right angles to X O X. Suppose the crank at the outer dead centre, turning in the direction of the arrow. Let O $a = r$ be the eccentric radius, $a\,b$ the eccentric rod, which drives b connected to the valve along $b\,d$, parallel to X O X. Then Y O a is the angle of advance θ of the eccentric. It is required to find the travel of the point b for any movement of the eccentric. Join O b and let X O $b = \beta$. If O Z is at right angles to O b, then Z O $a = \theta + \beta$.

If the point b were driven by O a along O $b\,c$, the travel $b\,c = \xi$, for any movement of the eccentric would be found by drawing an ordinary polar valve circle, for an eccentric radius O $a = r$, the line of stroke being O $b\,c$ and the angle of advance Z O $a = \theta + \beta$. The actual movement of b along O′ $b\,d$ will be found very approximately by taking O d = O c. Then since O $c\,d$ is very nearly a right angle, $b\,d = b\,c/\cos\beta$. Hence the actual travel along O′ $b\,d$ is very approximately $\zeta = \xi\,/\cos\beta$. In other words, the actual travel of a valve driven by an eccentric O a obliquely is the same as if it were

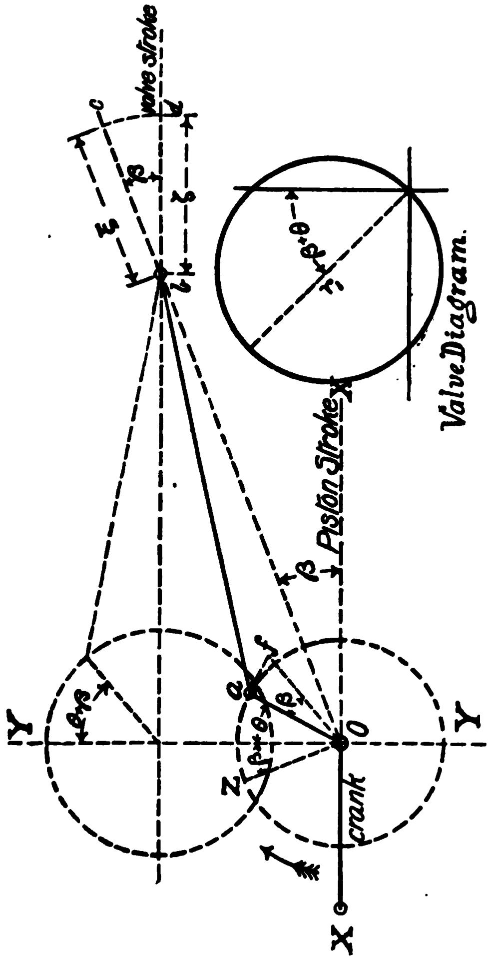


Fig. 244

driven by an eccentric O′ a' directly, the radius O′ $a' = r_1 = r \,/\, \cos\beta$, and the angle of advance Y O a' = Z O a $= \theta + \beta$.

Take a O $f = \beta$, draw af at right angles to O a. Then O $f = r \,/\, \cos\beta$ is the virtual eccentric radius and Y O $f = \theta + \beta$ the virtual angle of advance.

The construction in fig. 244, is for the case of open rods (as in fig. 240), where X O $b = \beta$ is measured from the line of piston stroke reverse to the direction of motion of the crank. For crossed rods (as in fig. 241), where β is measured from the line of stroke in the direction of motion of the crank, β is negative and the virtual angle of advance is $\theta - \beta$. Then af is to be drawn backwards towards Y O Y.

261. *Approximate designing of a Stephenson link motion. Equivalent eccentric for any position of the block in the link.*—If the ends of the link are supposed to move parallel to the line of stroke of the valve rod, then the method of § 260 may be used to find an eccentric which would give the valve a motion nearly identical with that which it receives from the two eccentrics driving the link.

In fig. 245 let O E be the crank at the dead point. O f = O $b = r$, the two eccentric radii, here drawn with equal angles of advance Y O f = Y O $b = \theta$. Let the angles β_1 β_2, figs. 240, 241, be determined from a drawing of the link motion, which for convenience may generally be on a smaller scale than the construction in fig. 245. Take f O $a = \beta_1$ for open rods. For crossed rods the angle would be taken on the other side of O f. Draw fa perpendicular to O f. Then O a, as in fig. 244, is the eccentric which, driving the end of the link directly, would give it the same motion as O f driving it obliquely, O $a = r_1 = r \,/\, \cos\beta_1$. Similarly find O $d = r_2 = r \,/\, \cos\beta_2$ for the other eccentric.

Let m and n, figs. 240, 241, be the distances of the slide-block pin from the lines of stroke of the ends of the link, the whole length of link being $2c$. Then the motion

given to the block by the forward eccentric will be less than that at the end of the link in the ratio $n / 2c$, and that given by the backward eccentric in the ratio $m / 2c$. Take $o g = n r_1 / 2c$ and $o h = m r_2 / 2c$. Complete the parallelogram $o g k h$, then $o k = \rho$ is the equivalent

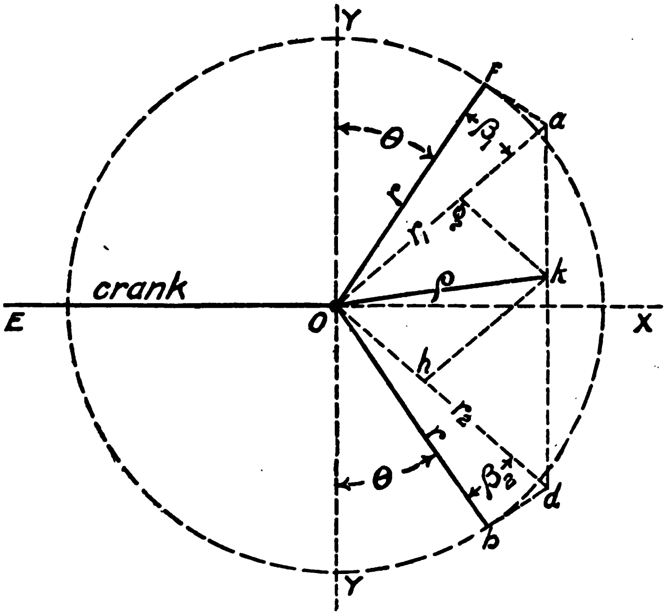


Fig. 245

eccentric. Valve circles drawn for an eccentric of radius ρ and angle of advance $\text{Y} o k$ will give very approximately the motion of the valve.

If $a d$ is joined and divided in the same ratio that the block divides the link, the point k is found even more simply.

262. *Valve circles for link motion with open rods.*—Let fig. 246 represent the link gear with the crank at the outer dead centre, o *f* being the forward, and o *b* the backward eccentric, and θ_1 θ_2 the corresponding angles of advance, which are generally but not always the same. Draw *f a*, *b d* perpendicular to the eccentric radii. Next from a drawing like fig. 240, find the angles β_1, β_2, made by the lines o *a*, o *d*, in that figure with the line of stroke, for any given position of the block in the link. Set off these angles at *f* o *a*, and *b* o *d*. Join *a d* and divide it at *k*, in the same

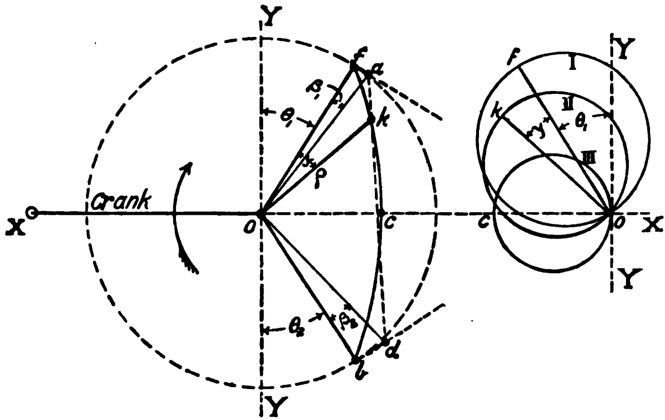


Fig. 246

ratio (*m*/*n*, fig. 240) as that in which the block divides the link. Then o *k* is the equivalent eccentric for that position of the block in the link. If points such as *k* are determined for other positions of the block in the link, they will be found to lie on a parabola *f k c b* concave to the crank shaft. When the block is in the centre of the link, usually $\beta_1 = \beta_2$ and the equivalent eccentric is o *c*. If the point *c* is found the parabola *f k c b* does not sensibly differ from a circular

arc through f, c, and b, and the position of k for any link-block position is easily found by taking $f\,k/k\,b = m/n$.

In fig. 246, valve circles I, II, III, are drawn for full gear forward, and for the link-block positions corresponding to O k, O c. For O k the angle of advance is Y O $k = \theta_1 + \gamma$. For O c the angle of advance is Y O $c = 90°$. The angles of advance are set off from O Y, reverse to the direction of motion, and the diameters of the valve circles are equal to the eccentric radii. There will be another set of valve circles for backward gear found in the same way.

263. *Macfarlane Gray's method.*—A simple approximate way of finding the equivalent eccentric for a link motion

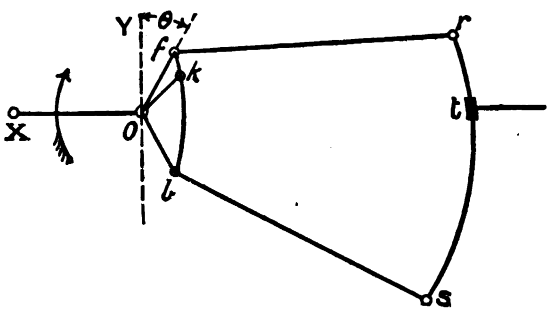


Fig. 247

with open rods has been given by Mr. Macfarlane Gray. Let fig. 247 represent the link motion with the crank at the outer dead centre, O f, O b, being the forward and backward eccentrics and t being the link block in any position. Through f and b draw an arc of radius $(f\,b \times f\,r)/2\,r\,s$. Take k so that $f\,k\,/\,k\,b = r\,t\,/\,t\,s$. Then O k is the equivalent eccentric, and Y O k its virtual angle of advance.

264. *Valve circles for link motion with crossed rods.*—The construction is precisely the same as for open rods with this exception, that $f\,a$, $b\,d$ are drawn towards the crank shaft, and β_1, β_2 are set off negatively (fig. 248).

o k is the equivalent eccentric radius for the given position of the link block, and ak/kd is the ratio in which the block divides the link between the eccentric-rod pins. Points such as k in this case lie on a parabola $fkcb$ convex to the crank shaft, and o c is the equivalent eccentric for mid gear. The valve circles (fig. 248) are drawn as for open rods, the angles of advance, θ_1, $\theta_1 + \gamma$ and 90° being

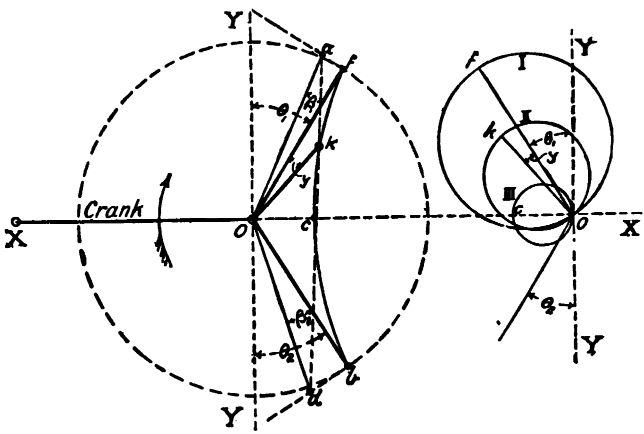


Fig. 248

set off towards the crank, and the diameters of the circles being equal to the eccentric radii.

265. *Determination graphically of the exact travel of the valve for any crank position.*—It remains to indicate how, if a link gear has been designed provisionally, the movement of the valve can be determined exactly, with a view to amendment if necessary. To obtain accuracy enough the movement of the valve must be determined full size or larger, and this makes it impossible to draw the whole link gear. A method is wanted not involving the drawing of the complete gear to so large a scale.

Let fig. 249 represent the gear to be examined, o c being the crank in any position, and o *f*, o *b* being the corresponding positions of the forward and backward eccentrics. The position of the link will be determined if the positions of three points *a*, *g*, *d* can be determined. As to *g* there is no difficulty, for in all positions of the link *g* lies on the arc *g e*, struck from centre *h* with radius equal to the length of the suspending rod. The point *a* lies on an arc *a m* struck from *f* with radius *f a*, and the point *d* on an arc *d n* struck from *b* with radius *b d*. The arcs *a m*, *d n*, *g e* being drawn, a tracing

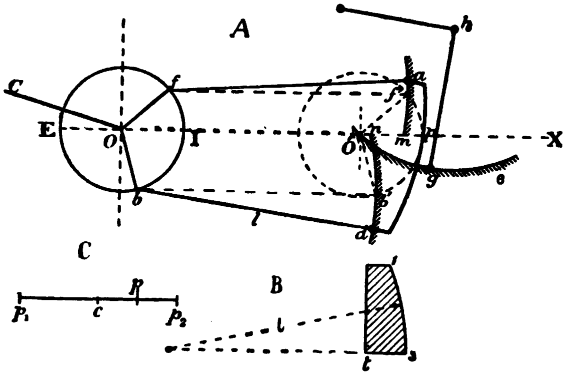


Fig. 249

of the link can be adjusted so that the points *a*, *g*, *d* fall on the director arcs, and then the intersection *p* of the centre line of the link with the line of stroke o x of the valve rod determines the position of the valve. By taking eight or twelve positions of the eccentrics, a corresponding number of positions of the valve can be determined by pricking through the tracing of the link. These can then be used to draw a valve ellipse, showing completely the action of the valve for the given point of suspension *h*, and the movement for any other notch can be ascertained in the same way.

There is no great difficulty in drawing the link full size,

but there would be a difficulty in drawing arcs with radius equal to the length of the eccentric rods. It remains to be seen how the director arcs *a m*, *n d* can be determined without making use of the eccentric circle on the left of fig. 249.

Take o o′ = *f a* = *b d*. With o′ as centre draw a circle of radius equal to the eccentric radius and draw o′ *f′*, o′ *b′* parallel to o *f*, o *b*; then *f′* and *b′* are points on the director arcs *a f′ m* and *d b′ n*. Further, *b′ b* and *f′ f*, parallel to o o′ will be radii of those arcs. Suppose an eccentric circle drawn and two simultaneous positions o′ *f′*, o′ *b′* of the eccentrics given. Let a templet *r s t* (fig. 249, B) be prepared, by using a pear-wood curve, for instance, of the radius *l* = *f a* = *b d*, and having the side *s t* in the direction of a radius. Placing this templet with *t s* coinciding with *f′ f* and *b′ b*, the arcs *a m*, *d n* can be drawn, and then with a tracing of the link the point *p* is determined.

The use of a templet in this way is due to MM. Coste and Maniquet. Although the process is cumbrous to describe, it is really a very easy one. Suppose that round the eccentric circle eight or twelve positions of *f′* are first marked and numbered, then eight or twelve corresponding positions of *b′* also numbered. The director arcs *a m*, *d n* can then be all drawn and numbered, and these arcs suffice for all positions of the suspending link. Next one of the arcs, *g e*, can be drawn and the link tracing used to mark off twelve valve positions for that notch. If the positions of *a* and *d* are pricked through (or, more strictly, the ends of the link centre line) twelve points are determined in the peculiar curves described by the ends of the link, and termed *slip curves*.

A new position of the director arc *g e* can then be taken and a new set of values of the valve travel determined, the same director arcs for the points such as *a* and *d* being used as for the previous notch.

To set off the valve travels thus determined properly on

an ellipse diagram, the true position of the centre of the cylinder face must be known. Usually the valve is set to equal linear leads for one notch ; for instance, for full gear forward. That is, the travel of the valve reckoned from centre of cylinder face is $o + e$ with the crank at the interior and exterior dead points for that notch. Let p_1, p_2, fig. 249 C, be two positions of the point p for the given notch and for the crank at the dead points. Bisect $p_1 \, p_2$ in c. Then c is the true centre of the cylinder face relatively to the position p of the valve, determined by the construction for that and all the other notches. $c p_1 = c p_2 = o + e$ for the notch at which the leads are equal. If p is any other position of p for any notch, $c p$ is the travel of the valve relatively to the centre of the cylinder face.

266. *Example of a link-motion valve gear.*—It may be useful as a guide in designing link motions to give some details of a carefully designed gear for a locomotive engine. The following data are from a gear designed by Mr. William Adams as a standard for outside cylinder engines on the North London Railway.[1]

The engine has cylinders 17 ins. diam. and 24 ins. stroke. The steam ports are $14\frac{1}{2} \times 1\frac{1}{4}$ ins., or 18·1 sq. ins. in area. This is a little less than 1/12th of the piston area. The exhaust port is $14\frac{1}{2} \times 3\frac{1}{2}$ ins. The bars are 1 in. wide. There is no inside lap and the outside lap is 1 in. The area of the back of the valve is about 140 sq. ins., so that the frictional resistance to sliding, calculated in the ordinary way, might be taken at about 1,680 lbs., and this is the straining force which the parts of the link motion have to overcome. The eccentrics are $15\frac{1}{4}$ ins. diameter with straps $2\frac{5}{8}$ ins. wide. At 40 miles per hour the engine would make about 210 revolutions per minute. Hence the width of the eccentric is about

$$b = \frac{\text{P N}}{148{,}000}.$$

[1] 'Engineering,' vol. viii. p. 112.

TABLE A.

Piston stroke, 24 ins.; radius of eccentric, $3\frac{1}{4}$ ins.; lap of valve, 1 in.

Notch	Travel of Valve	Lead		Maximum opening of Port		Fraction of Stroke at which Steam is cut off		Fraction of Stroke at release		Maximum Slip of Block in Link
		Front Port	Back Port	Front Port	Back Port	Stroke towards Crank	Stroke from Crank	Stroke towards Crank	Stroke from Crank	
Forward	in.	in.	in.	in.	in.					in.
1st . .	2·56	·28	·28	·28	·28	·11	·11	·58	·58	·12
2nd. .	2·62	·28	·28	·31	·31	·22	·23	·65	·65	·12
3rd . .	2·74	·28	·25	·37	·37	·31	·32	·72	·71	·19
4th . .	2·91	·25	·25	·44	·47	·40	·41	·77	·76	·25
5th . .	3·06	·25	·22	·50	·56	·48	·49	·82	·80	·31
6th . .	3·31	·22	·19	·62	·69	·57	·57	·86	·84	·35
7th . .	3·44	·22	·19	·72	·72	·64	·61	·88	·86	·37
8th . .	3·81	·19	·16	·84	·92	·70	·67	·90	·88	·44
9th . .	4·09	·19	·12	·97	1·12	·75	·71	·92	·90	·50
10th . .	4·34	·16	·09	1·09	1·25	·79	·75	·94	·92	·56
Backward										
1st . .	2·56	·28	·25	·28	·28	·11	·11	·58	·57	·03
2nd. .	2·62	·28	·25	·31	·31	·23	·23	·65	·65	nil
3rd . .	2·76	·28	·25	·37	·39	·31	·32	·73	·72	·03
4th . .	2·91	·25	·22	·44	·47	·40	·41	·78	·77	·09
5th . .	3·09	·22	·22	·53	·56	·48	·49	·82	·81	·12
6th . .	3·31	·22	·19	·62	·69	·57	·57	·86	·85	·19
7th . .	3·56	·19	·19	·75	·81	·64	·64	·89	·88	·25
8th . .	3·84	·16	·16	·87	·97	·70	·69	·90	·90	·31
9th . .	4·12	·12	·16	1·00	1·12	·75	·73	·92	·92	·37
10th . .	4·37	·09	·12	1·12	1·25	·79	·78	·94	·94	·44

Comparing this with the rule on p. 184, it will be seen that a smaller bearing surface is allowed than the rule there given provides for. It is true also, however, that the ordinary friction of the slide valve is probably little more than half that calculated above, that an engine is not always running at so high a speed, and that even in full gear the leverage of the link

TABLE B.

Stroke of piston, 24 ins. ; radius of eccentric, 3¼ ins. ; lap, 1 in. Set with all rods 5/64ths in. short. 5/32nds more opening in front port than back.

Notch	Travel of Valve	Lead		Maximum opening		Fraction of Stroke at cut off		Fraction of Stroke at release		Maximum Slip of Block Link
		Front Port	Back Port	Front Port	Back Port	Stroke towards Crank	Stroke from Crank	Stroke towards Crank	Stroke from Crank	
Forward	in.	in.	in.	in.	in.					in.
1st . .	2·53	·34	·16	·34	·19	·19	·12	·61	·55	·12
2nd. .	2·56	·34	·16	·34	·22	·26	·19	·69	·61	·19
3rd . .	2·72	·34	·16	·44	·28	·35	·28	·75	·68	·25
4th . .	2·87	·31	·12	·50	·37	·44	·38	·80	·73	·28
5th . .	3·06	·31	·12	·56	·50	·52	·46	·84	·78	·31
6th . .	3·31	·31	·09	·69	·62	·60	·53	·88	·81	·37
7th . .	3·53	·28	·09	·78	·75	·67	·59	·90	·84	·44
8th . .	3·78	·28	·06	·91	·78	·72	·65	·92	·86	·50
9th . .	4·03	·25	·03	1·03	1·03	·77	·69	·93	·89	·56
10th . .	4·35	·22	—	1·16	1·19	·80	·73	·95	·91	·57
Backward										
1st . .	2·50	·34	·16	·34	·16	·19	·13	·61	·55	·03
2nd. .	2·56	·34	·16	·34	·22	·26	·19	·68	·61	—
3rd . .	2·72	·34	·16	·44	·28	·34	·28	·74	·69	·06
4th . .	2·87	·31	·16	·50	·37	·43	·38	·80	·74	·09
5th . .	3·06	·31	·12	·56	·50	·52	·46	·84	·78	·12
6th . .	3·31	·28	·12	·69	·62	·59	·54	·87	·82	·19
7th . .	3·53	·25	·09	·78	·75	·66	·60	·90	·86	·25
8th . .	3·77	·22	·06	·91	·86	·71	·67	·92	·89	·31
9th . .	4·03	·19	·06	1·03	1·03	·76	·72	·93	·91	·44
10th . .	4·38	·16	·03	1·19	1·19	·80	·76	·95	·93	·50

reduces the thrust on the eccentric a little below the value assumed above. The link is 18 ins. long, and the slide block 4½ ins. long by 2⅔ ins. wide. This gives an area of surface to the block of 12 sq. ins., so that the maximum pressure on the wearing surface of the block is about 140 lbs. per sq. in.

The preceding tables show the action of the link gear. Table A shows the most equable distribution of steam which could be obtained by the gear; Table B shows the action of the gear, with the setting actually adopted. It was found in running that the action was better with the setting shown in Table B. Mr. Adams states that the setting in Table B is arrived at in this way:—The reversing lever is placed in mid gear and the crank at half-stroke. The length of the eccentric rods is then adjusted so that the valve closes the ports perfectly when the crank is on either the top or bottom centre.

Radial Gears

267. It is desirable that the action of a valve gear should be such that there is a fixed amount of lead, and equal port opening and cut-off at the same fraction of the stroke, in the outstroke and instroke of the piston. With an infinitely long connecting rod the motion of the piston would be the same in both strokes and symmetrical on either side of the mid position. But in consequence of the obliquity of the connecting rod, the motion of the piston is not simple harmonic motion. The piston is in advance of the harmonic position in the outstroke and behind it in the instroke. On the other hand the motion of a valve driven by an eccentric is very nearly harmonic, the obliquity of the eccentric rod being small. Hence if a valve is set with equal lead for in and out stroke, the point of cut-off is later in the outstroke than in the instroke and generally the port opening is unequal also. If the valve is driven by a link, further irregularities are introduced depending on the position of the block in the link. With the ordinary form of link there is excessive lead at early points of cut-off, and also excessive compression. With engines of high rotative speed this may be tolerable, but it is very objectionable with slower engines. With ordinary link-motion gear these difficulties are inherent, and the best that can be done is to make a compromise

between different evils. Very many gears have been invented with the object of obviating the defects of the link motion, some being in this respect quite successful, but on the other hand having the defect of complication. *Radial Gears* are as simple as the link motion and at the same time secure a better steam distribution. In these gears motion is derived from the connecting rod or crank, and the link work is arranged so that some point from which the valve is driven describes a nearly symmetrical oval curve. By altering the position of the axis of this oval, reversal or variable expansion is obtained.

268. *Joy's valve gear.*—A radial gear invented by Mr. David Joy has been largely used on locomotives. It is

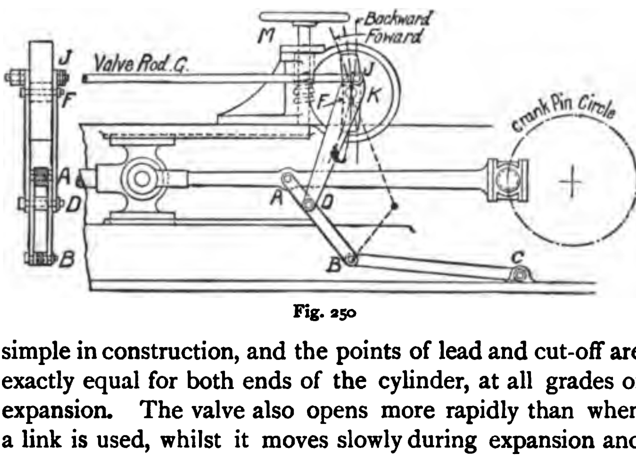


Fig. 250

simple in construction, and the points of lead and cut-off are exactly equal for both ends of the cylinder, at all grades of expansion. The valve also opens more rapidly than when a link is used, whilst it moves slowly during expansion and exhaust. Fig. 250 shows an ordinary form of Joy gear,[1] which, however, may be modified in several ways without much impairing its action.

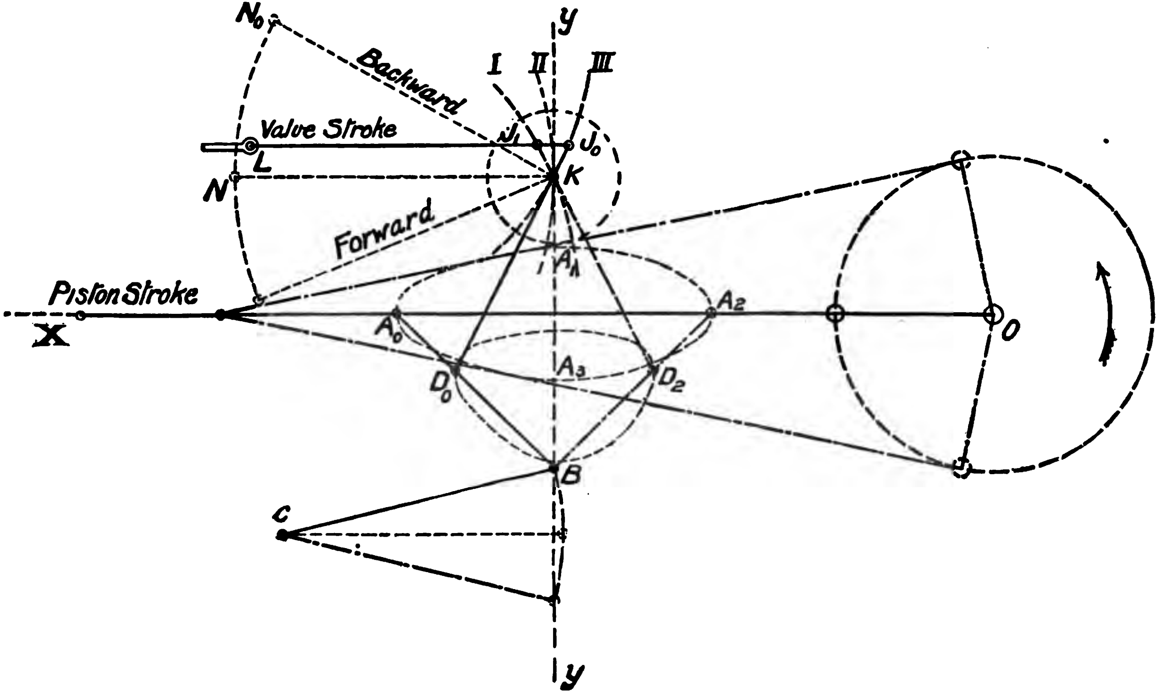
Motion is derived from a point A on the connecting rod, which describes an oval curve (shown in fig. 251), the vertical

[1] 'Proc. Inst. of Mech. Engineers,' 1880, p. 418. Also Harrison on 'Radial Gears,' 'Proc. Inst. of Civil Engineers,' cxiii. p. 470.

axis of which should be rather greater than twice the valve travel. The link A B is guided by a radius bar B C, so that B moves nearly on a vertical, bisecting at right angles the horizontal axis of the curve described by A. A point D is selected in A B, which also describes an oval curve (fig. 250), the vertical axis of which is the same as that of the curve described by A.

Attached at D is a link J K D guided by a block F sliding in a curved slot. Usually A D $= \frac{1}{3}$ A B nearly, but D is selected so that the guided point K moves equally above and below its position when the crank is at the dead point. The valve rod is attached at J. The curved slot can be rotated. In the arrangement shown it is formed in a sheave which can be rotated by a worm and wheel. By altering the position of the slot the valve obtains motion for forward or backward running, or for cutting-off at any fraction of the stroke.

269. *Laying out a Joy valve gear.*—Let fig. 251 represent an engine with the crank at the inner dead centre, O X being the line of piston stroke and J_0 L the line of valve stroke, which is fixed by the construction of the engine. In the connecting rod choose a point A, which should generally be such that it has a vertical motion greater than twice the valve travel. Let A_0 A_2 be the positions of A when the crank is at the dead points, and A_0 A_1 A_2 A_3 the oval it describes. Draw Y Y bisecting A_0 A_2 at right angles. Next choose a link A_0 B_0 of such a length that the angle A_0 B_0 A_2 is not greater than a right angle. The point B_0 is guided nearly in the vertical Y Y by a radius rod B_0 C, which may be as long as convenient, or it may be guided by a slide. The point C should be so chosen that the mid position of B_0 C is horizontal. On the line of valve stroke mark off from Y Y to J_0 J_1 distances each equal to lap + lead, in full gear. The position of the valve lever J_0 K D_0 is best selected by trial. Usually A_0 D_0 = about $\frac{1}{3}$ A_0 B_0, but the condition to be satisfied is that the fulcrum K of the lever should move



N0
Backward
y
I
II
III
Valve Stroke
J1
J0
L
N
K
Forward
A1
Piston Stroke
A0
A2
O
X
A3
D0
D2
B
c
y

Fig. 251

equally above and below the position shown when the crank is at the dead point. That is, K must move up and down a distance equal to half $A_1 A_3$. The circle drawn with K as centre has this distance as radius and marks the limit of travel of K. By shifting the point D this condition can be satisfied. D moves in an oval the vertical axis of which is equal to $A_1 A_3$. The point K where the valve lever intersects Y Y is taken as the fulcrum of the lever and the centre of oscillation of the curved slot which guides K.

Next choose the valve link $J_0 L$ which may be of any length. Whatever length is chosen must be taken as the radius of the curved slot in which K moves. Through K draw a horizontal, and with centre N and radius $N K = J_0 L$ draw the centre line II of the slot in mid gear. For full forward gear the slot must be turned into the position I, the arc passing through J_0 and K. For full backward gear the arc must be turned to the position III, the arc passing through J_1 and K. The corresponding centres of the arcs are easily found by drawing an arc from K with radius K N.

It is easily seen from the figure that the valve will have moved an amount equal to lap + lead at the beginning of either instroke or outstroke, and since K will be at the centre of oscillation of the slot at the beginning of a stroke whatever the inclination of the slot, the lead will be constant for all ratios of expansion. In *mid gear* the valve opens each port alternately by the amount of the lead, because the curve of the slot being to the radius of the valve link, the movement of the block in the slot does not move the valve. If, however, the slot is inclined, then the vertical movement of the block in the slot will move the valve horizontally by an amount greater than lap + lead, and equal for either out or in stroke if the vertical travel is equal on either side of the centre of oscillation of the slot. The forward or backward inclination of the slot alters the valve motion precisely in the same way as turning the eccentric through 180° in a simple valve motion.

270. *Hackworth gear.*—In the Hackworth gear, fig. 252, motion is obtained from an eccentric O E keyed at 0° or 180° from the crank. The eccentric rod E A G is at right angles to the line of piston stroke. The end G is guided in a straight slot, pivoted at F, which can be set in the positions I, II, III, for forward gear, mid gear, and backward gear. In intermediate positions the valve cuts off earlier. A link to the valve rod A B is attached to the eccentric rod at the

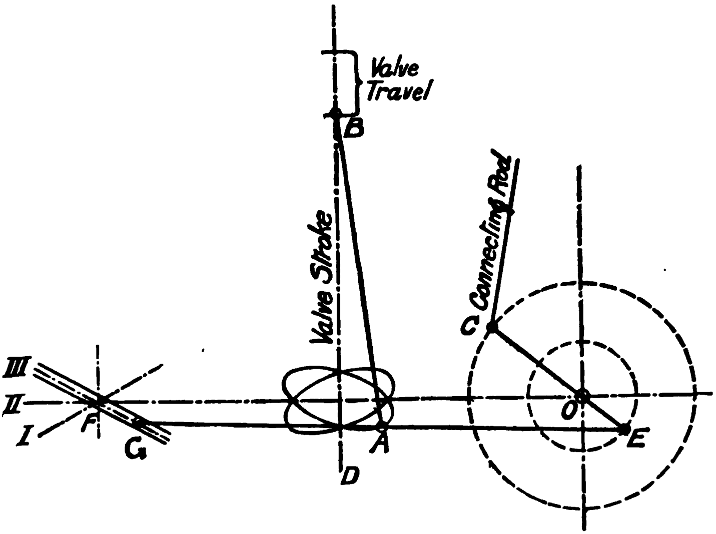


Fig. 252

point A, which describes an oval curve nearly an ellipse. According to the position of the slot F G, the longer axis of the ellipse is inclined as shown in the figure, and the valve receives motion for forward or backward gear. The motion of the valve is very exact for both strokes and for all grades of expansion, but it was found that the wear of the slide block and slot was objectionable, the obliquity of the slide in full gear being nearly 30°.

271. *Marshall's gear.*—Marshall's gear, which is largely used in marine engines, is a modification of Hackworth's. In place of a guiding slot a radius bar is used, fig. 253. The point corresponding to G, fig. 252, then travels in an arc which does not differ much from a straight line, and the motion given to the valve is very nearly the same as in Hackworth's gear. By moving over the centre of rotation of this radius bar, which in the example shown is done by a worm and wheel, the incli-

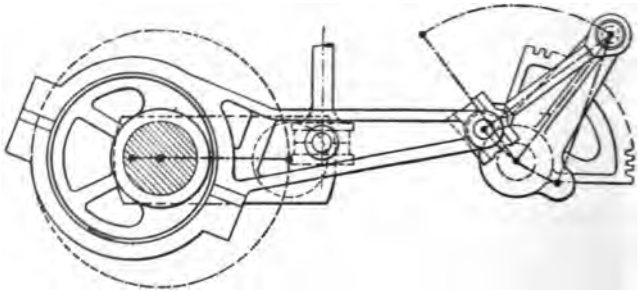


Fig. 253

nation of the path of G to the line of stroke of the eccentric is altered just as in Hackworth's gear. Applied to a vertical inverted engine the Marshall gear, in consequence of the curvature of the path of G, gives an earlier cut-off and compression in the down than in the up stroke, but in this particular case that is not a disadvantage, as the excess of mean pressure below the piston partly neutralises the effect of the weight of the moving parts.

CHAPTER XVII

LUBRICATORS

272. The amount of the frictional resistance of machine parts which slide on each other depends on the smoothness of the surfaces and on their lubrication. A lubricant is a substance which, interposed between the rubbing surfaces, reduces the friction. The diminution of friction diminishes the work wasted, the wear of the rubbing parts, and the amount of heat developed. Surfaces which run perfectly cool and well, if properly lubricated, heat and even seize, if the lubrication fails. Seizing is the cohering of the parts with force great enough to cause fracture or stoppage of the machine.

An efficient lubricant should possess the following qualities:—(*a*) It should wet the rubbing surfaces. (*b*) It must not evaporate or decompose while in use. (*c*) At the temperature at which it is employed it should have enough, and only enough, viscosity to remain between the surfaces. (*d*) It must contain no acids or other constituents capable of acting on the rubbing surfaces. (*e*) It must be free from grit or other foreign matter.

Air or water are good lubricants, when the velocity is great enough to carry in a layer between the rubbing surfaces. For instance, water is the lubricant for propeller shaft bearings in the stern tube, the bearing surfaces being strips of lignum vitæ. (See Part I. p. 325.)

Lubricants are sometimes solid at ordinary temperatures, as tallow or railway grease, more commonly fluid, as vegetable, animal, or mineral oils. Metaline, plumbago, and

some other materials are used without lubricants, and act themselves as lubricants.

Of vegetable oils, olive, palm, rape, and others are used. Of animal oils, sperm is one of the very best, but lard, neat's foot, seal, and other oils are used. Of mineral oils, some are derived from the distillation of shale, and others from petroleum wells. Heavy mineral oils have now largely superseded all other oils for purposes of lubrication. Railway grease, which is a solid or semi-fluid lubricant, is a mixture of tallow, palm oil, water, and a portion of caustic soda.

To provide for the proper lubrication of rubbing parts, a reservoir of the lubricant must be provided, so arranged, if possible, that it delivers the lubricant continuously or at regular short intervals in small quantities. The lubricant flows through an aperture to the rubbing surfaces, one of which is generally provided with channels for its suitable distribution. Lastly, in many cases a vessel must be provided to catch the lubricant which flows off the rubbing surfaces after it has done its work.

273. *Cylinder lubrication with superheated steam.*—With ordinary superheated steam, superheated about 100° or 150° F., the only care necessary is to use an oil which stands a high temperature, and various mineral oils, some of them distilled, or charcoal filtered, are available. The flash-points of such oils are from 500° to 700° F.

274. *Lubrication by graphite.*—In the case of air-compressor cylinders the use of oil lubrication has sometimes given rise to explosions of gas generated by the decomposition of the oil. In the case of some steam engines it is desirable not to introduce oil into the cylinders, because the condensed steam is to be returned to the boilers, and oil in the boilers does mischief. In such cases flake graphite is now used as a lubricant and answers well. The graphite must be pure. The graphite is fed by an ordinary lubricator, sometimes mixed with a little oil, but more generally dry. Even sight-feed lubricators are made for graphite.

275. *Cup lubricators.*—The simplest lubricator is a cup which can be filled from time to time, and which has a cock, by turning which the lubricant is permitted to flow to the rubbing surfaces.

Fig. 254 shows the ordinary form of steam cylinder oil-cup or tallow-cup. It is intended to serve for introducing oil to the cylinder without causing an escape of steam. It consists of a vessel having two cocks. Closing the lower one and opening the upper one, it can be filled ; closing the

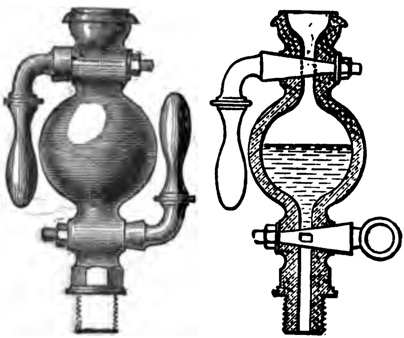


Fig. 254

upper and opening the lower, the oil or melted tallow is admitted to the cylinder. It is often placed directly on the steam cylinder, sometimes on the steam pipe, where the rapid current of steam carries forward the oil to the working parts.

276. *Siphon lubricator.*—Fig. 255 shows an ordinary siphon lubricator for oil. This has a tube rising above the surface of the oil in which a cotton wick is placed. The oil is slowly siphoned by the capillary action of the wick and drops on to the bearing. Various arrangements are adopted for closing the reservoir to keep out dirt. Sometimes there is a hinged or screwed cap. In the example shown there is a rotating plate with a hole. By rotating the plate this hole can be brought over a hole in a lower plate

or over a blank part of the lower plate. The amount of cotton wick necessary to supply sufficient oil is ascertained by trial. In the case of connecting rods, eccentric straps, etc., the siphon lubricator is generally in one piece with the

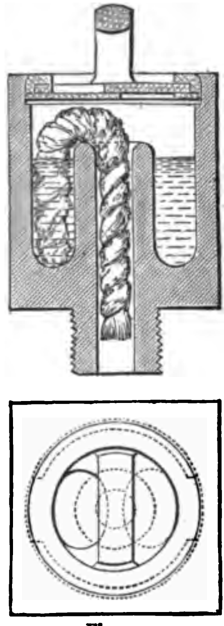


Fig. 255

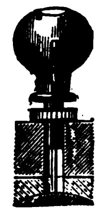


Fig. 256

Fig. 257

part to be lubricated, instead of being a separate piece screwed on. The filling-hole is then conveniently closed by an inside valve and spring, which can be pressed open by the spout of the oil can.

277. *Needle lubricator.*—Fig. 256 shows Lieuvain's needle lubricator, oftenest used for the bearings of shafting. It consists of a glass reservoir having a wooden plug. This is filled and inverted over the bearing. A wire needle or pin passes through the plug, loosely, and rests on the journal. When the shaft is running, the vibration of the needle causes

a slow descent of the oil. When the shaft is at rest, the capillary attraction stops the flow of the oil. The rate of supply of oil may be regulated by making the needle thicker or thinner.

278. *Stauffer lubricator.*—Fig. 257 shows another kind of lubricator, in which a semi-fluid or grease lubricant is used. The cap can be screwed down on a fine pitched screw. This forces the lubricant down the tube to the bearing. The tube may even be of considerable length if convenient, the grease flowing under pressure like a fluid.

279. *Displacement and sight-feed lubricators.*—Fig. 258 shows the displacement lubricator invented by Ramsbottom. The steam, condensing on the surface of the oil, forms a drop which sinks down through the oil and displaces a small quantity of oil, which then flows down the bent pipe into the cylinder. The plug at the top serves for filling the cistern, that at bottom for removing the condensed water. Numerous forms of sight-feed lubricators have been introduced. These are displacement lubricators, so arranged that each drop of oil when displaced ascends through a glass tube filled with water and can be seen. Then the rate of lubrication is adjusted by noting the number of oil drops delivered per minute. Commonly one to three drops per minute are sufficient. When a displacement lubricator is attached direct to the steam cylinder, it only works when the engine is running. When it is placed on the steam pipe oil, is economised by cutting off the steam supply when the engine is standing.

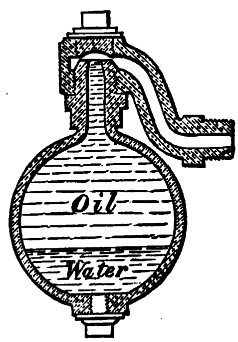


Fig. 258

280. *Positively driven lubricators.*—Occasionally displacement lubricators give trouble, and hence lubricators

in which a mechanically driven piston displaces the oil have come into extensive use. Fig. 259 shows such a lubricator. E is the oil reservoir and D the plunger which slowly displaces the oil, delivering it at F. The plunger D is driven by the screw, which in turn is rotated by a worm-wheel C, driven by the ratchet-wheel A and ratchet lever B.

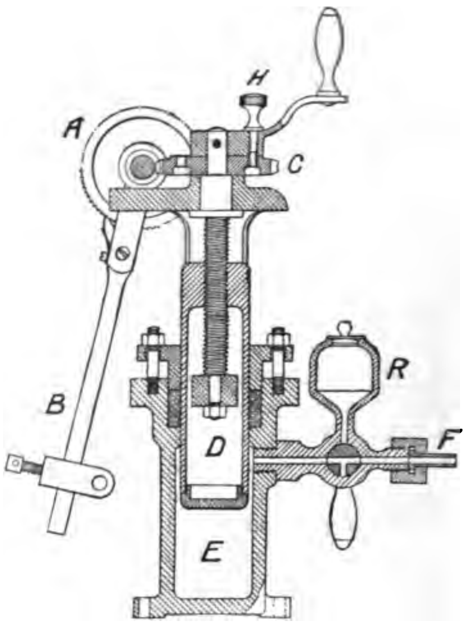


Fig. 259

The rate of oil feed is adjusted by moving the sliding block on the lever B, which is reciprocated from some convenient moving part of the engine. To charge the reservoir, oil is filled into the reservoir R, the two-way cock being turned to put R and E in communication. The screw is then turned reversely by the winch handle at top, the worm-wheel being disconnected by withdrawing the pin H.

INDEX

Spottiswoode & Co., Ltd., Printers, New-street Square, London.